Name ____

Student number _______________________________________

Lecture section (L01-Dr. Y. Wang; L02-Dr. V. Dimitrov) _________

University of Calgary
Schulich School of Engineering
Fall 2015

ENGG 407 - Numerical Methods

Midterm

Saturday, Oct. 24, 2015
14:00-15:30
ST148 (L01) / ST140 (L02)

1. Examination is *open book* and *notes*.
2. The exam encompasses 45 multiple choice questions where each question has equal value. Total marks for the exam is 100.
3. Write the answers on *both* the separate bubble sheet and the exam booklet.
4. No wireless and electronic devices are allowed during exam.
5. *No calculators* are allowed.
6. An extended aid sheet is provided on D2L, which you may bring into the exam site.
7. You do not need to simplify the numerical expressions unless stated.
8. All angles are in radians. For example sin(x), x is assumed to be in radians.

Circle the correct answer below each question. Then, mark your answers on the standard answer sheet. Each question has equal value.

1. Which of the following statements best expresses the characteristics of *numerical methods*?

a) Using approximation techniques to find solutions for analytic functions
b) Using iterative and/or recursive algorithms
c) Suitable for problems that cannot be or difficult to be solved analytically
d) All the above

2. Three termination conditions are adopted in the iterative methods for solving nonlinear functions. Which of the following is *NOT* a typical option? (ε is a small positive number.)

a) $\frac{b^i - a^i}{2} \le \varepsilon$

b) $\left| \frac{x^i - x^{i-1}}{x^{i-1}} \right| \le \varepsilon$

c) $f(x^i) = 0$

d) $\left| \frac{f(x^i) - f(x^{i-1})}{f(x^{i-1})} \right| \le \varepsilon$

3. Following Question (2), which of the following is with the *lowest* evaluation cost?

a) $\frac{b^i - a^i}{2} \le \varepsilon$

b) $\left| \frac{x^i - x^{i-1}}{x^{i-1}} \right| \le \varepsilon$

c) $f(x^i) = 0$

d) $\left| \frac{f(x^i) - f(x^{i-1})}{f(x^{i-1})} \right| \le \varepsilon$

4. Following Question (2), which of the following is with the *highest* evaluation cost?

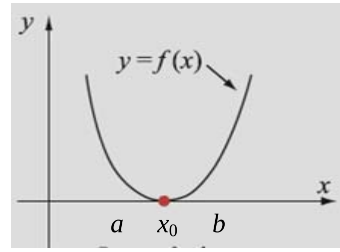
a) $\frac{b^i - a^i}{2} \le \varepsilon$

b) $\left| \frac{x^i - x^{i-1}}{x^{i-1}} \right| \le \varepsilon$

c) $f(x^i) = 0$

d) $\left| \frac{f(x^i) - f(x^{i-1})}{f(x^{i-1})} \right| \le \varepsilon$

5. Why does Newton's method not work on the special case of a parabola as given in the figure? It's because:



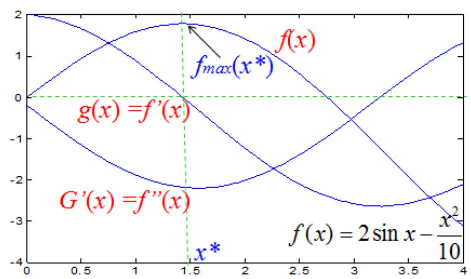
a) $f(a)\bullet f(b) \not< 0$

b) $f(x_0) = 0$

c) $f'(a)\bullet f'(b) < 0$

d) $f'(x_0) = 0$

6. Which pair of curves in the figure determines the maxima (x^*) in Newton's method for *optimization*?



a) $f'(x)$ and $f''(x)$

b) $f(x)$ and $f'(x)$

c) $f(x)$ and $f''(x)$

d) Undeterminable

7. Following Question 6, which pair of curves in the above figure determines the root (x_0) in Newton's method for *root finding*?

