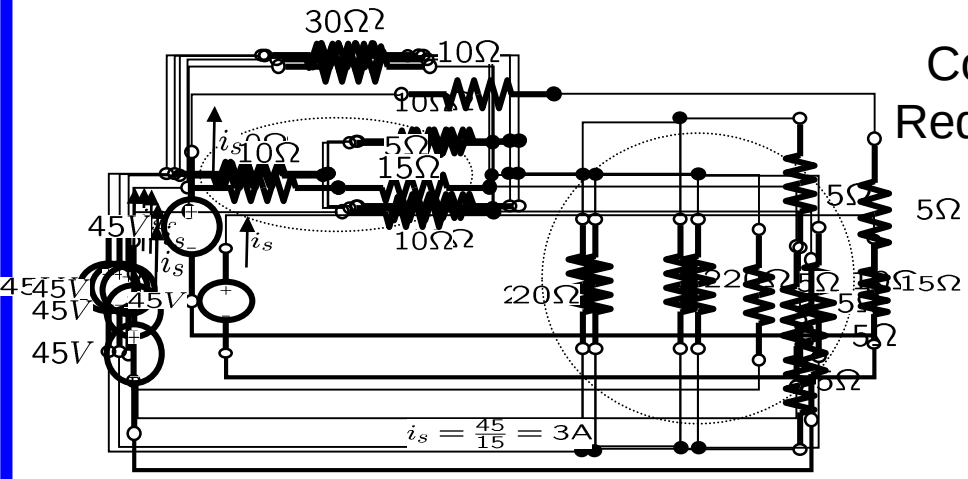
Module 3

Methods of Analysis of Resistive Circuits

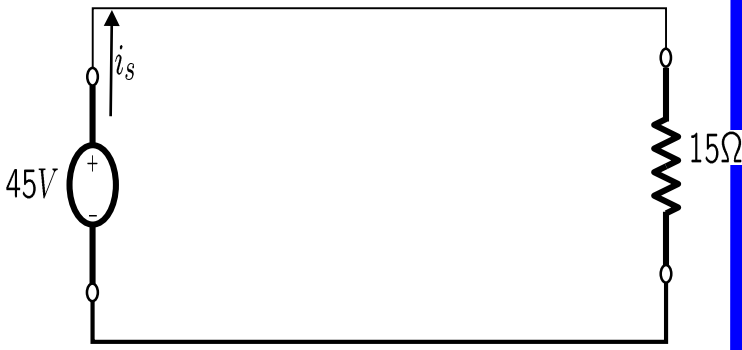
Chapter 4 of Dorf and Svoboda's book

Analysis of general resistive circuits

In the examples we had seen so far, we were able to do our circuit analysis ONLY by simplifying the circuit, using the notion of parallel and series resistors. For example,



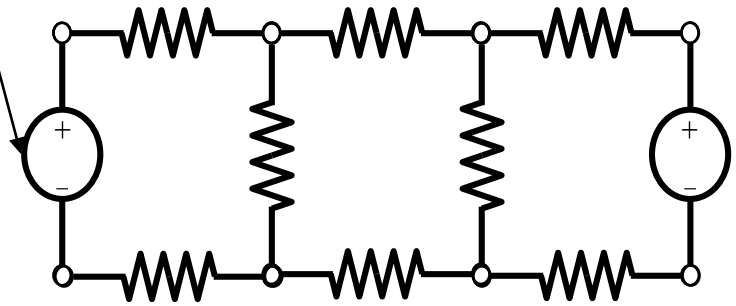
Could be Reduced by



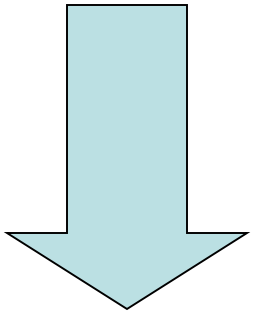
$$i_s = \frac{45}{15} = 3A$$

Not all the circuits can be treated likewise. See the following circuit as an example

This circuit does not have any two or more parallel or series resistors. Therefore, it does not allow any further simplification.



What we need for analysis is



Systematic procedure to analyze general resistive circuits

In this module we will study two main approaches

Node Voltage Analysis Method

Mesh Current Analysis Method

Our plan in this module

1

Node Voltage Analysis Method

Systematic Analysis Steps for circuits

- Independent Current Sources
- Independent Current + Voltage Sources
- Dependent Current + Voltage Sources

→→→→ Examples

2

Mesh Current Analysis Method

Systematic Analysis Steps for circuits

- Independent Voltage Sources
- Independent Current + Voltage Sources
- Dependent Current + Voltage Sources

→→→→ Examples

3

Comparison between the two methods

→ Which method would it be easier to use if we are given some circuit

Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — **Identify the number of nodes in the circuit**

Step 2 — **Designate one node as a reference node**

Step 3 — **Express current in the circuit elements in terms of node voltages**

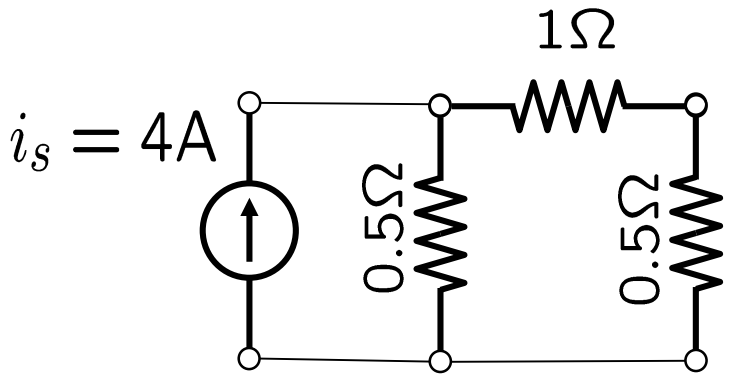
Step 4 — **Apply KCL Law at each node except the reference node**

Step 5 — **Solve the equations resulting from KCL to get node voltages**

- Direct Substitution
- Matrix Algebra

Step 6 — **Using the node voltages Find the currents in the elements**

We are going to use this Circuit as an example to illustrate the steps



Node Voltage Analysis Method

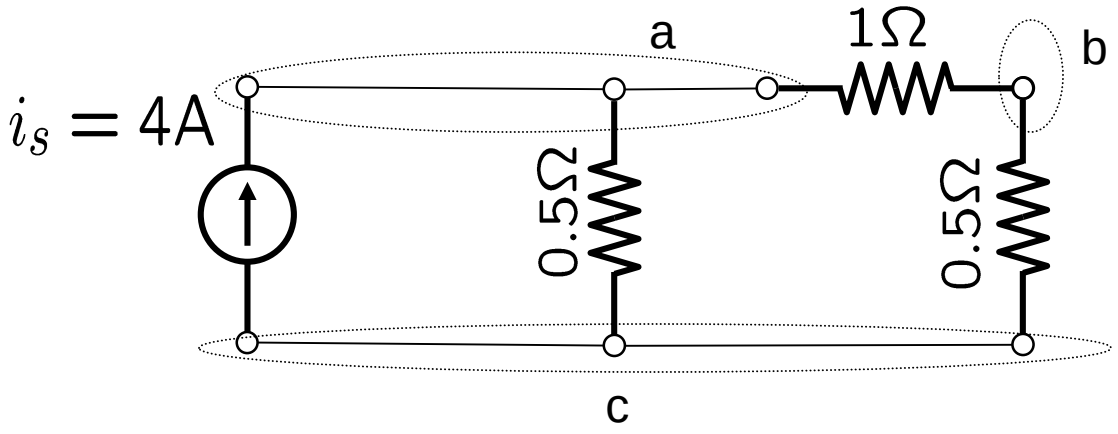
Circuits with independent Current Sources - ***Step (1/6)***

Identify the number of nodes in the circuit

Easy

Assume the number of nodes is n

In the example circuit we have 3 nodes. a, b and c.



Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — Identify the number of nodes in the circuit

Step 2 — **Designate one node as a reference node**

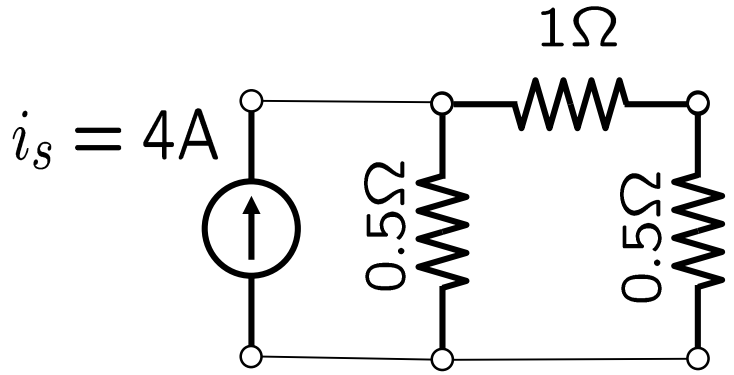
Step 3 — Express current in the circuit elements in terms of node voltages

Step 4 — Apply KCL Law at each node except the reference node

Step 5 — Solve the equations resulting from KCL to get node voltages
- Direct Substitution
- Matrix Algebra

Step 6 — Using the node voltages Find the currents in the elements

We are going to use this Circuit as an example to illustrate the steps



Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (2/6)***

Step 2

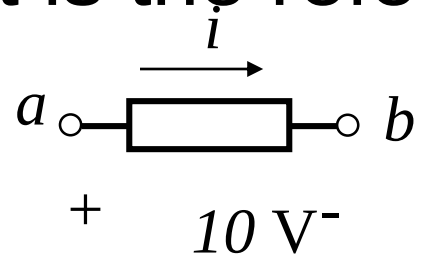
Designate one node as a reference node

What is the reference node?!

Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (2/6)***

What is the reference node?!



In case of a single element which has a voltage difference of 10 volts between its terminals as shown above, we can choose either point as a reference point. So if we choose point "**a**" as a reference point we indicate that by connecting it to the ground symbol.

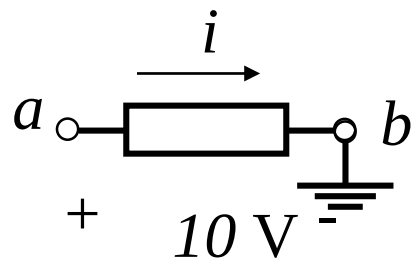
Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (2/6)***

What is the reference node?!

If point b is chosen a reference node, then one can say that node a has an absolute voltage of 10 volts when ***referred to point b***.

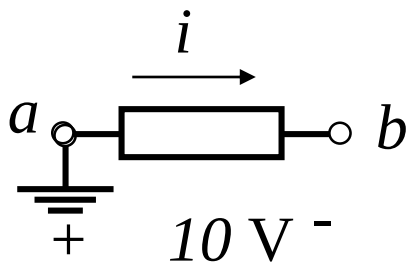
$v_a = 10$ Volts



$v_b = 0$ Volts

If point a is chosen a reference node, then one can say that node b has an absolute voltage of -10 volts when ***referred to point a.***

$v_a = 0$ Volts



$v_b = -10$ Volts

Node Voltage Analysis Method

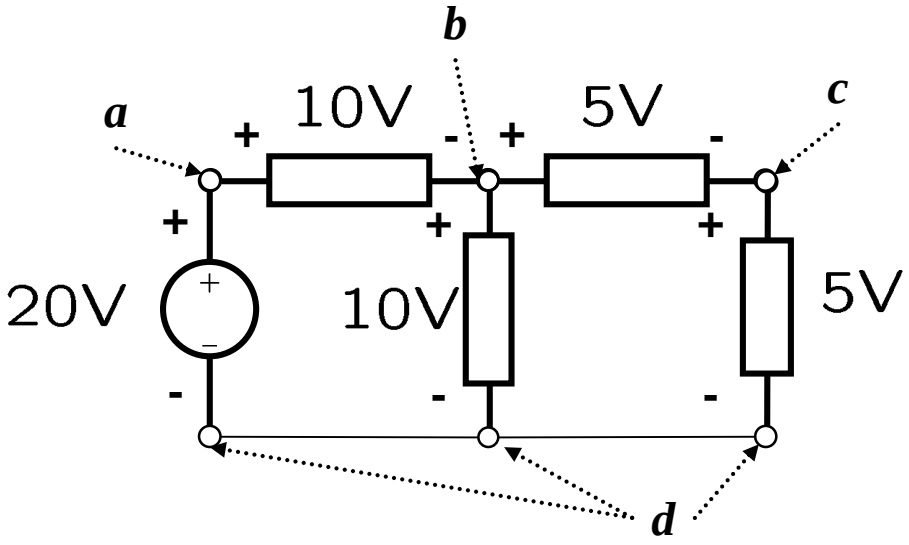
Circuits with independent Current Sources - ***Step (2/6)***

What is the reference node?!

A reference node is: A node to which every other node voltage is referred to it.

An Example

For the circuit shown →→→



Find all the node voltages assuming that one node is a reference.

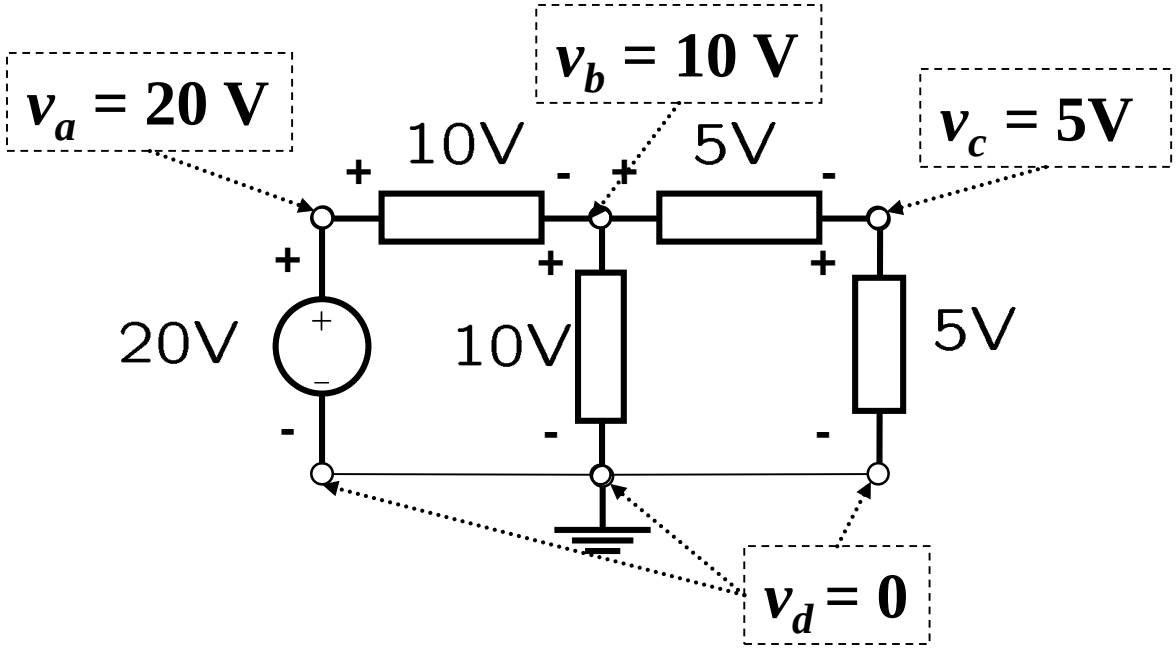
Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (2/6)***

What is the reference node?!

