MAT 1320 3x Summer 2014 JUNE 5th, 19:00 Prof. C. Rada

TEST #1

Max = ~~20~~ 26

SOL.

Student Number: ____________________

- Time: 80 min.
- Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability. Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite. Graphs and sketches may be drawn in pencil.
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

1. [2 points] Solve: $\ln(x-2)+\ln(x)=\ln 3$.

$$\begin{cases} x-2>0 \\ x>0 \end{cases} \Rightarrow \begin{matrix} x>2 \\ x>0 \end{matrix} \Rightarrow \boxed{x>2}$$ 0.5p

0.5p $\ln\left((x-2)(x)\right)=\ln 3 \Rightarrow (x-2)x=3$

$\Rightarrow x^2-2x-3=0 \Rightarrow (x-3)(x+1)=0$

$$\begin{cases} x=3 \\ x=-1 \end{cases}$$

0.5p

ONLY $x=3$ WORKS (i.e. 3 is the ONLY SOLUTION)

0.5p

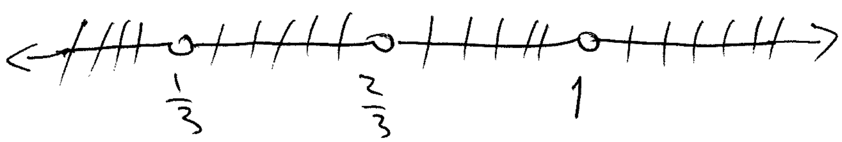
2. [2 points] Find the domain of the function: $h(x)=\frac{3}{\ln|3x-2|}$.

- $3x-2\neq 0 \Rightarrow \boxed{x\neq \frac{2}{3}}$ 0.5p
- $\ln|3x-2|\neq 0 \Rightarrow |3x-2|\neq e^0=1 \Rightarrow \begin{cases} 3x-2\neq 1 \\ 3x-2\neq -1 \end{cases}$

so $\boxed{\begin{cases} x\neq 1 \\ x\neq \frac{1}{3} \end{cases}}$ 0.5

So: $(-\infty,\frac{1}{3})\cup(\frac{1}{3},\frac{2}{3})\cup(\frac{2}{3},1)\cup(1,\infty)$ 1p

or



3. [2 points] If $f(x)=x^3-3x+6$, find $f'(1)$.

$f'(x)=3x^2-3+0=3x^2-3$ 1p

$f'(1)=3\cdot 1^2-3=0$ 1p

4. [5 points] a) Give the formula for $f'(a)$, the derivative of $f(x)$ at a.

b) Use a) to find $f'(a)$ if $f(x) = \frac{1}{x+1}$.

c) Find the equation of the tangent line to the graph of $f(x)$ at $x = 1$, where $f(x) = \frac{1}{x+1}$.

a) 1p $\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$ $\overset{\text{or}}{=} \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

b) 1p $\boxed{f'(a) = \lim_{h \to 0} \frac{\frac{1}{a+h+1} - \frac{1}{a+1}}{h}} = \boxed{\lim_{h \to 0} \frac{\frac{a+1-a-h-1}{(a+h+1)(a+1)}}{h}}$ 0.5

$= \lim_{h \to 0} \frac{-h}{h(a+h+1)(a+1)} = \boxed{\lim_{h \to 0} \frac{-1}{(a+h+1)(a+1)}}$ 0.5

$= \boxed{-\frac{1}{(a+1)^2}}$ 0.5

c) SAY it is: $\boxed{y = mx + n.}$ 0.5

$m = f'(1) = -\frac{1}{(2)^2} = \boxed{-\frac{1}{4}}$. 0.5 So $y = -\frac{1}{4}x + n$

Since $(1, \frac{1}{2})$ is on the line $\Rightarrow$ $\frac{1}{2} = -\frac{1}{4} \cdot 1 + n \Rightarrow$

$n = \frac{1}{2} + \frac{1}{4} = \boxed{\frac{3}{4}}$ 0.5

Hence: $y = -\frac{1}{4}x + \frac{3}{4}$

5. Find the limits:

a) $\lim_{x \to \frac{\pi}{2}} \ln(\sin(x))$

b) $\lim_{x \to \infty} \ln(\frac{3x^2}{x^2+1000})$

$1 \qquad 1 \qquad 0.5$

$$a) = \ln\left(\lim_{x \to \pi/2} \sin x\right) = \ln\left(\sin \frac{\pi}{2}\right) = \ln 1 = \boxed{0}$$

$$b) = \underbrace{\ln\left(\lim_{x \to \infty} \frac{3x^2}{x^2+1000}\right)}_{1p} = \underbrace{\ln\left(\lim_{x \to \infty} \frac{3}{1+\frac{1000}{x^2}}\right)}_{1p} =$$

$$= \underbrace{\ln(3)}_{0.5}$$

6. [5 points] Find the values (if any) of d such that the function is continuous.

$$f(x) = \begin{cases} 5x & if \quad x < 1 \\ (2x+d)^2 & if \quad x \geq 1 \end{cases}$$

We need to impose:

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1) \qquad 1p$$

$$\lim_{x \to 1^-} f(x) = \boxed{5 \cdot 1 = 5} \quad 1p$$

$$\lim_{x \to 1^+} f(x) = \boxed{(2 \cdot 1 + d)^2} \quad 1p$$

so: $5 = 4 + 4d + d^2$ or:

$$d^2 + 4d - 1 = 0$$

$$d_{1,2} = \frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2}$$

$$d_{1,2} = \frac{-4 \pm \sqrt{20}}{2}$$

$$\begin{cases} d_1 = \dfrac{-4 - 2\sqrt{5}}{2} = -2 - \sqrt{5} & 1p \\ d_2 = -2 + \sqrt{5} & 1p \end{cases}$$

7. [5 points] Find $f''(x)$, $g'(x)$ and $h'(x)$ if $f(x) = 3e^x + 43\sin x$;

$g(x) = \frac{2e^x}{\cos(x)}$;

$h(x) = 3x^{2014} - 3$.

a) $f'(x) = 3e^x + 43 \cdot \cos x$ 1p

$f''(x) = 3e^x + 43(-\sin x)$ 1p

b) $g'(x) = \dfrac{2e^x \cdot \cos x - 2e^x(-\sin x)}{\cos^2(x)}$ 1p 1p

c) $h'(x) = 3 \cdot 2014 \cdot x^{2013} - 0$ 1p

Rough Work