<u>**Tutorial #2**</u>

Question 1: Write the following function as a piecewise function: $f(x) = |x^2 + 6x + 5|$

Answer:

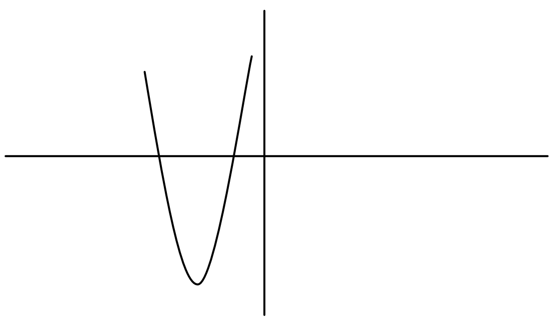
First we solve for the zeros of the inner part of the function:

$x^2 + 6x + 5 = (x + 5)(x + 1)$
$\therefore x = -1, -5$

Next we construct an interval table to see where there function is postive or negative:

	$-\infty$		-5		-1		∞
	|		|		|		|
x		-6		-2		0	
y		5		-3		5	

We could also do a quick sketch of the function to see this as well:



This means that $x^2 + 6x + 5 < 0 \; when - 5 < x < -1$, and $x^2 + 6x + 5 \geq 0 \; when \; x \leq -5 \; or \; x \geq -1$

This gives us the following piecewise function:

$$f(x) = \begin{cases} x^2 + 6x + 5, & x \leq -5 \; or \; x \geq -1 \\ -x^2 - 6x - 5, & -5 < x < -1 \end{cases}$$

Question 2: Consider an odd function $f(x) = \sin(x) + x^3$, determine if $[f(x)]^2$ is even or odd.

Answer:

If we consider $[f(-x)]^2 = f(-x)f(-x)$
$= \left(\sin(-x) + (-x^3)\right)\left(\sin(-x) + (-x^3)\right)$
$= (-\sin(x) - x^3)(-\sin(x) - x^3)$
$= (-1)(-1)\left(\sin(x) + x^3\right)\left(\sin(x) + x^3\right)$
$= \left(\sin(x) + x^3\right)\left(\sin(x) + x^3\right) = f(x)^2$

Thus this function is even.

Question 3: Determine the value of the following limit:

$$\lim_{h \to 8} \frac{2\sqrt{x+1}-6}{\sqrt{x-4}-2}$$

Answer:

$$= \lim_{h \to 8} \frac{2\sqrt{x+1}-6}{\sqrt{x-4}-2}\left(\frac{2\sqrt{x+1}+6}{2\sqrt{x+1}+6}\right)\left(\frac{\sqrt{x-4}+2}{\sqrt{x-4}+2}\right)$$

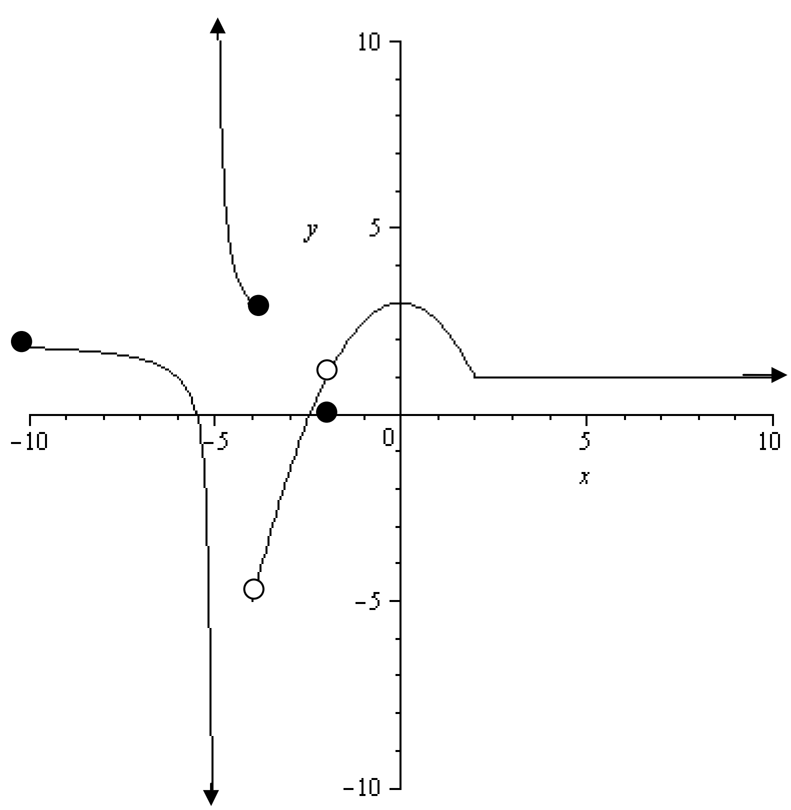
$$= \lim_{h \to 8} \frac{4(x+1)-36}{(x-4)-4}\left(\frac{\sqrt{x-4}+2}{2\sqrt{x+1}+6}\right)$$

$$= \lim_{h \to 8} \frac{4x-32}{x-8}\left(\frac{\sqrt{x-4}+2}{2\sqrt{x+1}+6}\right)$$

$$= \lim_{h \to 8} \frac{4(x-8)}{x-8}\left(\frac{\sqrt{x-4}+2}{2\sqrt{x+1}+6}\right)$$

$$= \lim_{h \to 8} 4\left(\frac{\sqrt{x-4}+2}{2\sqrt{x+1}+6}\right) = \frac{16}{12} = \frac{4}{3}$$

Question 4: Given the diagram below, state where the function is discontinuous. Also state the type of discontinuity.



Answer:

The function is discontinuous as follows:

X=-5 → Asymptote Discontinuity
X=-4 → Jump Discontinuity
X=-2 → Replaceable Discontinuity
X=-10 → Endpoint Discontinuity,

Note that the function is right continuous at -10, left continuous at -4. This wasn't asked, but would be good to know.

Question 5: Determine the limits and function values given below for the diagram shown in Question 4. (Getting at least 4 correct will award you a correct response in this question)

Answer:

$\lim_{x \to 0} f(x) = 3$

$\lim_{x \to \infty} f(x) = 1$

$\lim_{x \to -\infty} f(x) = \text{DNE}$

$\lim_{x \to -2} f(x) = 1$

$\lim_{x \to -4^-} f(x) = 3$

Question 6: Determine the limits and function values given below for the diagram shown in Question 4. (Getting at least 4 correct will award you a correct response in this question)

Answer:

$\lim_{x \to -4^+} f(x) = -5$

$\lim_{x \to -4} f(x) = \text{DNE}$

$\lim_{x \to -5^+} f(x) = \infty$

$f(-2) = 0$

$\lim_{x \to -5} f(x) = \text{DNE}$