Tutorial

Question 1: Evaluate the following limit: $\lim_{x\to\infty} \sqrt{2x+3} - 2\sqrt{x-1}$

Answer:

$= \lim_{x\to\infty} (\sqrt{2x+3} - 2\sqrt{x-1}) \left(\frac{\sqrt{2x+3}+2\sqrt{x-1}}{\sqrt{2x+3}+2\sqrt{x-1}}\right)$

$= \lim_{x\to\infty} \left(\frac{2x+3-4x+4}{\sqrt{2x+3}+2\sqrt{x-1}}\right)$

$= \lim_{x\to\infty} \left(\frac{-2x+3}{\sqrt{2x+3}+2\sqrt{x-1}}\right)$

The highest power in the denominator is $x^{½}$, we divide numerator and denominator by x to get:

$= \lim_{x\to\infty} \left(\frac{-\frac{2x}{x^{0.5}}+\frac{7}{x^{0.5}}}{\sqrt{\frac{2x}{x}+\frac{3}{x}}+2\sqrt{\frac{x}{x}-\frac{1}{x}}}\right)$

$= \lim_{x\to\infty} \left(\frac{-2x^{0.5}+\frac{7}{x^{0.5}}}{\sqrt{2+\frac{3}{x}}+2\sqrt{1-\frac{1}{x}}}\right)$

$= \frac{-\infty}{\sqrt{2}+2}$

$= -\infty$

Question 2: Evaluate the following limit: $\lim_{x\to\infty} \ln(2x^2+3x) - \ln(3x^2+5)$

Answer:

$= \lim_{x\to\infty} \ln\left(\frac{2x^2+3x}{3x^2+5}\right)$

The highest power in the denominator within the brackets is x^2 so divide numerator and denominator by x^2 to get:

$= \lim_{x\to\infty} \ln\left(\frac{\frac{2x^2}{x^2}+\frac{3x}{x^2}}{\frac{3x^2}{x^2}+\frac{5}{x^2}}\right)$

$= \lim_{x\to\infty} \ln\left(\frac{2+\frac{3}{x}}{3+\frac{5}{x^2}}\right)$

$= \ln\left(\frac{2}{3}\right)$

Question 3: Determine the derivative of the following function: $f(x) = 3x^2 - 5x + \frac{2}{x} - \sqrt[3]{x^2} - 2$

Answer:

$f(x) = 3x^2 - 5x + 2x^{-1} - x^{\frac{2}{3}} - 2$

$f'^{(x)} = 6x - 5 - 2x^{-2} - \frac{2}{3}x^{-\frac{1}{3}}$

Question 4: Determine the derivative of the follow (expand and simplify your result)

$f(x) = (x^2 - 2x)(2e^x - 3)$

Answer:

We have a product of two functions. Product rule tells us: $A'(x)B(x) + B'(x)A(x)$

$A(x) = x^2 - 2x$ $\quad B(x) = 2e^x - 3$

$A'(x) = 2x - 2$ $\quad B'(x) = 2e^x$

$f'(x) = (2x - 2)(2e^x - 3) + (x^2 - 2x)(2e^x)$

$f'(x) = 4xe^x - 4e^x - 6x + 6 + 2x^2e^x - 4xe^x$

$f'(x) = 2x^2e^x - 4e^x - 6x + 6$

Question 5: Determine the derivative of the follow (Do not expand and simply)

$g(x) = \frac{(x^3 - 5x^2 - 3)}{x - e^x}$

Answer:

We have a product of two functions. Quotient rule tells us: $\frac{T'(x)B(x) - B'(x)T(x)}{B(x)^2}$

$T(x) = x^3 - 5x^2 - 3$ $\quad B(x) = x - e^x$

$T'(x) = 3x^2 - 10x$ $\quad B'(x) = 1 - e^x$

$\frac{(3x^2 - 10x)(x - e^x) - (1 - e^x)(x^3 - 5x^2 - 3)}{(x - e^x)^2}$

Question 6: Determine the equation of the tangent line for the following function at $x = 0$

$g(x) = 3x^2 + 2x + 2 + 4e^x$

Answer:

Fist we find the derivative:
$g'(x) = 6x + 2 + 4e^x$

Next we find the slope when x=0
$g'(0) = 2 + 4e^0$
$g'(0) = 2 + 4$
$g'(0) = 6$

Next we find a point on the line when x=0:
$g(0) = 2 + 4(1)$
$g(0) = 6$

Subbing this in into y=mx+b gives:
$y = mx + b$
$6 = 6(0) + b$
$b = 6$

Thus the equation is $y = 6x + 6$