MAT 1320 3X Summer 2014 JULY 15th Prof. Catalin Rada

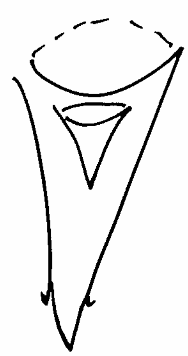
TEST #2

Max = 20

Student Number and Name: SOL

- Time: 80 min.
- Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability. Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. **Do not use any other paper!**
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

Study



0.25

1. [1 points] If $xy^4 + x = 82$ find the equation of the tangent line to the ~~curve~~ y at each point where $x = 1$ and $y = 3$.

0.25 $y = mx + n$; $m = y'(1)$, where: $1 \cdot y^4 + x \cdot 4y^3 \cdot y' + 1 = 0$

or $81 + 4 \cdot 27 \cdot y'(1) + 1 = 0$ or $\boxed{y'(1) = \frac{-82}{108}}$ 0.25

So: $y = -\frac{82}{108}x + n$. From $3 = -\frac{82}{108} * 1 + n \Rightarrow$

$\boxed{n = 3 + \frac{82}{108}.}$ 0.25 p

Hence: $y = -\frac{82}{108}x + 3 + \frac{82}{108}$

2. [3 points] Use logarithmic differentiation to find the derivative of $f(x) = \dfrac{x^{2014}\cos^{-1}x}{(x^2+1)^{2009}}$.

$\ln f(x) = \ln(x^{2014}) + \ln(\cos^{-1}x) - \ln(x^2+1)^{2009}$ 0.5 p

$\ln f(x) = 2014 \ln x + \ln(\cos^{-1}x) - 2009 \ln(x^2+1)$ 0.5 p

$$\frac{f'(x)}{f(x)} = 2014\frac{1}{x} + \frac{\frac{-1}{\sqrt{1-x^2}}}{\cos^{-1}(x)} - 2009\frac{2x}{x^2+1}$$ 1 p

$$f'(x) = \left[2014 \cdot \frac{1}{x} + \frac{\frac{-1}{\sqrt{1-x^2}}}{\cos^{-1}(x)} - 2009 \cdot \frac{2x}{x^2+1}\right] \cdot \frac{x^{2014} \cdot \cos^{-1}x}{(x^2+1)^{2009}}$$

1p

3. [4 points]

(a) Find the following limit: $\lim_{x\to 0} \frac{1}{\sin(x)} - \frac{1}{x}$

(b) Find f if $f''(x) = \cancel{\frac{-2017}{x+2}}\, e^x + x^{30} + x^{6.9}$.

1p

a) $\lim_{x\to 0} \frac{x - \sin(x)}{x \cdot \sin(x)} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to 0} \frac{1-\cos(x)}{\sin(x) + x\cdot(\cos x)} \overset{\frac{0}{0}}{\underset{L'H}{=}}$

$= \lim_{x\to 0} \frac{\sin(x)}{\cos(x) + \cos(x) + x\cdot(-\sin x)} = \frac{0}{2} = 0$ 1p

b) $f'(x) = \int e^x + x^{30} + x^{6.9}\, dx = e^x + \frac{x^{31}}{31} + \frac{x^{7.9}}{7.9} + c$

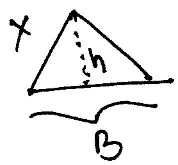
1p where c #

$f(x) = \int e^x + \frac{x^{31}}{31} + \frac{x^{7.9}}{7.9} + c \quad dx =$

$= e^x + \frac{x^{32}}{32\cdot 31} + \frac{x^{8.9}}{(8.9)\cdot(7.9)} + cx + d$, where c, d #s

1p

4. [4 points] The side length of an equilateral triangle is increasing at a rate of 3 cm/min. How fast is the area increasing at the moment when area is 10 cm^2?



Let x = side of the triangle

1p $\frac{dx}{dt} = 3 \; (cm/min)$

Area of triangle : $A = \frac{B \times h}{2} = \frac{x}{2}\sqrt{x^2 - \left(\frac{x}{2}\right)^2}$

1p $= \frac{x}{2}\sqrt{x^2 - \frac{x^2}{4}} = \frac{x}{2}\sqrt{\frac{3x^2}{4}} = \frac{x^2\sqrt{3}}{4}$

1p $\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt} = \frac{2x\sqrt{3}}{4} \cdot 3$.

when $A = 10 \; cm^2 \Rightarrow \frac{x^2 \cdot \sqrt{3}}{4} = 10 \Rightarrow x = \sqrt{\frac{40}{\sqrt{3}}}$

1p So:

$\frac{dA}{dt} = 2 \cdot \sqrt{\frac{40}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{4} \cdot 3$ when $A = 10 \; cm^2$

$(1-x)^{\frac{1}{2}}$

5. [6 points]

(i) Find the linear approximation of $f(x) = \sqrt{1-x}$ at $a = 0$, and use it to estimate $\sqrt{0.98}$.

(ii) Use Newton's method to find x_3. (Give the answer to four decimal places.) The equation is $\frac{x^3}{3} + \frac{x^2}{2} + 3 = 0$ and the initial guess(approximation) is $x_1 = -3$.

(i) $L(x) = f(0) + f'(0)(x-0) = 1 + f'(0)\cdot x$

Note: $f'(x) = \frac{1}{2}(1-x)^{-\frac{1}{2}} \cdot (-1)$ by $\left\{ \begin{array}{l} \text{Chain Rule} \\ \text{Power Rule} \end{array} \right.$ 1p

$L(x) = 1 + \left(-\frac{1}{2}\right)\cdot x$

$\sqrt{0.98} \overset{!}{=} f(0.02) \approx L(0.02) = 1 - \frac{1}{2}(0.02) =$

$= \boxed{0.99}$ 2p

ii) $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + 3$; $x_1 = -3$; $\boxed{f'(x) = x^2 + x}$ 1p

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \boxed{-2.75}$ 1p

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx \boxed{-2.7186}$ 1p

More space for question 5.

6. [2 points] Find $f(x)$ if it is known that $f''(x) = \cos(x) + 4$ and $f(0) = -2$, $f(\frac{\pi}{2}) = 0$.

0.5p

$$f'(x) = \int f''(x)\,dx = \int \cos x + 4 \quad dx = \sin x + 4x + C$$

C#

$$f(x) = \int f'(x)\,dx = \int \sin x + 4x + C \;\; dx =$$ 0.5p

$$= -\cos(x) + 2x^2 + Cx + d$$ d, #

From: $-2 = f(0) = -1 + d \Rightarrow \boxed{d = -1}$ 0.5p

From $0 = f(\frac{\pi}{2}) = 2 \cdot \frac{\pi^2}{4} + C \cdot \frac{\pi}{2} - 1 \Rightarrow$

$$\boxed{C = \frac{1 - \frac{\pi^2}{2}}{\frac{\pi}{2}}}$$ 0.5p.

Rough work