Solution:

1. **Assuming node *d* is the reference**



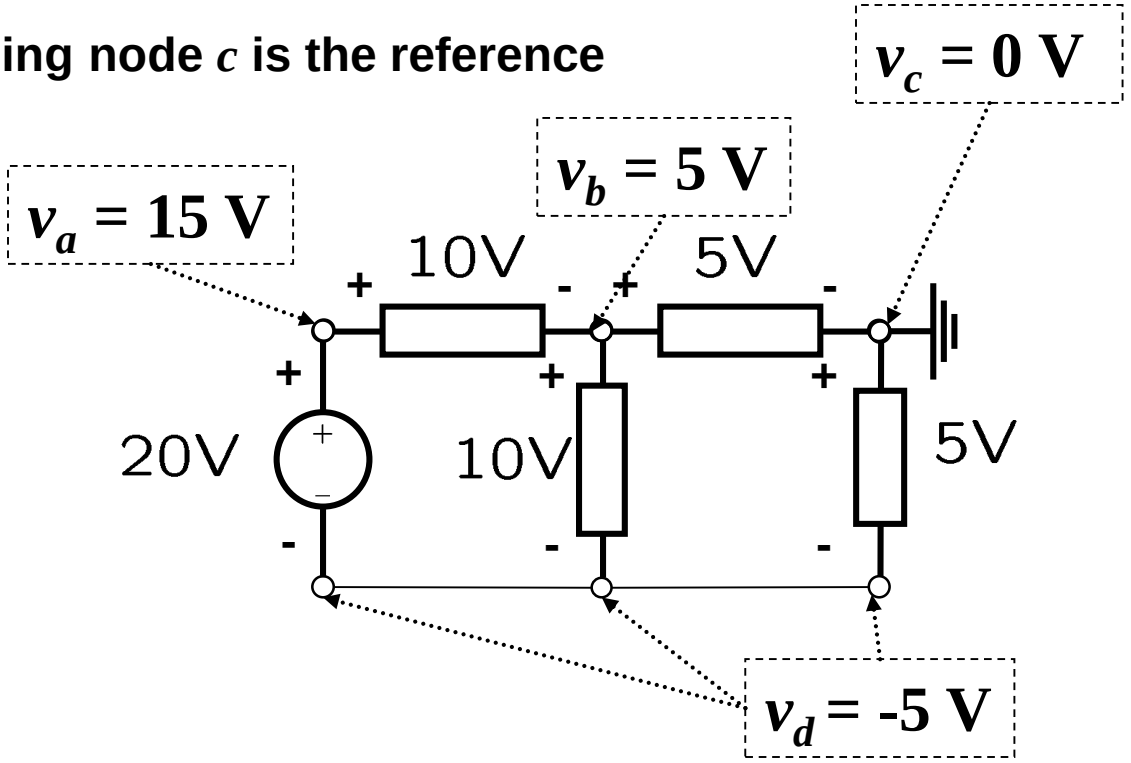
Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (2/6)***

What is the reference node?!

Solution:

2. **Assuming node c is the reference**



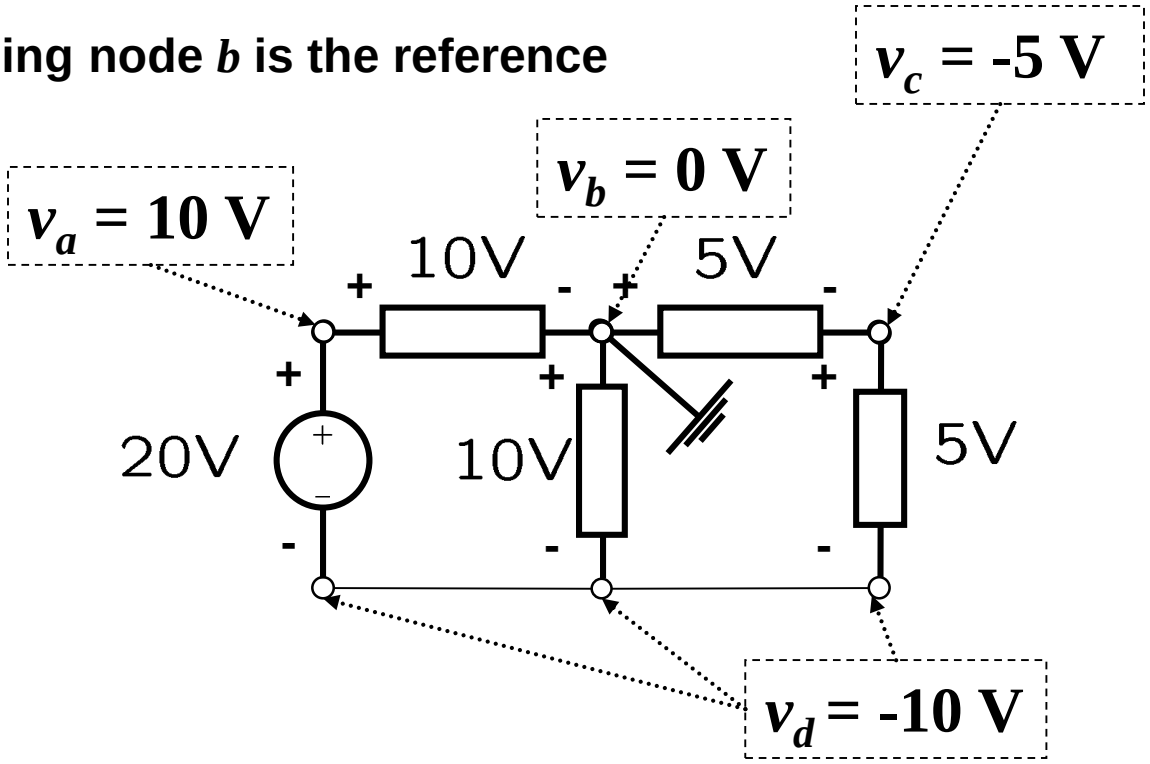
Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (2/6)***

What is the reference node?!

Solution:

3. **Assuming node *b* is the reference**



Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (2/6)***

What is the reference node?!

A good practice is to use the reference node in the negative terminal of the independent voltage source or the ingoing terminal of the independent current source. This makes analyzing circuits more convenient.

Sometimes, the problem is given where a reference node has been indicated on the circuit schematic. In this case, we will have to use the reference node as indicated.

Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — Identify the number of nodes in the circuit

Step 2 — Designate one node as a reference node

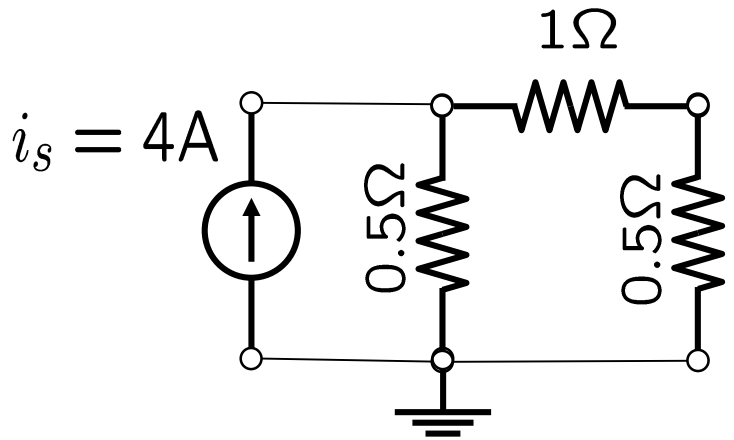
Step 3 — ***Express unknown currents in circuit elements in terms of node voltages***

Step 4 — Apply KCL Law at each node except the reference node

Step 5 — Solve the equations resulting from KCL to get node voltages
- Direct Substitution
- Matrix Algebra

