UNIVERSITY OF CALGARY

11 Dec 2007

FINAL EXAMINATION
(Fall, 2007)
ENGG 201 - Behaviour of Liquids, Gases and Solids

December 13, 2007 Time Allowed: 180 minutes.

1. Attempt all six (6) questions. Marks are noted in the left page margin. Total marks = 100.
2. Only sanctioned calculators are permitted. <u>Notes and textbooks are not allowed.</u>
3. A separate handout listing formulas and charts is provided.
4. You must show all of your work and specify units in order to obtain full marks.
5. Print your name and lecture section clearly in the space provided on the cover page. Write your ID No. on all other pages in the provided space.

Surname: [redacted] **Given Name(s):** [redacted] **Section*** [redacted]

***L01** (Dr. Sen); **L02** (Dr. Javadpour); **L03** (Dr. Abedi).

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an acceptable alternative I.D. e.g. one with a printed name and photograph, are allowed to write the examination.

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(e) attempting to read other student's examination papers.
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(8) The candidate is to write his/her name on each answer book as directed and is to number each book.
(9) A candidate must report to a supervisor before leaving the examination room.
(10) Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
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Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such a request will be denied.
(12) Smoking during examinations is strictly prohibited.

RO 93-02

Question No.	Maximum Mark	Mark Obtained
I	16	13
II	20	17
III	16	14.5
IV	16	6
V	16	14
VI	16	16
Total (out of 100)		~~78.5~~

80.5

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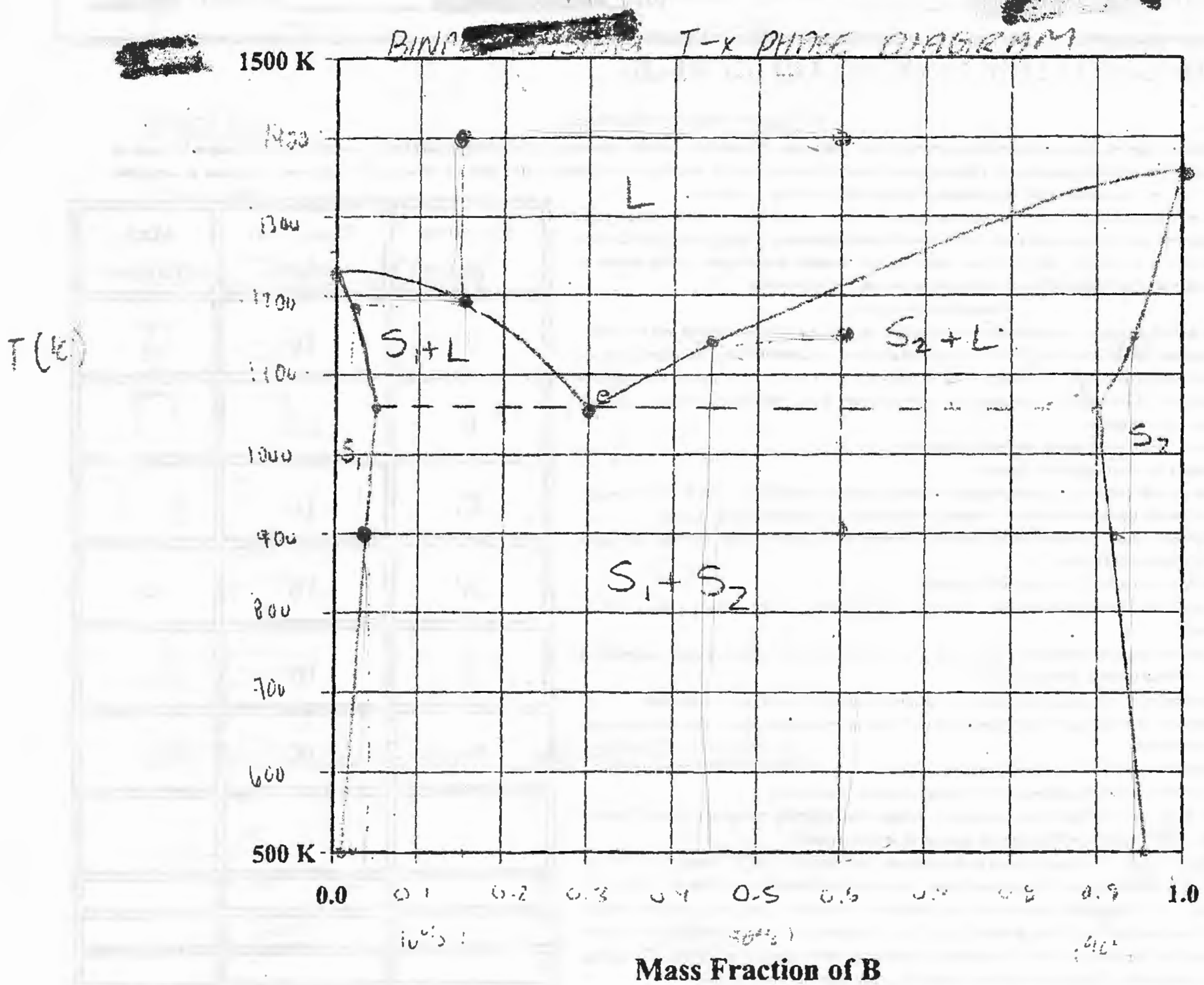
Question 1 ***(16 marks)***

