Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2013
TEST 5

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name :

Student Number:

Tutorial Section (A1, A4, ...):

PART I: Multiple Choice Questions
(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[3 marks]** Evaluate $\int_{-1}^{1} \frac{1}{x^2-4}\,dx$.

   (a) 0, (b) $\ln 3$, (c) $-\frac{1}{2}\ln 3$, (d) 1, e) none of these.

2. **[3 marks]** Evaluate $\int_0^{\pi/4} 2\tan x \sec^2 x\,dx$

   (a) -3, (b) 2, (c) 0, (d) 1, e) none of these

3. **[3 marks]** Evaluate $\int \frac{1}{(x-1)(x-3)}\,dx$

   (a) $\frac{1}{2}\ln|x-3|-\frac{1}{2}\ln|x-1|+C$, (b) $\frac{3}{2}\ln|x-3|+\frac{1}{2}\ln|x-1|+C$, (c) $\frac{1}{2}\ln|x-3|+C$, (d) $\frac{1}{4}\ln|x-3|-\frac{1}{4}\ln|x-1|+C$, e) none of these

4. **[3 marks]** Evaluate $\int_0^{\pi/2} \sin^2 x\,dx$

   (a) π, (b) $\frac{\pi}{4}$, (c) 0, (d) 1, e) none of these

5. **[3 marks]** $\int_0^{\pi/4} \cos^2 x \sin^2 x\,dx = \frac{\pi}{32}$

   (a) TRUE, (b) FALSE,

PART II: Show all work here and give details.
No additional pages will be accepted

6. **[5+5 marks]** a) Evaluate the definite integral $\int_0^{\pi/4} \sec^4 x \tan^4 x\,dx$.

   b) Evaluate the indefinite integral $\int \frac{dx}{(x-1)(x-2)^2}$.

a) $I = \int \sec^4 x \tan^4 x\,dx = \int \sec^2 x \tan^4 x \cdot \sec^2 x\,dx$ (1)

$= \int (1+\tan^2 x)\tan^4 x \sec^2 x\,dx$ (1)

$= \int (1+u^2)u^4\,du$ ← (1) $\quad \{u = \tan x,\ du = \sec^2 x\,dx$

$= \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7}$ ← (1)

$\therefore \int_0^{\pi/4} \sec^4 x \tan^4 x\,dx = \left(\frac{\tan^5 x}{5} + \frac{\tan^7 x}{7}\right)\Big|_0^{\pi/4} = \frac{1}{5}+\frac{1}{7} = \frac{12}{35}$ (1)

verso →

6) $\int \frac{dx}{(x-1)(x-2)^2} = I$. Use Partial Fractions.

$$\frac{1}{(x-1)(x-2)^2} = \boxed{\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}} \leftarrow (1)$$

$\therefore 1 = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$

(Find A, B, C using any 3 values of x.)

(1)

say $x=1 \Rightarrow \boxed{A=1.}$ ← (1)

$x=2 \Rightarrow \boxed{C=1.}$ ← (1)

$x=0 \Rightarrow 1 = 4A + 2B + (-C) = 4 + 2B - 1 \Rightarrow \boxed{B=-1}$

$$\therefore I = \int \frac{dx}{x-1} - \int \frac{dx}{x-2} + \int \frac{dx}{(x-2)^2}$$

$$= \underbrace{\ln|x-1| - \ln|x-2| - \frac{1}{x-2} + C}_{(1)}$$

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7). $I = \int \frac{3t^3+6}{t^2(t-1)(t^2+1)}\,dt$. FORM only!

$$\frac{3t^3+6}{t^2(t-1)(t^2+1)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t-1} + \frac{Dx+E}{t^2+1}.$$

(1) (1) (1) (1) (1)

(where A, B, C, ... are to be determined).