Step 6 — Using the node voltages Find the currents in the elements

We are going to use this Circuit as an example to illustrate the steps



Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (3/6)***

Step 3

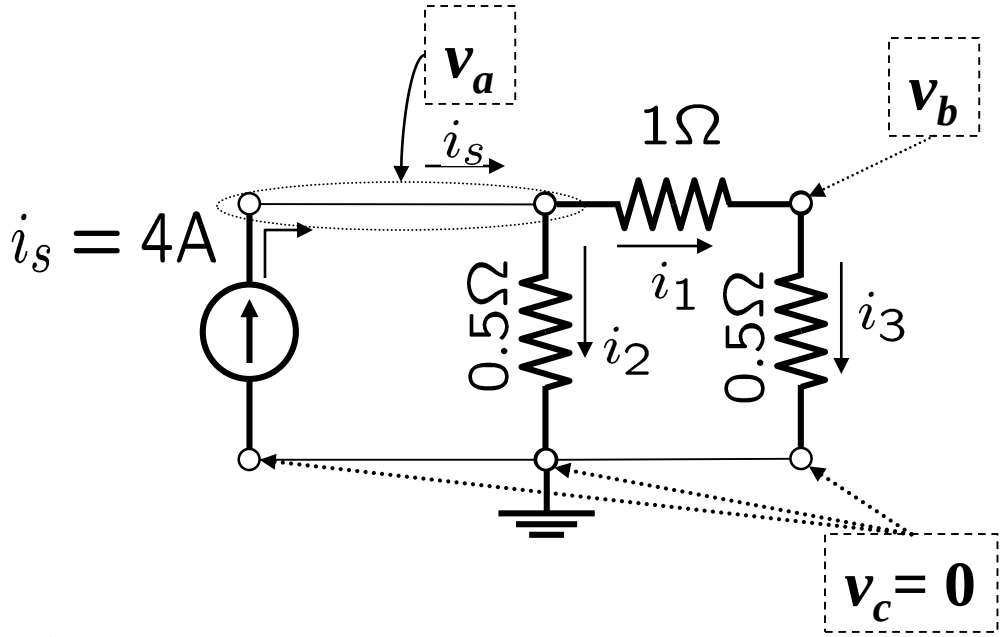
Express unknown currents in circuit elements in terms of node voltages

Unknown Currents

$i_1\ i_2\ i_3$

Node Voltages

v_a, v_b, v_c



$$i_1 = \frac{v_a - v_b}{1}$$

$$i_2 = \frac{v_a - v_c}{0.5} = \frac{v_a}{0.5}$$

$$i_3 = \frac{v_b - v_c}{0.5} = \frac{v_b}{0.5}$$

Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — Identify the number of nodes in the circuit

Step 2 — Designate one node as a reference node

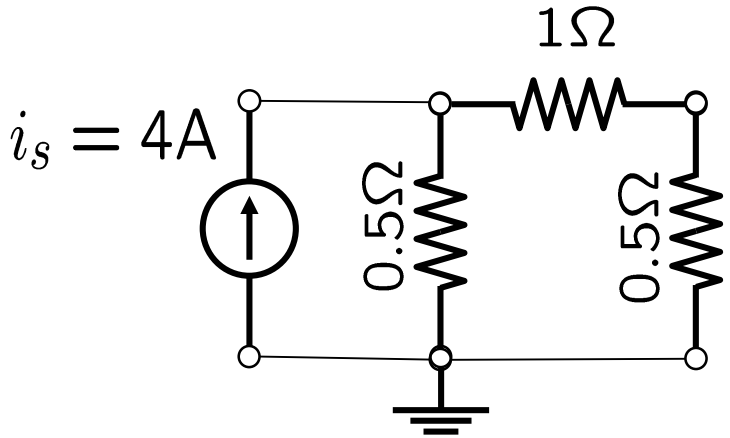
Step 3 — Express unknown currents in circuit elements in terms of node voltages

Step 4 — ***Apply KCL Law at each node except the reference node***

Step 5 — Solve the equations resulting from KCL to get node voltages
- Direct Substitution
- Matrix Algebra

Step 6 — Using the node voltages Find the currents in the elements

We are going to use this Circuit as an example to illustrate the steps

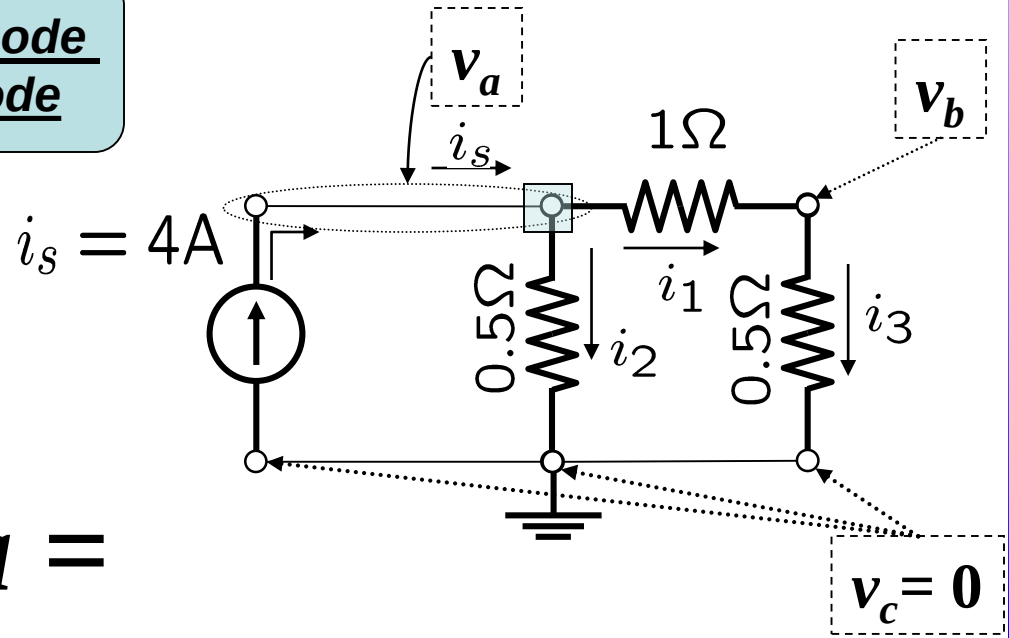


Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (4/6)***

Step 4

Apply KCL Law at each node except the reference node



KCL at node a

$$\sum \textit{Currents entering } a = \sum \textit{Currents exiting from } a$$

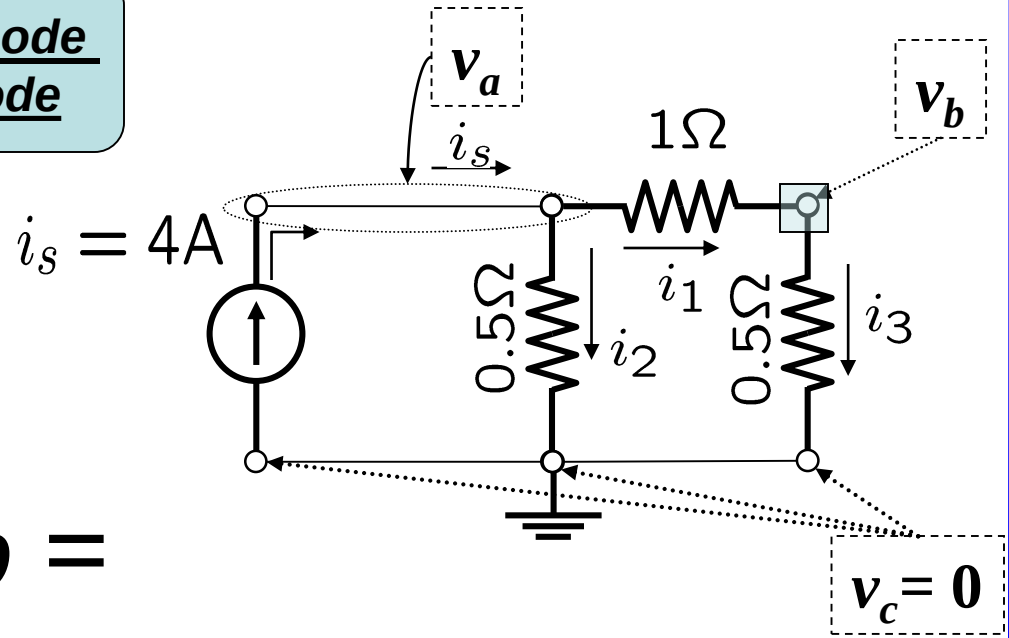
$$i_s = i_1 + i_2$$

Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (4/6)***

Step 4

Apply KCL Law at each node except the reference node



KCL at node a

$$\sum \textit{Currents entering } b = \sum \textit{Currents exiting from } b$$

$$i_1 = i_3$$

Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — Identify the number of nodes in the circuit