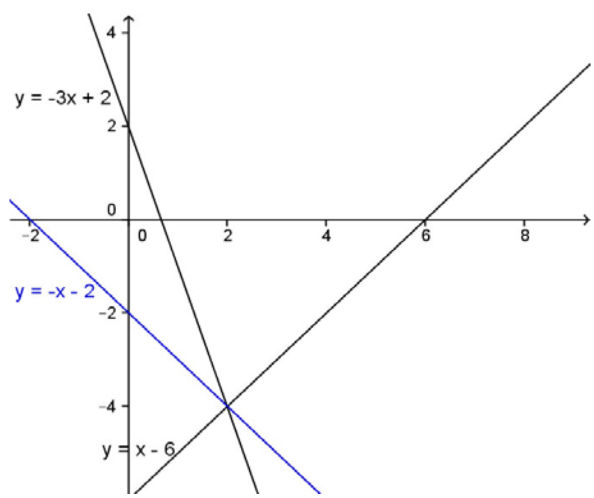
a) $f'(x)$ and $f''(x)$

b) $f(x)$ and $f'(x)$

c) $f(x)$ and $f''(x)$

d) Undeterminable

8. How many lines do you need to uniquely solve the system of linear equations as given in the figure?



a) 3
b) 1
c) 2
c) > 3

9. Following Question 8, what is the solution (x, y) for the given system of linear equations?

a) (-2, -4)
b) (2, -2, -6)
c) (-2, 0.5, 6)
d) (2, -4)

10. In the *bisection* method, the root of $f(x)$ is to be determined in a given initial bracket $[a,b]$. After *n* iterations, what is the length of the bracket ($L_n = b_n - a_n$)?

a) $L_n = (b-a) / n$
b) $L_n = (b-a) * 2^n$
<u>c)</u> $L_n = (b-a) / 2^n$
d) $L_n = (b-a) * n$

11. Why do most of systems of linear equations (SLEs) elicited from real-world problems have a *square* coefficient matrix *A* in engineering problem solving?

a) m (#rows or #functions) ≥ n (#columns or #variables)
b) Determined by problem-specific reasons
c) Squire matrix possesses special properties
d) It meets the least sufficient and necessary condition for solutions

12. The given SLE $\begin{cases} 3x_1 - 10x_2 + 5x_3 = 3 \\ 2x_1 + 0x_2 + 8x_3 = -2 \\ 20x_1 + 3x_2 + 2x_3 = 6 \end{cases}$ can be represented in the matrix form **AX** = **b**, i.e.: $\begin{bmatrix} 3 & -10 & 5 \\ 2 & 0 & 8 \\ 20 & 3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$. The *advanced pivoting* technology can be adopted not only for eliminating zero pivots but also for selecting the largest pivot element among the rows in order to obtain more accurate and stable results in the given SLE. Which of the following solutions for **A'X** = **b'** is an advanced pivoting that reserves the system invariant.

a) $\mathbf{A}'\mathbf{X} = \mathbf{b}' \Rightarrow \begin{bmatrix} 3 & -10 & 5 \\ 2 & 0 & 8 \\ 20 & 3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$

b) $\mathbf{A}'\mathbf{X} = \mathbf{b}' \Rightarrow \begin{bmatrix} 3 & -10 & 5 \\ 20 & 3 & 2 \\ 2 & 0 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -2 \end{bmatrix}$

<u>c)</u> $\mathbf{A}''\mathbf{X} = \mathbf{b}'' \Rightarrow \begin{bmatrix} 20 & 3 & 2 \\ 3 & -10 & 5 \\ 2 & 0 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix}$

d) $\mathbf{A}''\mathbf{X} = \mathbf{b}'' \Rightarrow \begin{bmatrix} 20 & 3 & 2 \\ 3 & -10 & 5 \\ 2 & 0 & 8 \end{bmatrix}\begin{bmatrix} x_3 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix}$

13. Given Newton-Raphson's method for solving system of nonlinear functions with multiple variables as $\mathbf{x}^{i+1} = \mathbf{x}^i - \mathbf{J}^i \setminus \mathbf{F}^i$, what is the single variable form known as Newton's method reduced based on Newton-Raphson's method?

