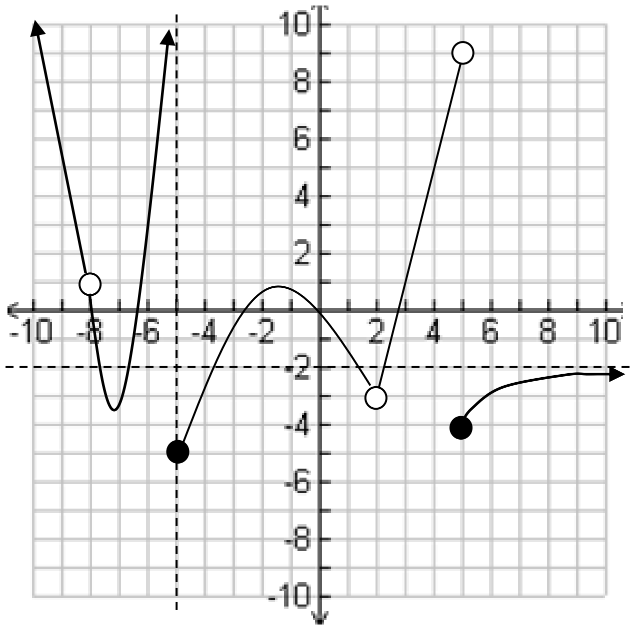
Test #2 (____ /30) **Name (print):** ____________________ **Student Number:** ____________________

Part A: Fill In The Blank/Multiple Choice:

Question 1: Using the diagram below, determine the value of the limits [3 Marks (0.5 Marks each)]



$\lim_{x\to\infty} f(x) = -2$
$\lim_{x\to -5^-} f(x) = \infty$
$\lim_{x\to -8} f(x) = 1$
$\lim_{x\to 5^-} f(x) = 9$
$\lim_{x\to 5^+} f(x) = -4$
$\lim_{x\to 5} f(x) =$ DNE

Question 2: Which x value(s) represents a replaceable discontinuity ____x = -8 and x = 2________.

Question 3: Give two examples of odd functions: _$x^{odd\ number}$_, Sin(x), the sum of any of these_____.

Question 4: $\lim_{x\to -1^+} \frac{x+1}{x} = ?$

a) ∞ b) $-\infty$ c) 0 d) 1 e) 2

Part B: Short Answer (Show all work)

Question 1: Determine the value of the given limits. If a limit DNE, state weather it is ∞ or $-\infty$, or DNE

$a)\ \lim_{x\to 1} \frac{x^2-4x+3}{x^3-1}$ [2 Marks]

$$= \lim_{x\to 1} \frac{(x-3)(x-1)}{(x-1)(x^2+x+1)}$$

$$= \lim_{x\to 1} \frac{(x-3)}{(x^2+x+1)} \qquad [1\ Mark]$$

$$= -2/3 \qquad [1\ Mark]$$

$b)\ \lim_{x\to\infty} \frac{x^2+\sqrt{x^2+1}}{x-\sqrt[3]{x^6+x^2+1}}$ [4 Marks]

Highest power of x in the denominator is $x^2 = \sqrt[3]{x^6}$. Dividing this in gives: [1 Mark]

$$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2}+\sqrt{\frac{x^2}{x^4}+\frac{1}{x^4}}}{\frac{x}{x^2}-\sqrt[3]{\frac{x^6}{x^6}+\frac{x^2}{x^6}+\frac{1}{x^6}}} \qquad [1\ Mark]$$

$$= \lim_{x\to\infty} \frac{1 + \sqrt{\frac{1}{x^2} + \frac{1}{x^4}}}{\frac{1}{x} - \sqrt[3]{1 + \frac{1}{x^4} + \frac{1}{x^6}}}$$ [1 *Mark*]

$$= -1$$ [1 *Mark*]

c) $\lim_{x\to 10} \frac{\sqrt{x-1} - 3}{x - 10}$ [3 *Marks*]

$$= \lim_{x\to 10} \frac{\sqrt{x-1} - 3}{x - 10} \left(\frac{\sqrt{x-1} + 3}{\sqrt{x-1} + 3}\right)$$ [1 *Mark*]