Step 2 — Designate one node as a reference node

Step 3 — Express unknown currents in circuit elements in terms of node voltages

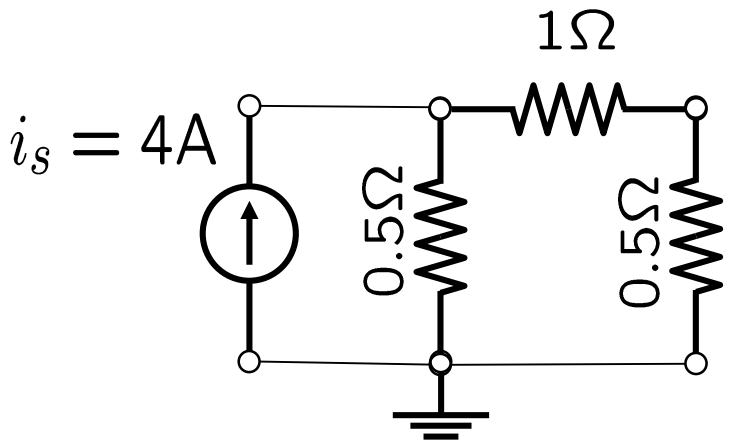
Step 4 — Apply KCL Law at each node except the reference node

Step 5 — ***Solve the equations resulting from KCL to get node voltages***

- Direct Substitution
- Matrix Algebra

Step 6 — Using the node voltages Find the currents in the elements

We are going to use this Circuit as an example to illustrate the steps



Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (5/6)***

Step 5

Solve the equations resulting from KCL to get node voltages → Direct Substitution

$$i_s = i_1 + i_2$$

$$i_1 = i_3$$

$$4 = \frac{v_a - v_b}{1} + \frac{v_a}{0.5}$$

$$\frac{v_a - v_b}{1} = \frac{v_b}{0.5}$$

$$4 = 3v_a - v_b$$

$$v_a = 3v_b$$

From ***Step 3, we have***

$$i_1 = \frac{v_a - v_b}{1}$$

$$i_2 = \frac{v_a - v_c}{0.5} = \frac{v_a}{0.5}$$

$$i_3 = \frac{v_b - v_c}{0.5} = \frac{v_b}{0.5}$$

$$i_s = 4A$$

Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (5/6)***

Step 5

Solve the equations resulting from KCL to get node voltages → Direct Substitution

$$4 = 3v_a - v_b$$

$$v_a = 3v_b$$

$$4 = 3 \times 3v_b - v_b$$

$$v_b = \frac{1}{2}\ \mathrm{V} \quad \Rightarrow \quad v_a = \frac{3}{2}\mathrm{V}$$

Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — Identify the number of nodes in the circuit

Step 2 — Designate one node as a reference node

Step 3 — Express unknown currents in circuit elements in terms of node voltages

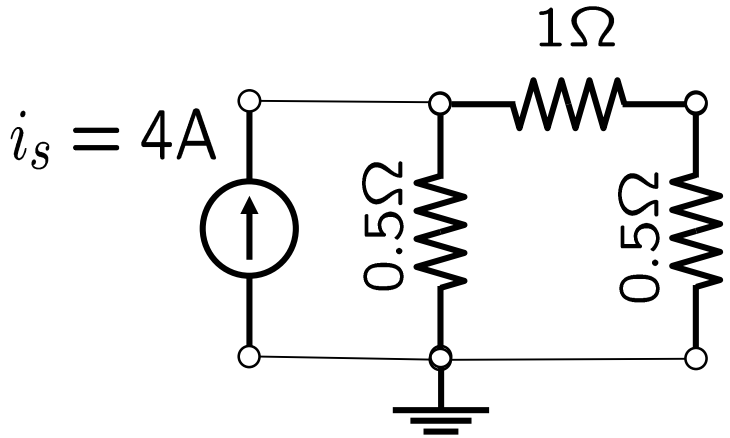
Step 4 — Apply KCL Law at each node except the reference node

Step 5 — Solve the equations resulting from KCL to get node voltages

- Direct Substitution
- Matrix Algebra

Step 6 — ***Using the node voltages Find the currents in the elements***

We are going to use this Circuit as an example to illustrate the steps



Node Voltage Analysis Method

Circuits with independent Current Sources - ***Step (6/6)***

Step 6

Using the node voltages
Find the currents in the elements

From ***Step 3, we have***

$$i_1 = \frac{v_a - v_b}{1}$$

$$i_2 = \frac{v_a - v_c}{0.5} = \frac{v_a}{0.5}$$

$$i_3 = \frac{v_b - v_c}{0.5} = \frac{v_b}{0.5}$$

From ***Step 5, we have***

$$v_a = \frac{3}{2}\text{V}$$

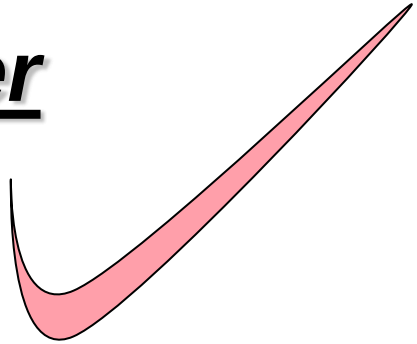
$$v_b = \frac{1}{2}\text{ V}$$

$$i_1 = 1\text{A}$$

$$i_2 = 3\text{A}$$

$$i_3 = 1\text{A}$$

Checking your answer



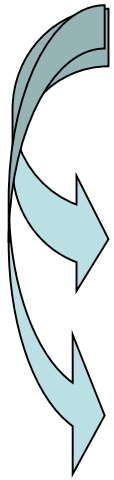
You can also

VERIFY

Your solution!!!

How to verify your solution??

The solution has to satisfy:



KCL at every node

KVL for every loop

How to verify your solution??

