Abdullah

Engineering 0138: Advanced control II — A. TAYEBI

Final Examination

Tuesday, April 18th, 2000
9:00AM To 12:00N
Closed book
Do all your work on sheets provided.

Q-1 [5]

Consider the following transfer function system

$$\frac{Y(s)}{U(s)} = \frac{s+3}{s^2+3s+2}$$

Obtain the state space representation of this system in

a) Controllable canonical form.
b) Observable canonical form
c) Diagonal or Jordan form

$X(s)[sI-A] = BU(s)$
$X(s) = BU(s)[sI-A]^{-1}$
$Y(s) = C[sI-A]^{-1}BU(s)$
$\frac{Y(s)}{U(s)} = C[sI-A]^{-1}B$

Q-2 [5]

Consider the system defined by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$C(\lambda I - A)^{-1}B$

Where

$$A = \begin{bmatrix} 1 & 3 \\ -4 & -6 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } C = [1 \;\; 1]$$

Transform the system into the controllable canonical form.

Q-3 [15]

Consider the system defined by

$B \;\; AB \;\; A B^2$

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Where

C
CA
CA^2

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ -1 & -3 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } C = [1 \;\; 0 \;\; 0]$$

a) Check the controllability and observability of the system.

$(\lambda - P_1)(\lambda - P_2)(\lambda - P_3) = 0$

$|\lambda I - (A - BK)| = K$

b) Design a state feedback tracking controller of the form $u = -\bar{K}x + l\, y_d$, in order to have the closed-loop poles at $(-4 \pm j), (-6)$ and the steady state error equal to zero for a step input. → Solve it

c) Design a state observer for the system with the following performances: $\lambda I - (A - LC)$

-Settling time at 2%=1s

-Damping ratio=0.7.

Q-4 [10]

Consider the system defined by

$$\dot{x} = Ax + Bu \qquad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$y = Cx$$

Where

$$A = \begin{bmatrix} 1 & 3 \\ -4 & -6 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

a) Design a state feedback stabilizing controller using LQR with

$$R = 1 \text{ and } Q = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

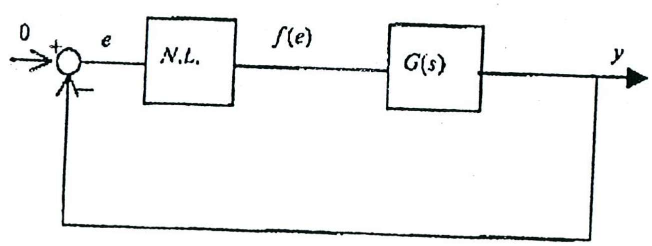
b) Give the minimum value of the cost function

c) Give the resulting closed loop system.

d) Solve the closed-loop system equations to determine the explicit expression of $y(t)$

Q-5 [10]

Consider the following closed loop system



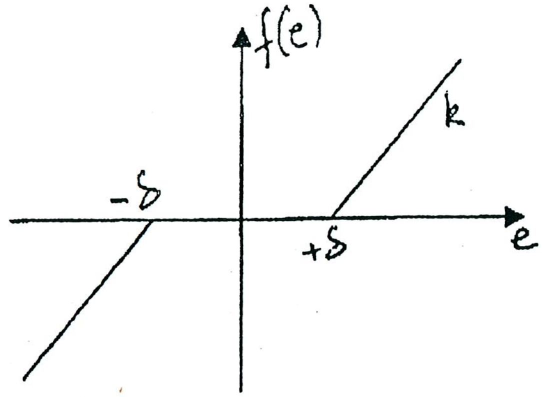
With $G(s) = \dfrac{K_1}{s(T_1 s + 1)(T_2 s + 1)}$.

The nonlinear element is a relay with the following describing function

$$N(A) = \frac{-4h}{\pi A}$$

a) Discuss the existence of limit cycles according to the parameters K_1, T_1 and T_2.

b) Derive the describing function for the following dead-zone nonlinearity



With the dead-zone width being 2δ and its slope k.
Assume that the sinusoidal signal amplitude $A \geq \delta$.

Q-6 [5]

Consider the nonlinear system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_1^2 + u$$

with $x_1 \in \Re$, $x_2 \in \Re$ and $u \in \Re$.
Using the following Lyapunov function candidate

$$V(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}(x_2 + k_1 x_1)^2, \text{ with } k_1 > 0,$$

design a state feedback stabilizing controller.

P.S.

Given Laplace transorms:

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \rightarrow \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n\left(\sqrt{1-\xi^2}\right)t\right)$$

$$\frac{s}{s^2 + 2\xi\omega_n s + \omega_n^2} \rightarrow \frac{-1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n\left(\sqrt{1-\xi^2}\right)t - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right)$$