MAT1341D-TS- Version A - Solution

1. Parametric equations of the line containing $(-5,\ 0,\ 1)$ and which is parallel to the two planes $2x-4y+z=0$ and $x-3y-2z=1$ are:

A. $x=5+11t,\ y=3t,\ z=1+2t,\ t\in\mathbf{R}$

B. $x=-5+11t,\ y=-3t,\ z=1+2t,\ t\in\mathbf{R}$

C. $x=5t,\ y=0,\ z=t,\ t\in\mathbf{R}$

D. $x=-5+5t,\ y=-5t,\ z=1-10t,\ t\in\mathbf{R}$

E. $x=-5+11t,\ y=5t,\ z=1-2t,\ t\in\mathbf{R}$

F. $x=-5t,\ y=0,\ z=t,\ t\in\mathbf{R}$

$$\vec{V}=(2,-4,1)\times(1,-3,-2)=(11,5,-2)$$

2. Which two of the following are vector parametric descriptions for the plane with equation $x+y-2z=4$?

I. $v=(0,0,0)+s\ (0,2,1)+t\ (2,0,1);\ s,t\in\mathbf{R}.$

II. $v=(4,0,0)+s\ (1,-1,0)+t\ (0,2,1);\ s,t\in\mathbf{R}.$

III. $v=(4,0,0)+s\ (1,1,1)+t\ (1,1,0);\ s,t\in\mathbf{R}.$

IV. $v=(0,0,-2)+s\ (1,-1,0)+t\ (2,0,1);\ s,t\in\mathbf{R}.$

$V=V_0+tV_1+sV_2$

A. I & II B. I & III C. I & IV D. II & III E. II & IV F. III & V

$\vec{n}=(1,1,-2)$

$V_1\cdot\vec{n}=0,\ V_2\cdot\vec{n}=0$

$(1,-1,0)\cdot(1,1,-2)=0,\quad (0,2,1)\cdot(1,1,-2)=0$

$(2,0,1)\cdot(1,1,-2)=0$

3. An equation for the plane parallel to the x–axis and passing through the points $(2,1,-1)$ and $(3,2,1)$ is:

A. $-3x+7y-2z=3$

B. $x-y=1$

C. $2y-z=3$

D. $2x-z=5$

E. $x+y-z=4$

F. $x+y+z=2$

Let $by+cz=d$, Sub $(2,1,-1)$ $\Rightarrow b(1)+c(-1)=d$

sub $(3,2,1) \Rightarrow b(2)+c(1)=d$

$\Rightarrow b=\frac{2}{3}d,\ c=-\frac{1}{3}d \Rightarrow \frac{2}{3}dy-\frac{1}{3}dz=d$

$\Rightarrow 2y-z=3$

4. Find an equation of the plane which passes through the point $(1,\ -7,\ 8)$ and which is perpendicular to the line whose (scalar) parametric equations are:

$$x=2+2t, \quad y=7-4t, \quad z=-3+t; \quad t\in\mathbf{R}.$$

A. $2x-4y+z=-38$

B. $-4x+2y+z=-10$

C. $-4x+2y+z=10$

D. $2x-4y+z=38$

E. $2x+7y-3z=-71$

F. $2x-4y+z=-28$

$\vec{v}=(2,-4,1)$

Let Π: $2x-4y+z=d$

sub $(1,-7,8)$: $d=38$

5. One of the following is an equation for the plane with vector parametric description

$$v = (2,0,3) + s(1,0,1) + t(0,-1,0);\ s,t \in \mathbf{R}.$$

Which is it?

A. $4x - 9y + z = 18$

B. $x + y - 2z = 14$

C. $x - 2y + 2z = 0$

D. $x + 2y - z = 0$

E. $x - z = -1$

F. $9x - 11y + 18z = -40$

$(1,0,1) \times (0,-1,0) = (1,0,-1)$

$= \vec{n}$

Let $x - z = d$

Sub $(2,0,3)$: $d = -1$

6. Find the polar form of

$$\frac{1-\sqrt{3}i}{-1+i}$$

A. $\sqrt{2}(\cos(-7\pi/12) + i\sin(-7\pi/12))$

B. $\sqrt{2}(\cos(5\pi/12) + i\sin(5\pi/12))$

C. $\sqrt{2}(\cos(-\pi/12) + i\sin(-\pi/12))$

D. $\sqrt{2}(\cos(\pi/12) + i\sin(\pi/12))$

E. $\sqrt{2}(\cos(-5\pi/12) + i\sin(-5\pi/12))$

F. $\sqrt{2}(\cos(11\pi/12) + i\sin(11\pi/12))$

$1 - \sqrt{3}i = 2e^{i\frac{5\pi}{3}}$

$-1 + i = \sqrt{2}e^{i\frac{3\pi}{4}}$

$\frac{1-\sqrt{3}i}{-1+i} = \frac{2}{\sqrt{2}} e^{i\left(\frac{5\pi}{3} - \frac{3\pi}{4}\right)}$