The following information is provided for a binary system comprised of substance A (M_A= 50 kg/kmol) and substance B (M_B = 90 kg/kmol) at a pressure of 1 atm:

- ~~Melting point of pure A is 1230 K~~
- ~~Melting point of pure B is 1350 K~~
- At 500 K the solubilities are approximately 5 mass % A in B and 1 mass % B in A.
- Maximum solubility of A in B is 10 mass %
- Maximum solubility of B in A is 5 mass %
- There is a eutectic at 1050 K with 30 mass % B

(a) Using the information provided, sketch a temperature-composition (T-x) phase diagram. Label all phase regions using the notation S_1, S_2, and L.

4



Question I ***(Contd.)***

(b) A 100 kg mixture containing 60 mass % B is held at a temperature of 900 K.

(i) List all the phases present at equilibrium.

1

solid I and solid II

(ii) What is the mass of each phase present?

2

S_1 :

Lever Rule.

~~$S_1 \rightarrow 28.16$ kg~~

~~$S_2 \rightarrow 71.84$ kg.~~

0.05 mass fraction B
= 36. 585

0.925 mass fraction B
= 63.415

$\frac{6.5\text{cm}}{10.25\text{cm}} = 0.63415 \; S_2$

S_1 — S_2

~~0.04~~ ~~0.6~~ ~~0.92~~

~~S_1 = 0.92 − 0.6 / 0.92 − 0.04~~

~~= 0.32 / 0.88~~

~~S_1 − 0.2816 × 100 kg~~

~~S_2 · 0.7184 × 100 kg~~

(iii) State the composition of the S_1 phase as a mol fraction.

S_1 : 0.04 mass% B.

2

$28.16\text{kg} \times 0.04 = \frac{1.1264 \text{ kg}}{90 \text{ kg/kmol}} = 0.0125$ kmol

$\times 0.96 = \frac{27.0336 \text{ kg}(A)}{50 \text{ kg/kmol}} = 0.541$ kmol

0.5535 kmol total

$B = \frac{0.0125 \text{ kmol}}{0.5535 \text{ kmol}} \rightarrow \boxed{0.02258}$

mol fraction B ⟸ be explicit!

(c) The temperature of the mixture in part (b) is raised to 1150 K. Determine the ~~composition of each phase~~ present, and calculate the ratio of their masses.

3

Composition

Solid (II):
0.95 (B)
0.05 (A)

mass fraction

Liquid:
0.45 (B)
0.55 (A)

L — S_2

0.45 0.6 0.95

$L = \frac{0.95 - 0.6}{0.95 - 0.45} = \frac{0.35}{0.5}$ 0.70

0.70 : 0.30

7 : 3

115

[redacted]

Question I ***(Contd.)***

(d) The mixture described in part (c) is heated to 1400 K. Substance A is then added until the mass composition of the mixture is 85% A. Calculate the mass of A added.

2

1400K 0.6 B
0.4 A (100kg)

60kg B
40kg A

(B) $0.15 = \frac{60\,kg}{100\,kg + n}$ $0.85 = \frac{40 + n}{100 + n}$

$15 + 0.15n = 60\,kg$ $85 + 0.85n = 40 + n$

$0.15n = \frac{45\,kg}{0.15}$ $\frac{45}{0.15} = 0.15n$

n =

$n = \cancel{300}\,kg$
200

$\boxed{300\text{ kg of A is added}}$

0.15 B

(e) The mixture in part (d) is now cooled until the first solid appears. Determine the composition of this solid.