a) $x^{i+1} = x^i - \frac{f'(x^i)}{f(x^i)}$

b) $x^{i+1} = x^i - \frac{f'(x^i)}{f''(x^i)}$

<u>c)</u> $x^{i+1} = x^i - \frac{f(x^i)}{f'(x^i)}$

d) Undeterminable

14. Given $f(x) = x^2+1$ in [0, 2], assume $x^{(0)} = 0.5$ for root finding using Newton's method. What is the numerical result of $x^{(1)}$ after the first iteration?

a) -1
b) 0.75
<u>c)</u> -0.75
d) 1.75

15. It's recognized that, given a nonlinear equation $f(x)=0$ in $[a,b]$, $f(a)\bullet f(b)<0$ is a *sufficient* (strong) condition for root(s) existence in the bracket. However, in an exceptional case as shown in Question 5, what is the *necessary* condition to predicate there is a root?

a) $\exists x_0, f'(x_0)=0$

b) $\exists x_0, f(a)\bullet f(b)<0$

c) $\exists x_0, f(x_0)=0 \vee f'(x_0)=0$

d) $\exists x_0, f'(x_0)=0 \wedge f(x_0)=0$

16. Given $f(x) = x^3 + x - 1 = 0$ and assuming that the *bisection* method will be applied in the initial interval [0, 2]. The first approximation of the root will be:

a) 2
b) 0.5
c) 0
d) 1.0

17. Given a general row of expression in an SLE as $L_i = (a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n) = b_i, 1 \le i, j \le n$, what is *NOT* a valid rule that keeps the system′s solutions invariant? (*k* and *k'* are a nonzero real number.)

a) $L_i \leftrightarrow L_j$

b) $kL_i \rightarrow L_i$

c) $kL_i + k'L_j \rightarrow L_j$

d) $kL_i \rightarrow L_j$

18. Based on Question 17, what is/are the rule(s) used in the method of Gaussian elimination with pivoting?

a) Rules (a) and (b)
b) Rules (a) and (c)
c) Rule (c)
d) Rule (d)

19. Given a polynomial $f(x) = x^3 - 3x + 1$ in [0, 2] for optimization, which of the following statements is true:

a) $f_{\max}(x \mid x = 1) = -1$
b) $f_{\min}(x \mid x = 0) = 0$
c) $f_{\min}(x \mid x = 1) = -1$
d) $f_{\max}(x \mid x = 2) = 2$

20. Let A = LU = $\begin{bmatrix} 2 & 0 \\ 1 & 0.5 \end{bmatrix}\begin{bmatrix} 1 & 1.5 \\ 0 & 1 \end{bmatrix}$. What is A?

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
b) $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$
c) $A = \begin{bmatrix} 2 & 1.5 \\ 3 & 0.5 \end{bmatrix}$
d) None of above

21. Which of the following expressions is *NOT* a valid operation for solving SLEs?

a) x = A \ b
b) x = b / A^T
c) x = $(A^TA)^{-1}A^Tb$
d) x = $\frac{1}{A}$b

22. Determine a pair of eigenvalues for the given matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$.

a) 1, 1
b) 2, -2
c) 1, -1
d) 0, 2

23. Given an SLE $Ax = b$, the condition number Cond(A) = $||A|| \, ||A^{-1}||$ can be an indicator for which of the following:

a) If Cond(A) >>1, the SLE is ill-conditioned

b) If Cond(A) ≈ 1, the relative error of the SLE is ordinarily predictable

c) $\forall A$, Cond(A) ≥ 1

d) All the above

24. Why do the error theories of numerical methods put emphases on truncation errors?

a) They are machine dependent
b) They are method and problem specific
c) They cannot be estimated by a Taylor series
d) They are predictable when a problem is divergent or ill-conditioned

25. For an arbitrary continuous function $f(x)$, what is the necessary condition which guarantees the existence of at least one optimum that is a minimum in the given bracket [a, b]?

a) $f'(x) = 0 \wedge f''(x) > 0$

b) $f'(x) = 0 \vee f''(x) < 0$

c) $f'(x) > 0 \vee f'(x) < 0$

d) $f'(x) \bullet f''(x) < 0$

26. The given iteration formula $x_{i+1} = x_i - \dfrac{f'(x_i)}{f''(x_i)}$ is suitable for:

a) Expressing the second order Taylor series of $f(x)$

b) Newton′s method for root finding
c) Nothing as the expression is meaningless
d) Newton’s method for optima finding