$= \sqrt{2}e^{i\frac{11\pi}{12}}$

7. What is the area of the triangle with vertices $(3,\ 0,\ -2)$, $(5,\ 2,\ -1)$ and $(5,\ 9,\ 0)$?

A. 13/2

B. 15/2

C. 17/2

D. 10

E. 13

F. 15

$$\vec{AB} = (5,2,-1)-(3,0,-2) = (2,2,1)$$

$$\vec{AC} = (5,9,0)-(3,0,-2) = (2,9,2)$$

$$\text{Area} = \frac{1}{2}\left|(2,2,1)\times(2,9,2)\right|$$

$$= \frac{1}{2}\left|(-5,-2,14)\right|$$

$$= \frac{1}{2}\sqrt{225} = 15/2$$

8. Let L be the line passing through $(1,\ 1,\ 0)$ and $(2,\ 3,\ 1)$. The point of intersection of L with the plane $x + y - z = 1$ is:

A. $(1/2,\ 1/2,\ 0)$

B. $(0,\ 1/2,\ -1/2)$

C. $(0,\ 1,\ 0)$

D. $(1/2,\ 0,\ -1/2)$

E. $(1,\ 0,\ 0)$

F. $(-1,\ 0,\ -1)$

L: $x = 1+t,\ y = 1+2t,\ z = t$

Sub into Π:

$$1+t+1+2t-t = 1$$

$$2t = -1,\quad t = -\frac{1}{2}$$

9. Express the following complex numbers in the form $a + bi$:

$$z_1 = \frac{i}{-1+i}$$

$$z_2 = (2+i)(1+i)$$

A. $z_1 = \frac{1}{2} + \frac{1}{2}i$; $z_2 = 1 - 3i$

B. $z_1 = \frac{1}{2} - \frac{1}{2}i$; $z_2 = 1 + 3i$

C. $z_1 = 1 - i$; $z_2 = 2 + 2i$

D. $z_1 = -1 + i$; $z_2 = 1 + 2i$

E. $z_1 = 2 - \frac{1}{4}i$; $z_2 = 3 - i$

F. $z_1 = 1 - i$; $z_2 = 2$

$$z_1 = \frac{i(-1-i)}{(-1+i)(-1-i)} = \frac{-i-i^2}{2}$$

$$= \frac{1}{2} - \frac{1}{2}i$$

$$z_2 = 2 + 3i + i^2 = 1 + 3i$$

10. If $v = (2,\ 2,\ 2)$ and $u = (-1, 0, -1)$ then $\text{proj}_u\, v =$

A. $\frac{4}{9}(2, 1, 2)$

B. $\frac{12}{7}(3,\ 3,\ 3)$

C. $\frac{4}{3}(2, 1, 2)$

D. $(2, 0, 2)$

E. $\frac{\sqrt{2}}{2}(1, 0, 1)$

F. $\frac{11}{7}(3,\ 3,\ 3)$

$$\text{proj}_u V = \frac{V \cdot u}{u \cdot u} u = \frac{-4}{2}(-1, 0, -1)$$

$$= (2, 0, 2)$$

11. Find the volume of the parallelepiped determined by the vectors $u = (1,\ 1,\ -1)$, $v = (2,\ 0,\ 1)$ and $w = (1,\ -1,\ 3)$.

A. 6

B. 8

C. 16

D. 2

E. 4

F. -2

$$V = |u \cdot (v \times w)|$$
$$= |(1,1,-1) \cdot (1,-5,-2)|$$
$$= |1-5+2| = 2$$

12. If $A = (1, 2, 1)$, $B = (2, 2, 1)$ and $C = (2, 2, 2)$, find the angle $\angle ACB$.

A. $\pi/4$

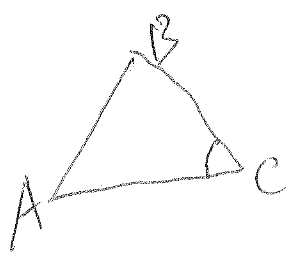
B. $\pi/6$

C. $3\pi/4$

D. $4\pi/3$

E. $\pi/2$

F. $\pi/3$



$$\cos\angle ACB = \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}|\,|\vec{CB}|}$$
$$= \frac{(-1,0,-1)\cdot(0,0,-1)}{|(-1,0,-1)|\,|(0,0,-1)|} = \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$