$$= \lim_{x\to 10} \frac{x - 1 - 9}{(x-4)(\sqrt{2x+1} + 3)}$$

$$= \lim_{x\to 10} \frac{(x-10)}{(x-10)(\sqrt{x-1} + 3)}$$ [1 *Mark*]

$$= \lim_{x\to 10} \frac{1}{(\sqrt{x-1} + 3)}$$

$$= \frac{1}{6}$$ [1 *Mark*]

d) $\lim_{x\to 0} \frac{2x\, Cos(x)}{Sin(4x)}$ [3 *Marks*]

$$= \lim_{x\to 0} 2Cos(x) * \lim_{x\to 0} \frac{x}{Sin(4x)}$$

$$= 2 \lim_{x\to 0} \frac{x}{Sin(4x)}$$ [1 *Mark*]

Let u = 4x in the first limit. This means that $u \to 0$ *as* $x \to 0$. It also means $x = \frac{u}{4}$. Substituting this in gives:

$$= 2\lim_{u\to 0} \frac{u}{4Sin(u)}$$ [1 *Mark*] (or other justification to get to $\frac{1}{3}$

$$= \frac{1}{2}$$ [1 *Mark*]

e) $\lim_{x\to\infty} \frac{2Sin(x) + 3}{x^2}$ [3 *Marks*]

$\lim_{x\to\infty} \frac{1}{x^2} \le \lim_{x\to\infty} \frac{2Sin(x)+3}{x^2} \le \lim_{x\to\infty} \frac{5}{x^2}$ [1 *Mark*] (since sin is bounded)

$0 \le \lim_{x\to\infty} \frac{2Sin(x) + 3}{x^2} \le 0$ [1 *Mark*]

$\lim_{x\to\infty} \frac{2Sin(x) + 3}{x^2} = 0$ [1 *Mark*]

$f)$ $\lim_{x \to 2^-} \frac{|x-2|}{|x+3|-5}$ $[4\ Marks]$

$$|x-2| = \begin{cases} x-2, & x \geq 2 \\ -(x-2), & x < 2 \end{cases} \qquad [1\ Mark]$$

$$|x+3| = \begin{cases} x+3, & x \geq -3 \\ -(x+3), & x < -3 \end{cases} \qquad [1\ Mark]$$

$$= \lim_{x \to 2^-} \frac{-(x-2)}{x-2} \qquad [1\ Mark]$$

$$= \lim_{x \to 2^-} -1$$

$$= -1 \qquad [1\ Mark]$$

Question 2: Determine the value of k that will make the following function continuous, or state that it is impossible to do so (make sure you show all conditions for a function to be continuous). [5 Marks]

$$f(x) = \begin{cases} 2x+k & x > 1 \\ 20 & x = 1 \\ kx^2 + k - 1 & x < 1 \end{cases}$$

For a piecewise function to be continuous, we require the following:

1. Each piece needs to be continuous, but the left side of 1 is a parabola (which is continuous) and the right side is a line (which is continuous). We will not have any endpoint, asymptotic, or replaceable discontinuities here as we do not have square roots or division to deal with. This means we must only worry about the potential jump discontinuity at x=1.

2. We need that the limit exists at x=1, this means the left hand limit must equal the right hand limit.
$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} kx^2 + k - 1 = k + k - 1 = 2k - 1$$
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2x + k = 2 + k$$
Since these must be equal to have continuity, we get: $2k - 1 = 2 + k$
$k = 3$

   This makes both of our limits **equal to 5.**

3. Next we check that the function is defined at x=1 (the middle piece)
$$f(1) = 20$$

4. Finally we note that the only k value that will make the limit exist is k = 3, but this limit is 5 (which is not the same as the function value which is 20). Thus there is no k that will make this function continuous.

[1 Mark to say that the left and right functions are continuous]
[2 Marks to show that the limit is the same on both sides and get k = 3]
[1 Mark to say that f(1) = 20]
[1 Mark to say that the limit ≠ f(2) and so is impossible to make it continuous]