Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2014

TEST 3

STUDIO 56 calculator ONLY permitted, 1 or more blank sheets permitted for roughs

Print Name : _____

Student Number: _____

Tutorial Section (A1, A2, A3, A4, or A5): _____

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** If $\log_{10} x = y$ what is x ?
   (a) $x = y^{10}$, (b) $x = 10$, (c) $x = e^y$, (d) $x = 10^y$, e) none of these

   (by definition)

2. **[2 marks]** Given that $y = 2^{x^2+1}$ calculate $\log_3 \frac{y}{2}$.
   (a) $\log_3(x^2+1)$, (b) $x^2 \log_3 2$, (c) $3^{y/2}$, (d) $(x^2+1)\log_2 3$, e) none of these

   $\frac{y}{2} = 2^{x^2} \Rightarrow \log_3\left(\frac{y}{2}\right) = \log_3(2^{x^2}) = x^2 \log_3 2$

3. **[2 marks]** Let $f(x) = e^{-\cos x}$. Find $f'(\pi/2)$, that is find the derivative of f at $x = \pi/2$.
   (a) 1, (b) -1, (c) 2, (d) π,

   $f'(x) = e^{-\cos x}(\sin x) \therefore f'(\pi/2) = 1 \cdot e^0 = 1$

4. **[2 marks]** Let $f(x) = (2^{3x})^{-1}$. Find $f'(0)$, that is find the derivative of f at $x = 0$.
   (a) $2\ln 3$, (b) $-3\ln 2$, (c) 1, (d) 0, e) none of these.

   $f(x) = 2^{-3x} \Rightarrow f'(x) = 2^{-3x} \ln 2 \cdot (-3) \Rightarrow f'(0) = -3\ln 2$

5. **[2 marks]** Let $f(x) = x(x^2-1)$. Then $f(x) < 0$ only when $0 < x < 1$.
   (a) TRUE, (b) FALSE,

   Also < 0 when $x < -1$.

PART II: Show all work here and give details. No additional pages will be accepted

6. **[5+5 marks]** a) Let $f(x) = x^{x^2}$. Evaluate $f'(1)$.
   b) Let f be defined by $f(x) = \log_2(3x+1)$. Evaluate the derivative of f at any point where $x > -1/3$.

a) $f(x) = e^{\ln(x^{x^2})} = e^{x^2 \ln x}$ (1) $\therefore f'(x) = e^{x^2\ln x} D(x^2 \ln x) = e^{x^2\ln x}\cdot\left(x^2\cdot\frac{1}{x} + 2x\ln x\right)$ (1)

$\therefore f'(1) = e^{1^2\ln 1}(1 + 2\cdot 1\cdot \ln 1) = e^0 = 1$ (2)

OR

$\ln f(x) = x^2 \ln x$ (1)

$\frac{1}{f(x)} f'(x) = x^2\cdot\frac{1}{x} + 2x\ln x = x + 2x\ln x$ (1)

$\therefore f'(1) = f(1)(1 + 2\cdot 1\cdot \ln 1) = f(1) = 1^1 = 1$ (2)

b) $D(\log_a \square) = \frac{D\square}{\square \ln a}$ (2)

$\therefore D(\log_2(3x+1)) = \frac{D(3x+1)}{(3x+1)\ln 2} = \frac{3}{(\ln 2)(3x+1)}$, $x > -1/3$.

(2) (1)

7. [5+5 marks]

a) • Solve the polynomial inequality $f(x) = x^3(x - 1 - x^2)(x^2 - 4) < 0$ for $-\infty < x < +\infty$.

b) • Solve the rational function inequality $\dfrac{3x}{x^2 - 1} > 0$ for $-\infty < x < +\infty$.

a). Break points at $x = 0, -2, +2$. as $x - 1 - x^2 < 0$ all x

	x^3	$x-2$	$x+2$	$-1+x-x^2$	$f(x)$ sign.
$(-\infty, -2)$	−	−	−	−	+
$(-2, 0)$	−	−	+	−	−
$(0, 2)$	+	−	+	−	+
$(2, \infty)$	+	+	+	−	−

$\therefore f(x) < 0$ when x is in either $(-2, 0)$ or $(2, \infty)$.

b). Break pts at $x = 0, \pm 1$. Let $r(x) = \dfrac{3x}{x^2 - 1}$.

	$3x$	$x-1$	$x+1$	Sign of $r(x)$.
$(-\infty, -1)$	−	−	−	−
$(-1, 0)$	−	−	+	+
$(0, 1)$	+	−	+	−
$(1, \infty)$	+	+	+	+

So $r(x) > 0$ when x is in either $(-1, 0)$ or $(1, \infty)$.