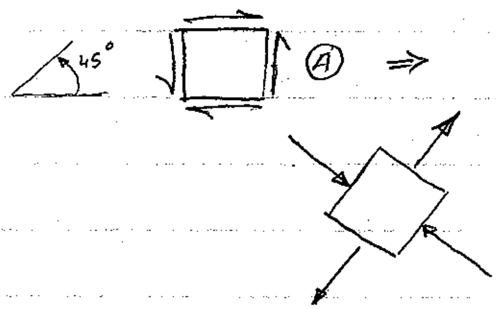
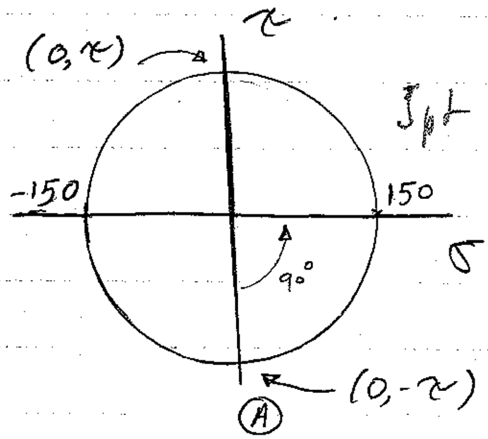
ME260 2008
Final Solutions

1. a) In section AB:
Take an element:



$\Rightarrow$



1pt

Thus $\tau_{AB,max} = 150$ MPa

Similarly: $\tau_{BC,max} = 150$ MPa

$\Rightarrow \tau_{max} = 150$ MPa

2pt

Equilibrium: $\Sigma T = 0 \quad \Rightarrow \quad T_{AB} + T_{BC} = T \quad (1)$ 2pt

Kinematics: $\phi_{AB} = \phi_{BC} \rightarrow \dfrac{2T_{AB}L}{GJ_{AB}} = \dfrac{T_{BC}L}{GJ_{BC}}$

note: $r_{AB} = 2r_{BC}$

$$\frac{2T_{AB}}{\frac{1}{2}\pi(2r_{BC})^4} = \frac{T_{BC}}{\frac{1}{2}\pi(r_{BC})^4} \quad \Rightarrow \quad T_{AB} = 8T_{BC} \quad (2)$$

2pt 2pt

In section AB:

3pt

$$\tau_{max} = \frac{T_{AB}C_{AB}}{J_{AB}} \Rightarrow T_{AB} = \frac{\frac{1}{2}\pi(2r_{BC})^4\tau_{max}}{2r_{BC}} \qquad T_{AB_{max}} = 1885 \text{ N.m} \quad (3)$$

In Section BC:

$$\tau_{max} = \frac{T_{BC}C_{BC}}{J_{BC}} \Rightarrow T_{BC} = \frac{\frac{1}{2}\pi(r_{BC})^4\tau_{max}}{r_{BC}} \qquad T_{BC_{max}} = 235.6 \text{ N.m} \quad (4)$$

But, T_{AB} and T_{BC} must follow equation (2), and each not exceed the max values given by (3) & (4).

$\longrightarrow$

Solution with L_1 and L

$T_{AB} = 16\, T_{BC}$

in AB $\tau_{max} = \frac{T_1 C_1}{J_1} \Rightarrow T_{1Max} = \frac{J_1 \tau_{max}}{C_1} = \frac{\frac{1}{2}\pi (2r)^4 \tau_{max}}{2r} = 4\pi r^3 \tau_{max}$

$T_{1max} = 4\pi \left(\frac{10}{1000}\right)^3 150 \times 10^6$ $\boxed{T_{1max} = 1885 \text{ N.m}}$ (3)

In BC $T_{2max} = \frac{\frac{1}{2}\pi (r)^4 \tau_{max}}{r} = \frac{1}{2}\pi r^3 \tau_{max}$

$T_2 = \frac{1}{2}\pi \left(\frac{10}{1000}\right)^3 \times 150 \times 10^6$ $\boxed{T_{2max} = 235.6 \text{ N.M}}$ (4)

But T_1 & T_2 must follow equations 1, 3 and 4.

If T_1 is limiting $T_2 = \frac{T_{1max}}{16} = \frac{1885}{16} = 117.8$ N.M < 235.6 OK

If T_2 is limiting $T_1 = T_{2max} \times 16 = 3769.6$ N.m > 1885 N.m not ok

[redacted]s. Thus $\boxed{T_{max} = T_{1max} + T_2 = 1885 + 117.8 = 2002.8 \text{ N.M}}$

1\. Continued: Check:

If T_{AB} is limiting:

$$T_{BC} = \frac{T_{AB,max}}{8} = \frac{1885}{8} = 235.6 \equiv T_{BC,max}\text{ , OK}$$

If T_{BC} is limiting:

$$T_{AB} = 8\,T_{BC,max} = 8 \times 235.6 = 1885 \equiv T_{AB,max}\text{ , OK}$$

Both conditions are satisfied (by chance!), so sections AB an BC are equally "stressed", and neither solely control the torque applied.