$v_a = \frac{3}{2}\text{V}$

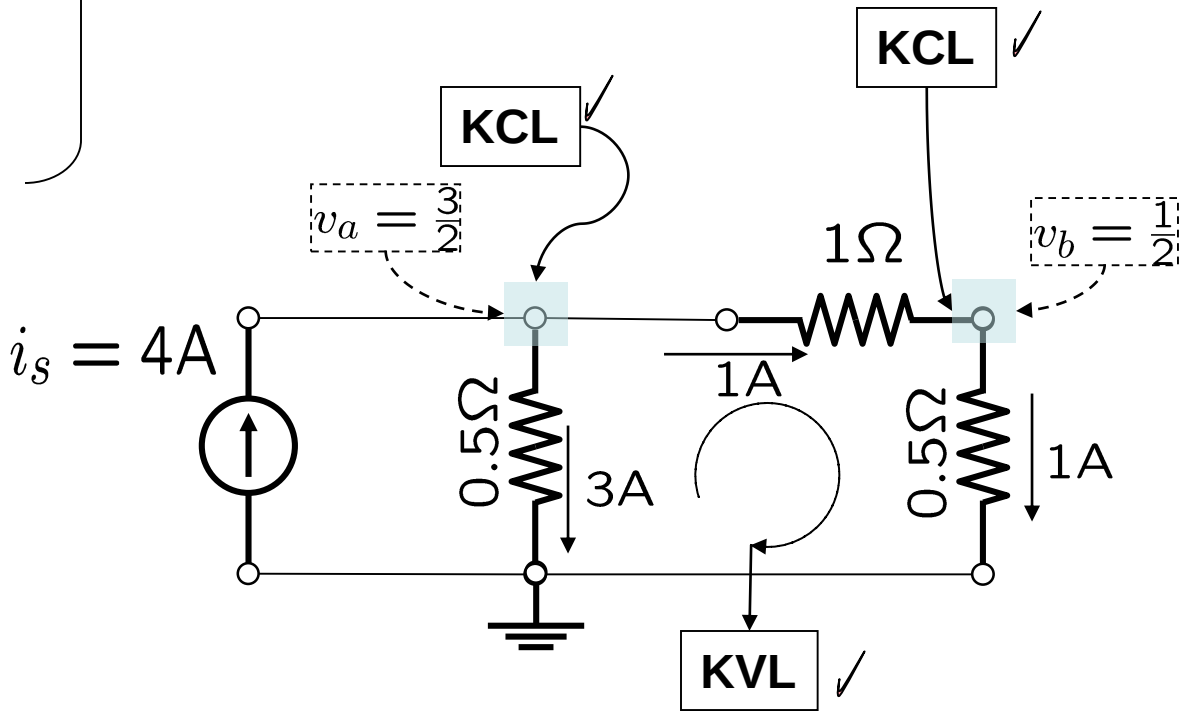
$v_b = \frac{1}{2}\text{ V}$

$i_1 = 1\text{A}$

$i_2 = 3\text{A}$

$i_3 = 1\text{A}$

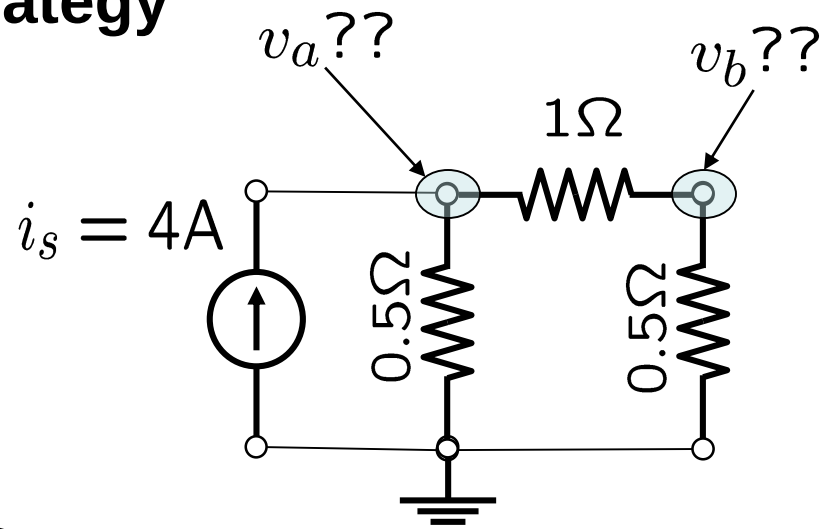
Solution of the previous example



Node Voltage Analysis Method

WHY and HOW does it work ??

Node Voltage Analysis Method
WHY and HOW?
The basic strategy



Consider node voltages as unknowns except for the reference node

To solve for a number of unknowns we need the same number of equations

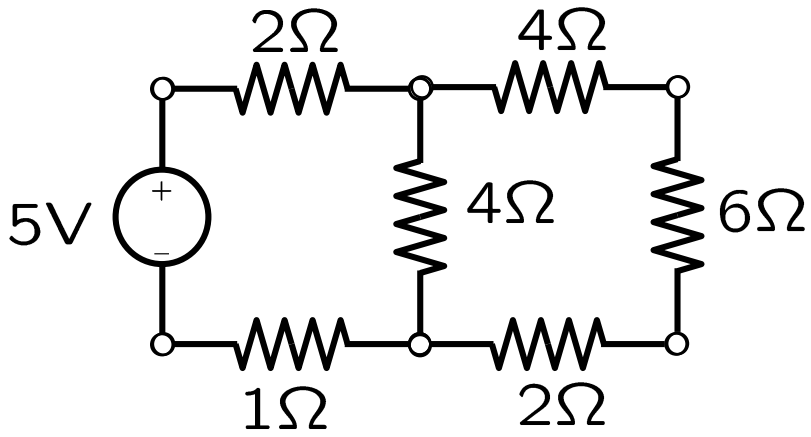
Generate the required equations by applying KCL at each node

Solve the resulting equations to obtain the unknown nodes voltages

Using Calculated Nodes voltages, calculate the currents

Another Example

Use the node voltage method to calculate all voltages and currents in the circuit shown below.



Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — **Identify the number of nodes in the circuit**

Step 2 — **Designate one node as a reference node**

Step 3 — **Express current in the circuit elements in terms of node voltages**

Step 4 — **Apply KCL Law at each node except the reference node**

Step 5 — **Solve the equations resulting from KCL to get node voltages**

- Direct Substitution
- Matrix Algebra

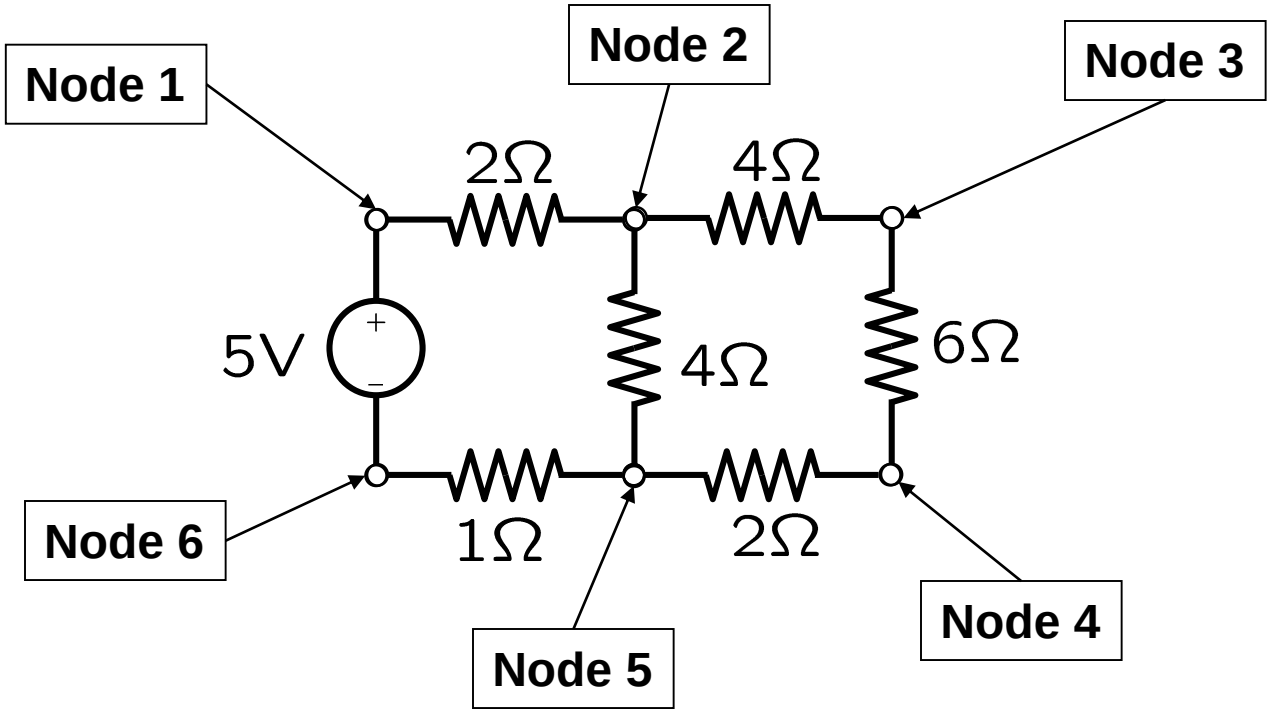
Step 6 — **Using the node voltages Find the currents in the elements**

