Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2013
TEST 3

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name : _____

Student Number: _____

Tutorial Section (A1, A4, ...): _____

PART I: Multiple Choice Questions
(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Let $f(x) = e^{-x^2}$. Then f has a relative maximum at:
   (a) $x = 0$, (b) $x = 1$, (c) $x = -1$, (d) $x = 2$, e) none of these.

2. **[2 marks]** Find all the critical points of the function $f(s) = \dfrac{2s}{s^2+4}$
   (a) $s = 1/2$ only, (b) $s = \pm 1$, (c) $s = \pm 2$, (d) $s = 0$ only, e) none of these

3. **[2 marks]** What is the most general antiderivative of the function $f(x) = 2xe^{x^2}$?
   (a) $xe^{x^2} + C$, (b) $e^{x^2} + C$, (c) $x^2 + C$, (d) e^{-x^2}, e) none of these

4. **[2 marks]** The function $f(x) = xe^{-x}$ has a horizontal asymptote given by $y = L$ where L is equal to:
   (a) 1, (b) e^{-1}, (c) e, (d) 0, e) none of these

5. **[2 marks]** Let $f(x) = x^4 + x - 1$. Then f has a point of inflection at $x = 0$.
   (a) TRUE, (b) FALSE,

PART II: Show all work here and give details.
No additional pages will be accepted

6. **[5+5 marks]** a) Evaluate the definite integral $\displaystyle\int_0^{\pi/2} \sqrt{\sin x}\,\cos x\,dx$. $= I$.

   a) Evaluate the definite integral $\displaystyle\int_0^1 \frac{t^2}{1+t^3}\,dt$. $= J$

a) $\square = \sin x$, $\frac{d\square}{dx} = \cos x$ (1)

$$\int_0^{\pi/2} \sqrt{\sin x}\cos x\,dx = \int \sqrt{\square}\cdot\frac{d\square}{dx}\,dx = \int \square^{1/2}\,d\square \quad (1)$$

$$= \frac{2}{3}\square^{3/2} = \frac{2}{3}(\sin x)^{3/2} = F(x)$$

is an antiderivative

$$\therefore I = F(\pi/2) - F(0) = \frac{2}{3}(\sin\pi/2)^{3/2} - \frac{2}{3}(\sin 0)^{3/2}$$

$$= \frac{2}{3} - 0 = \boxed{\frac{2}{3}} \quad (2)$$

b) $J = F(1) - F(0)$ where F is an antiderivative of $\frac{t^2}{1+t^3}$.

how $\square = 1+t^3$ (1)

$\frac{d\square}{dt} = 3t^2 \Rightarrow t^2 = \frac{1}{3}\frac{d\square}{dt}$ (1)

$$\therefore F(t) = \int \frac{1}{3}\frac{1}{\square}\cdot\frac{d\square}{dt}\,dt$$

$$= \frac{1}{3}\int \frac{1}{\square}\frac{d\square}{dt}\,dt \quad (1)$$

$$= \frac{1}{3}\ln|\square| = \boxed{\frac{1}{3}\ln(1+t^3)}$$

$$\therefore J = \frac{1}{3}\ln(1+t^3)\Big|_0^1 = \frac{1}{3}\ln 2 - \frac{1}{3}\ln 1 = \boxed{\frac{1}{3}\ln 2} \quad (2)$$