MATH1007 A – Test 3 – 1:35 pm - 2:25 pm, Nov. 9

Name: Student No.: Tutor. sect'n (PMC students only):

Total: 15 marks (for 7 questions). Closed book, no calculator! You may write on **both sides**.

1. [1 pt] Multiple choice. No partial marks.

(i) Let $f(x) = \arctan x$. Find $f'(1) =$. (a) $\frac{1}{2}$, (b) $-\frac{1}{2}$, (c) $\frac{1}{4}$, (d) $-\frac{1}{4}$, (e) $\frac{\pi}{4}$

[0.5]

(ii) Let $f(x) = 2x - x^{\frac{3}{2}}$. Then $\lim_{x\to\infty} f'(x) =$. (a) ∞, (b) $-\infty$, (c) 2 (d) $-\frac{3}{2}$

[0.5]

2. [2] Let $f(x) = \frac{x+1}{x^2+1} + x$. Find (i) $f'(x)$ and $f'(1)$; (ii) the slope of the normal line of the curve at $x = 1$.

(i) $f'(x) = \dfrac{-x^2-2x+1}{(x^2+1)^2} + 1$ [1]

$f'(1) = \dfrac{-2}{4} + 1 = \frac{1}{2}$ [0.5]

(ii) Let k be the slope of the normal line.

$k \cdot f'(1) = -1 \Rightarrow k = -2$. [0.5]

3. [3] Differentiate the following functions:

(i) $f(x) = e^{2\sin(2x)}$

$f'(x) = \underbrace{e^{2\sin(2x)} \cdot (2\sin 2x)'}_{[1]} = \underbrace{4\cos(2x) \cdot e^{2\sin(2x)}}_{[0.5]}$

(ii) $f(x) = \cos\sqrt{x+1}$

$f'(x) = \underbrace{(-\sin\sqrt{x+1})(\sqrt{x+1})'}_{[1]} = \dfrac{-\sin\sqrt{x+1}}{2\sqrt{x+1}}$ [0.5]

4. [2] Let $f(x) = xe^x$. Find $f'(x)$ and the second derivative $f''(x)$.

$f'(x) = (x)'e^x + x(e^x)' = e^x + xe^x$

$= (1+x)e^x$ [1]

$f''(x) = 1 \cdot e^x + (1+x)(e^x)'$

$= (2+x)e^x$ [1]

5. [3] Let the curve C be determined by the equation $x^2 + 2xy + 2y^2 = 5$. Find the tangent at the point $(1, 1)$. Hint: implicit differentiation.

$2x + 2(y + xy') + 4yy' = 0$ [1.5]

$\Rightarrow x + y + xy' + 2yy' = 0$

$\Rightarrow y' = -\dfrac{x+y}{x+2y}$ [0.5]

At $(1,1)$, $y' = -\frac{2}{3}$. [0.5]

The tangent:

$y - 1 = -\frac{2}{3}(x-1)$. [0.5]

6. [1] Evaluate the approximate value of $\sqrt{4.1}$ (give detail).

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(4) = \frac{1}{2}\frac{1}{\sqrt{4}} = \frac{1}{4}$

$\sqrt{4.1} = \underbrace{f(4) + f'(4)\cdot(0.1)}_{[0.5]} = 2 + \frac{1}{4}(0.1) = 2.025$

7. [3] Consider $f(x) = x^3 - 3x^2 + 1, 1 \le x \le 3$. Find (i) the critical point(s) on $[1, 3]$; (ii) the global maximum and minimum.

(i) $f'(x) = 3x^2 - 6x = 0$ [0.5]

[1] $\Rightarrow x^2 - 2x = 0$ $x = 0$ or $x = 2$. [0.5]

The critical point is $x = 2$. [0.5]

(ii) $\underbrace{\{f(1), f(2), f(3)\}}_{[0.5]} = \{-1, -3, 1\}$

g. max $= 1$ [0.5]

g. min $= -3$ [0.5]