27. Applying the Jacobi iteration method on the SLE as given below, which of the following systems is equivalent and will converge according to the sufficient condition of convergence for iterative methods?

$$\begin{cases} 3x_1 + 8x_2 - 14x_3 = 5 \\ 2x_1 + 7x_2 + x_3 = -9 \\ 12x_1 + 5x_2 + x_3 = 11 \end{cases}$$

a) $\begin{bmatrix} 3 & 8 & -14 \\ 2 & 7 & 1 \\ 12 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ 11 \end{bmatrix}$

b) $\begin{bmatrix} 12 & 5 & 1 \\ 2 & 7 & 1 \\ 3 & 8 & -14 \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 11 \\ -9 \\ 5 \end{bmatrix}$

c) $\begin{bmatrix} 12 & 5 & 1 \\ 2 & 7 & 1 \\ 3 & 8 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11 \\ -9 \\ 5 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 8 & -14 \\ 12 & 5 & 1 \\ 2 & 7 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ -9 \end{bmatrix}$

28. How many eigenvalues may be found in Question 27?

a) 1
b) 2
c) 0
d) 3

29. For a given SLE as $\begin{cases} 3x_1 + x_2 + x_3 = 1 \\ 2x_1 + 5x_2 - 2x_3 = 2 \\ -x_1 + 4x_2 - 6x_3 = 1 \end{cases}$, the Jacobi method will:

a) depend on the initial values
b) definitely converge
c) undeterminable
d) definitely diverge

30. Assume $x^{i+1} = x_l^i + (x_u^i - x_l^i)r^i$ is the iterative formula for 1-D optimizations using the *random search* method where r is a random number, $0 \le r \le 1$. How may it be extended to a 2-D problem $f(x, y)$ in $[x_l,\ x_u]$ and $[y_l,\ y_u]$?

a) $\begin{cases} x^{i+1} = x_l^i + r_x^i \\ y^{i+1} = y_l^i + r_y^i \end{cases}$

b) $\begin{cases} x^{i+1} = x_l^i - (x_u^i - x_l^i)r^i \\ y^{i+1} = y_l^i - (y_u^i - y_l^i)r^i \end{cases}$

c) $\begin{cases} x^{i+1} = x_l^i r^i + x_u^i \\ y^{i+1} = y_l^i r^i + y_u^i \end{cases}$

d) $\begin{cases} x^{i+1} = x_l^i + (x_u^i - x_l^i)r_x^i \\ y^{i+1} = y_l^i + (y_u^i - y_l^i)r_y^i \end{cases}$

31. What is the first order Taylor expansion of the function $f_1(x, y) = \sin(xy)$ at the given expansion point (x_0, y_0)?

a) $f_1(x, y) = \sin(x_0 y_0) + \cos(x_0 y_0)(x - x_0) + \cos(x_0 y_0)(y - y_0)$

b) $f_1(x, y) = \sin(x_0 y_0) + y_0 \cos(xy)(x - x_0) + x_0 \cos(xy)(y - y_0)$

c) $f_1(x, y) = \sin(x_0 y_0) + y_0 \cos(x_0 y_0)(x - x_0) + x_0 \cos(x_0 y_0)(y - y_0)$

d) $f_1(x, y) = \sin(xy) + y\cos(xy)(x - x_0) + x\cos(xy)(y - y_0)$

32. According to Taylor's theorem for an arbitrary continued function f(x) as given below:

$$f(x)=\sum_{i=0}^{n}\frac{f^{(i)}(a)}{i!}(x-a)^i+R_n \text{ where the } \textit{remainder } R_n=\frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1},\xi\in(a,x)$$

what is the order (maximum) of the truncation error for the 3rd order expansion of f(x)?