Finally, $T = T_{AB} + T_{BC} = 1885 + 235.6$

$$T = 2120.6 \text{ N.m}$$

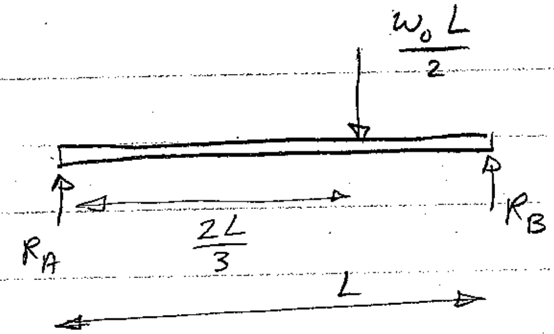
b) Use either of $\phi_{AB} = \dfrac{T_{AB}\,2L}{GJ_{AB}}$ or $\phi_{BC} = \dfrac{T_{BC}L}{GJ_{BC}}$

$$\phi_{AB} = \frac{1885 \times 2L}{G\,\frac{1}{2}\pi\left(\frac{20}{1000}\right)^4}$$

$$\phi = 6\times10^{10}\frac{L}{G} \text{ [Rad]}$$

with L & G in appropriate metric units

2. FBD:

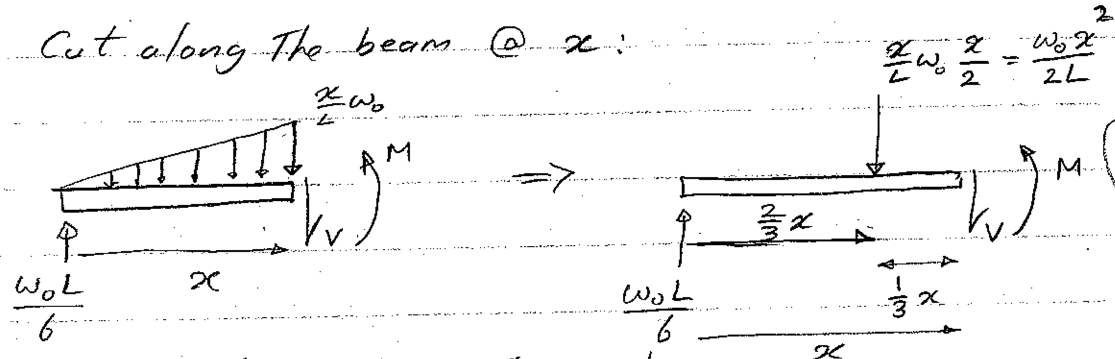


3pt

Reactions: $\sum M_A = 0$ ⟹ $R_B = \frac{w_0 L}{3}$

$\sum M_B = 0$ ⟹ $R_A = \frac{w_0 L}{6}$ 3pt

Cut along the beam @ x:



3pt

For the figure on the right:

$\sum F_y = 0 \downarrow^{+} \quad V + \frac{w_0 x^2}{2L} - \frac{w_0 L}{6} = 0$ $\quad V = \frac{w_0 L}{6} - \frac{w_0 x^2}{2L}$ 3pt

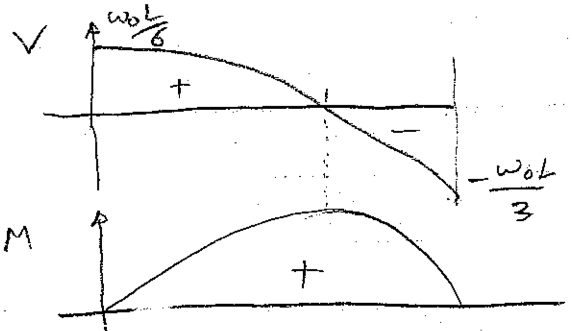
(Just check @ $x = L$ $\quad V = \frac{-w_0 L}{3} = -R_B$) ✓✓

7pt

$\sum M = 0 \quad M + \frac{w_0 x^2}{2L} \frac{x}{3} - \frac{w_0 L}{6} x$ $\quad M = \frac{w_0 L x}{6} - \frac{w_0 x^3}{6}$

(Just a check @ $x = L$ $\quad M = 0$) ✓✓

5pt



5pt

Not required, but if $V(x)$ is set equal to zero, the location of M_{max} & its Mag. can be found.

3. The deflection of point C can be determined, <u>but</u> the students must recast the Appendix C equations with x axis pointing from the right support towards the left. This may need more time, and could be a bit confusing. <u>Instead</u>, we ask the students to determine the deflection of point "B", which is still "half" challenging:

a) Determine the deflection of point B.

Deflection of point B is the results (algebraic sum) of two deflections. One due to upward P, and the other due to downward P.