Node Voltage Analysis Method

Step (1/6)

Step 1

Identify the number of nodes in the circuit



Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — **Identify the number of nodes in the circuit**

Step 2 — **Designate one node as a reference node**

Step 3 — **Express current in the circuit elements in terms of node voltages**

Step 4 — **Apply KCL Law at each node except the reference node**

Step 5 — **Solve the equations resulting from KCL to get node voltages**

- Direct Substitution
- Matrix Algebra

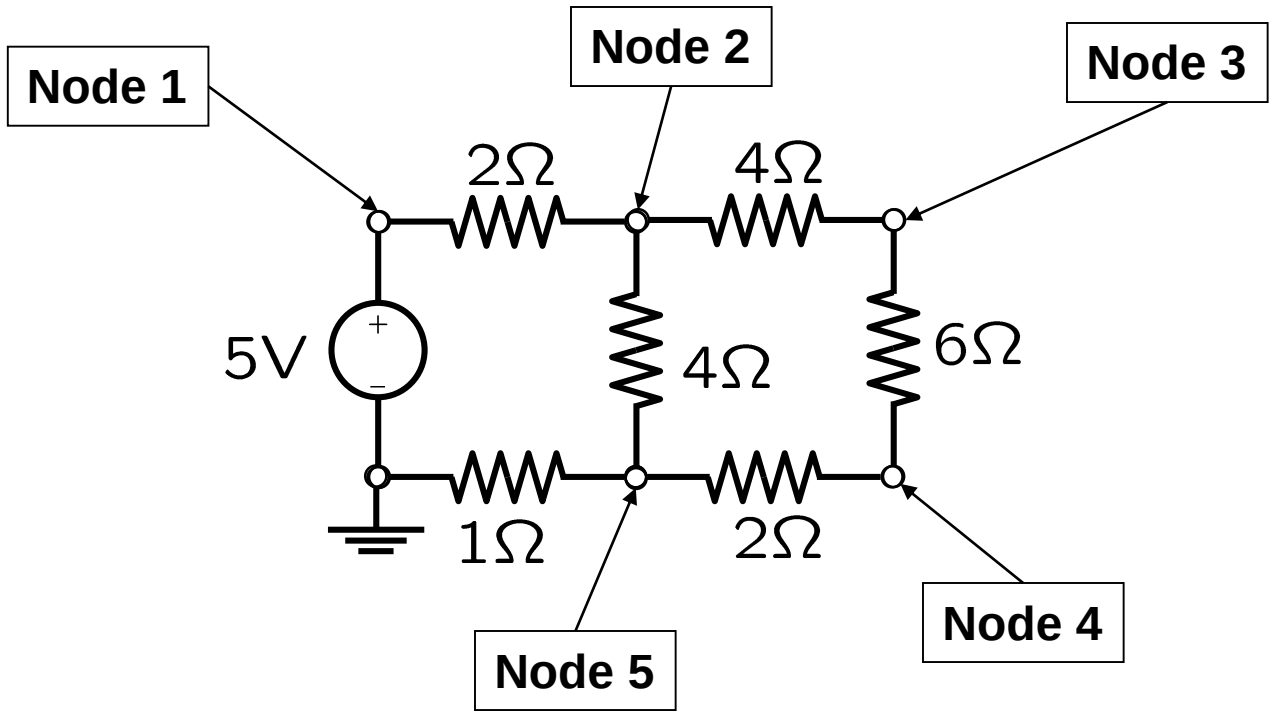
Step 6 — **Using the node voltages Find the currents in the elements**

Node Voltage Analysis Method

Step (2/6)

Step 2

Designate one node as a reference node



Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — **Identify the number of nodes in the circuit**

Step 2 — **Designate one node as a reference node**

Step 3 — **Express current in the circuit elements in terms of node voltages**

Step 4 — **Apply KCL Law at each node except the reference node**

Step 5 — **Solve the equations resulting from KCL to get node voltages**

- Direct Substitution
- Matrix Algebra

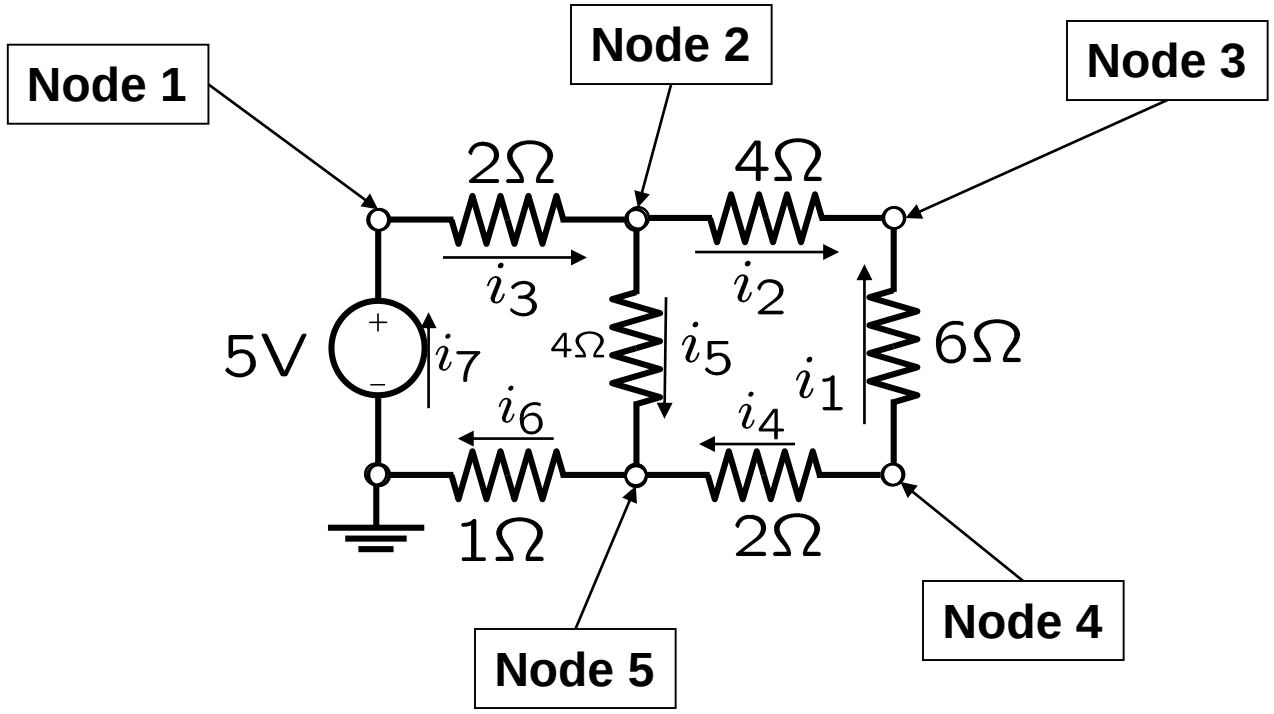
Step 6 — **Using the node voltages Find the currents in the elements**

Node Voltage Analysis Method

Step (3/6)