a) $R_3=\frac{f^{(3)}(x)}{3!}(x-a)^3$

b) $R_3=\frac{f^{(4)}(a)}{4!}(x-a)^4$

c) $R_3=\frac{f^{(4)}(\xi)}{4!}(x-a)^4,\xi\in(a,x)$

d) $R_3=\frac{f^{(3)}(\xi)}{3!}(x-a)^3,\xi\in(a,x)$

33. How can the Gaussian method be enhanced as a generic method for dealing with any SLEs *except*:

a) Simple pivoting
b) LU decomposition
c) Advanced pivoting
d) Gauss-Jordan with pivoting

34. Which of the following is *NOT* a sign that indicates a numerical method for solving root(s) for nonlinear functions is converging?

a) A given tolerance for the independent variable is reached
b) The value of the function is approaching to 0
c) The length of the bracket is decreasing
d) The values of the function in two ends of the bracket become the same sign

35. The convergent condition for solving root(s) for a nonlinear functions $f(x)$ in [a, b] is determined by the following *except*:

a) $(b-a)/2$ < tolerance
b) $f(x_i)$ < tolerance
c) $f'(x_i) = 0$
d) $f(x_i) = 0$

36. In the *bisection* method, the root of $f(x)$ is to be determined given an initial bracket of $[a,b]=[0,10]$. After 3 iterations, what is the length of the bracket ($L_3 = b_3 - a_3$)?

a) 2.50
b) 1.25
c) 5.00
d) 1.75

37. The *Jacobi iterative method* for solving SLEs is a special case of the *Gauss-Seidel iterative method*. Given the general iteration formula of the latter as $x_i = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j)$, what is the iterative expression of the former?

a) $x_i^{k+1} = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j)$

b) $x_i^{k} = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j^{k})$

<u>c)</u> $x_i^{k+1} = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j^{k})$

d) None of above

38. For a generic SLE as $\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ ... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{cases}$, what is the condition for obtaining unique solutions?

a) $m > n$
b) $n \neq m$
c) $n \geq m$
<u>d)</u> $m \geq n$

39. How does the trend of a nonlinear function *f(x)* as being descending at x_0 towards $x_0+\Delta x$ be detected where Δx is a small positive real number close to the machine epsilon?

<u>a)</u> $f(x_o) > f(x_o + \Delta x)$
b) $f(x_o) < f(x_o + \Delta x)$
c) $|f(x_o)| > |f(x_o + \Delta x)|$
d) $|f(x_o)| < |f(x_o + \Delta x)|$

40. In the following characteristics of iterative methods for solving an SLE, such as those of Gauss-Seidel and Jacobi, which statement is *NOT* true?

a) The sufficient convergent condition needs always to be tested
b) May not converge in iteration when the sufficient convergent condition is not met
c) Its algorithm complexity is higher than those of direct methods
d) Cannot be carried out when zero pivoting elements are involved without pivoting treatment

41. Given a function $f(x) = 2\sin(x) + 1$, what is the first order Taylor expansion of the function at the point $x_0 = \pi$?

a) $f(x) = f(x_0) + 2\cos(x_0)(x - x_0)$
b) $f(x) = f(x_0) + 2\cos(x)(x - x_0)$
c) $f(x) = 1 + 2(x - \pi)$
d) $f(x) = 1 - 2(x - \pi)$

42. Which of the following methods is *NOT* suitable for solving a 2-D optimization problem?

a) The random search method
b) The regula falsi method
c) The sliced reduction method
d) All of above

43. When minimizing the function $f(x) = x^2 - x + 1$ in the interval [a, b] = [0, **2**] using the golden section optimization method, the interval [a, b] will be reduced to which after the first iteration?

a) [2*0.618, 1]
b) [0, 0.618]
c) 2[0.382, 0.618]
d) [0, 2*0.618]

44. One of the strategies for solving an SLE can be described as x = [A, b] = [I, b′] ⇒ x = b′. What is the name of this method?

a) Newton
b) Gauss-Seidel
c) Gauss-Jordan
d) LU decomposition

45. When the residual of the Taylor series expansion for $f(x, y) = x^2 y$ will reach zero?

a) The zero order
b) The 1^{st} order
c) The 2^{nd} order and beyond
d) The 3^{rd} order and beyond