First, V_1 due to upward P:
Referring to Appendix C; and using the column "deflections", and that $a = \frac{L}{3}$ & $b = \frac{2L}{3}$:

(+)

$$V_{1,B} = \frac{+\frac{2L}{3}\cdot\frac{L}{3}}{6EIL}\left[L^2 - \left(\frac{2L}{3}\right)^2 - \left(\frac{L}{3}\right)^2\right] \Rightarrow V_{1,B} = \frac{4PL^3}{243EI}$$

Second V_2 due to upword P:
Referring to Appendix C; and this time using the column "Elastic curve", and that $a = \frac{2L}{3}$ & $b = \frac{L}{3}$

(−)

$$V_2 = \frac{-P\frac{L}{3}x}{6EIL}\left[L^2 - \left(\frac{L}{3}\right)^2 - x^2\right]$$

3.
Continued

$$\nu_{2,B} = \nu_2 \left(x = \frac{L}{3}\right)$$

$$\nu_{2,B} = \frac{-P \frac{L}{3}\frac{L}{3}}{6EIL}\left(L^2 - \frac{L^2}{9} - \frac{L^2}{9}\right) \Rightarrow \nu_{2,B} = \frac{-7PL^3}{486EI}$$

$$\nu_B = \nu_{1,B} + \nu_{2,B}$$

$$\nu_B = \frac{4PL^3}{243EI} - \frac{7PL^3}{486EI}$$

$$\nu_B = \frac{PL^3}{486EI}$$

.00206

Check: looking at individual ν_1 and ν_2 and The directions of the forces, The magnitude of ν_B and its direction seem reasonable. ✓

b) Slope at A is due to upward and downward forces.

For upward force: From Appendix C under column slope:

$$\theta_1 = \frac{\overset{(+)}{+}Pab(L+b)}{6EIL} \quad a = \frac{L}{3},\ b = \frac{2L}{3} \Rightarrow \theta_{1,A} = \frac{+P\frac{L}{3}\frac{2L}{3}\left(L+\frac{2L}{3}\right)}{6EIL}$$

$$\frac{5}{81}\frac{PL^2}{EI}$$

For downward force:

$$\theta_2 = \frac{-Pab(L+b)}{6EIL} \quad a = \frac{2L}{3},\ b = \frac{L}{3} \Rightarrow \theta_{2,A} = \frac{-P\frac{2L}{3}\frac{L}{3}\left(L+\frac{L}{3}\right)}{6EIL}$$

$$\theta_A = \theta_{1,A} + \theta_{2,A} = \quad \Rightarrow \quad \theta_A = \frac{PL^2}{81EI}$$

4. $D_o = 4^{cm}$ $\quad R_o = 2^{cm}$ $\quad t = 4^{mm}$

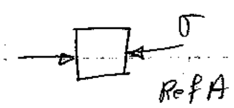
X-sec area can be calculated by either of the following

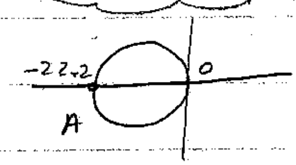
$$A_{exact} = \pi (R_o^2 - R_i^2) \Rightarrow A_{exact} = 4.5 \times 10^{-4} [m^2]$$

or

$$A_{approx} = \pi D_o t \Rightarrow A_{approx} = 5 \times 10^{-4} [m^2]$$

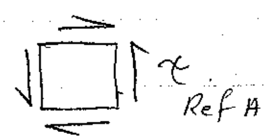
a) $\sigma = \frac{F}{A}$ $\quad \sigma = \frac{10000}{4.5 \times 10^{-4}}$ $\quad \sigma = 22.2$ MPa Comp.

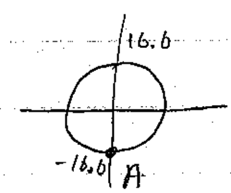




8 pts

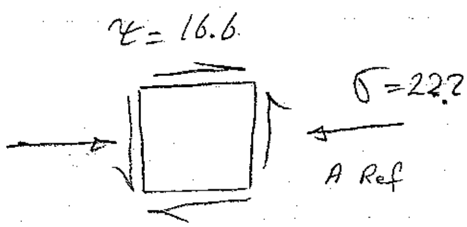
b) $\tau = \frac{TC}{J} = \frac{TR_o}{J} = \frac{75 \frac{2}{100}}{\frac{\pi}{2}\left(\left(\frac{2}{100}\right)^4 - \left(\frac{1.6}{100}\right)^4\right)}$ $\quad \tau = 16.6$ MPa

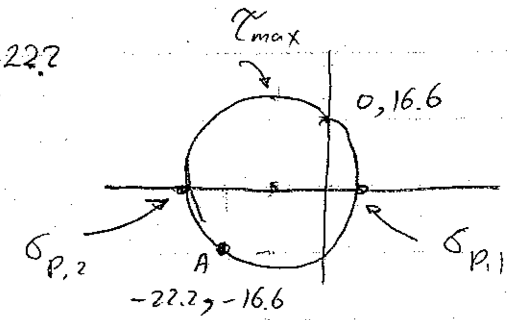




8 pts

c)





8 pts

Using equation (9-5) & (9-7).

$\sigma_{1,p} \approx \cancel{8.8}$ 3.97 MPa $\quad \sigma_{2,p} \approx \cancel{31}$ 26.0 MPa

$\tau_{max} \approx \cancel{19.9}$ 19.0 MPa