Consider the following Linear Programming problem, in which X and Y denote the number of units of products X and Y to be produced, respectively, and Z denotes the overall resulting profit

Objective Function

Maximize Z = $4X + $5Y

Subject To

$X + 2Y \leq 10$ (labor available, in hours)

$6X + 6Y \leq 36$ (material available, in pounds)

$8X + 4Y \leq 40$ (storage available, in square feet)

$X, Y \geq 0$

The Excel Sensitivity Report for this problem is given below. Answer the following questions and provide calculations of your answers. Each question is independent of the others.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C5	X	2	0	4	1	1.5
D5	Y	4	0	5	3	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E7	Labor	10	1	10	2	2
E8	Material	36	0.5	36	4	6
E9	Storage	32	0	40	1E+30	8

a) How much excess labor and storage capacity there are in the optimal solution?
b) What would be the company's total profit if it had 5 additional pounds of material?
c) How does the total profit change if you give up 1 hour of labor?
d) How does the total profit change if the profit contribution of product X was changed to $4.75?
e) Would the optimal solution remain the same if the profit contribution of product Y was changed to $2.5?
f) How does the total profit change if you decided to introduce a new product that has a profit contribution of $2 per unit, given that each unit of this product will use 1 pound of material?

a) Solution: Excess Labor = 10 – 2 – 2x4 = 0 **[1 mark]**

Excess storage = 40 – 8x2 – 4x4 = 8 **[1 mark]**

b) Solution: Maximum allowable increase in material is 4 pounds. 5 additional pounds of material is an infeasible increase outside of the sensitivity range. **[1 mark]**
In order to calculate how the profit would change with 5 additional pounds of material, the problem needs to be solved again with the material constraint set to 41. **[1 mark]**

c) Solution: The changes in the labor constraint is within the allowable decrease rang, so the shadow price still applies. **[1 mark]**

The profit will change by -1 × $1 = - $1. The profit will drop by $1. **[1 mark]**

d) Solution: The changes in the objective coefficient is within the sensitivity range (less than 1). So the optimal solution remains unchanged. **[1 mark]**

The profit would be 4.75 × 2 + 5 × 4 = $29.5, an increase of $1.5. **[1 mark]**

e) Solution: No. **[1 mark]**
The optimal solution would change because the change in objective coefficient exceeds allowable decrease. **[1 mark]**

f) Solution: The material requirement of the new product is considered a decrease in the available material for products X and Y. A decrease of 1 pound of material is within the allowable decrease in the material constraint. So the shadow prices still apply. **[1 mark]**

Profit Change = Additional profit from new product - shadow price decrease = $2 – 1 × 0.5 = 0.5. The profit would increase by $1.5 **[1 mark]**