Step 3

Express current in the circuit elements in terms of node voltages

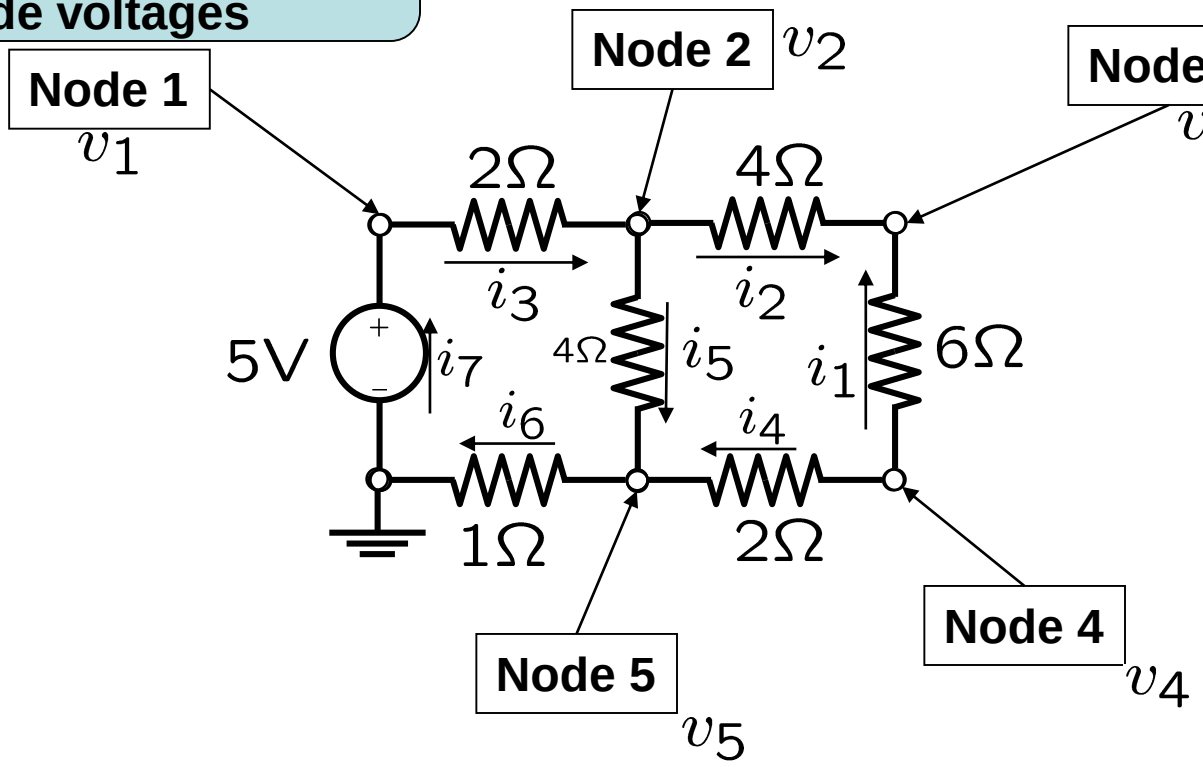


Node Voltage Analysis Method

Step (3/6)

Step 3

Express current in the circuit elements in terms of node voltages



$$i_1 = \frac{v_4 - v_3}{6}$$

$$i_2 = \frac{v_2 - v_3}{4}$$

$$i_3 = \frac{v_1 - v_2}{2}$$

$$i_4 = \frac{v_4 - v_5}{2}$$

$$i_5 = \frac{v_2 - v_5}{4}$$

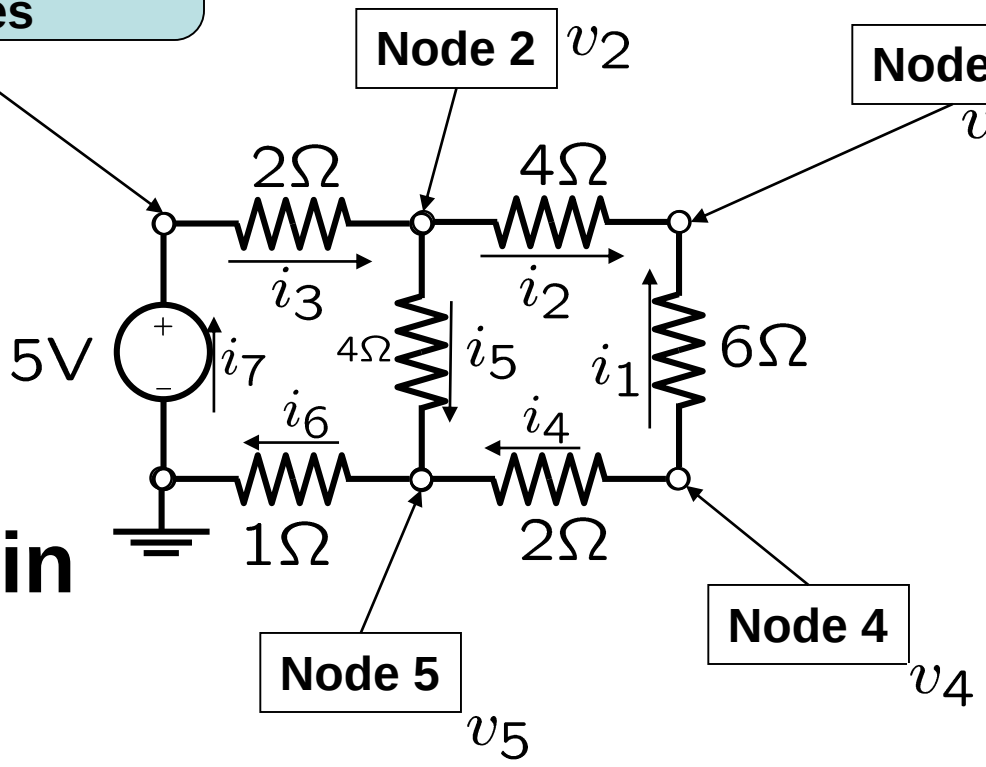
$$i_6 = \frac{v_5 - 0}{1}$$

Node Voltage Analysis Method

Step (3/6)

Step 3

Express current in the circuit elements in terms of node voltages



i_7

Can not be written in terms of node voltages

Node Voltage Analysis Method
Circuits with independent Current Sources

Step 1 — **Identify the number of nodes in the circuit**

Step 2 — **Designate one node as a reference node**

Step 3 — **Express current in the circuit elements in terms of node voltages**

Step 4 — **Apply KCL Law at each node except the reference node**

Step 5 — **Solve the equations resulting from KCL to get node voltages**

- Direct Substitution
- Matrix Algebra

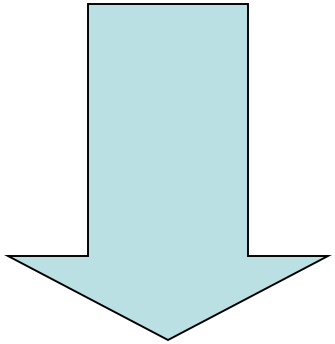
Step 6 — **Using the node voltages Find the currents in the elements**

These steps will

FAIL

if we have independent Voltage Source

Node Voltage Analysis Method
Circuits with independent Current Sources



Should be modified to handle independent Voltage Source

Our plan in this module

1

Node Voltage Analysis Method

Systematic Analysis Steps for circuits

- **Independent Current Sources**
- **Independent Current + Independent Voltage Sources**
- **Dependent Current + Voltage Sources**

→→→→ Examples

2

Mesh Current Analysis Method

Systematic Analysis Steps for circuits

- **Independent Voltage Sources**
- **Independent Current + Voltage Sources**
- **Dependent Current + Voltage Sources**

→→→→ Examples

3

Comparison between the two methods

→ Which method would it be easier if we are given some circuit