1 0.03 mass fraction (B)

$\approx$ 3 mass% B

(f) Determine (show all work) the number of degrees of freedom at the eutectic point.

1

c = 2 $F = C + 2 - P$

P = 3 $F = 2 + 2 - 3$

$= 1$ (except pressure/temperature of system is already fixed)

this one degree of freedom is already used up

∴ 1 − 1 = 0 degrees of freedom

Question II ***(20 marks)***

There are two rigid tanks in a gas plant. Tank 1 contains 3200 kg of gas A and 8800 kg of gas B at 296.9 K and 1976 kPa. You have been given the following information about gases A and B:

	Component	Molar Mass (kg/kmol)	T_c (K)	P_c (atm)	ω
methane	Gas A	16	190.6	45.4	0.008
CO_2 / [illegible]	Gas B	44	304.2	72.8	0.225

Tank 2 contains only Gas C which is held at a very low pressure (0.100 Pa) and a temperature of 0 °C. The collision diameter of gas C is $\sigma = 5.22\times10^{-10}$ m and its molar mass is 30.07 kg/kmol.

(a) Calculate the volume (m^3) of Tank 1 using the ideal gas law.

2 · Tank 1 $m = \frac{3200 \text{ kg } (A)}{16 \text{ kg/kmol}} \Rightarrow 200$ kmol (A)

T = 296.9 K. P = 1976 kPa $m = \frac{8800 \text{ kg } (B)}{44 \text{ kg/kmol}} \Rightarrow 200$ kmol (B)

$pV = nRT$

$V = \frac{nRT}{P} \Rightarrow V = \frac{(400 \text{ kmol})(8.314 \text{ m}^3\cdot\text{kPa/kmol-K})(296.9 \text{ K})}{1976 \text{ kPa}}$

$= 499.68 \text{ m}^3$

$= \boxed{500 \text{ m}^3}$

(b) Determine the pseudocritical temperature (K), pseudocritical pressure (kPa), and pseudocritical acentricity for the gas mixture in Tank 1.

3

	y_i	y_iT_c (K)	y_iP_c ($\times10^6$ Pa)	$y_i\omega$
Gas A	0.5	95.3	2.30	0.004
Gas B	0.5	152.1	3.695	0.1125
pseudocritical		247.4	5.995	0.1165

$T_{pc} = 247.4$ K

$P_{pc} = 5.99 \times 10^6$ Pa

$\omega_{pc} = 0.1165$

Question II *(Contd.)*

(c) Calculate the volume (m^3) of Tank 1 using the compressibility factor chart.

3 $T_{pc} = 247.4\ K \Rightarrow T_r = \frac{T}{T_c} = \frac{296.9K}{247.4K} \Rightarrow 1.20$ (Kay's method)

$P_{pc} = 5.99 \times 10^6\ Pa \Rightarrow P_r = \frac{P}{P_c} = \frac{1976\,000\ Pa}{5980000\ Pa} \Rightarrow 0.32999$

$\downarrow$ 0.33

Z = 0.95

$PV = nZ_m RT$

$V = \frac{(400\ kmol)(0.95)(8.314\ m^3 \cdot kPa/kmol \cdot K)(296.9K)}{1976\ kPa}$

$= 474.6974\ m^3$

$\Rightarrow \boxed{475\ m^3}$

(d) Calculate the volume (m^3) of Tank 1 using the Pitzer-Curl method.

4 $T_r = 1.20$

$P_r = 0.33$

$\omega_{pc} = 0.1165$

Z^0	0.2	0.33	0.4
T_r 1.20	0.963	0.9357	0.921

$Z_m = Z^0 + \omega_{pc} Z^1$

$= (0.9357) + (0.1165)(0.01485)$

$Z_m = 0.93743$

$\frac{0.4 - 0.2}{0.921 - 0.963} = \frac{0.33 - 0.2}{x - 0.963}$

$\frac{0.2}{-0.042} = \frac{0.13}{x - 0.963}$

$Z^0 = x = 0.9357$

$PV = nZ_m RT$

$V = \frac{(400 kmol)(0.94)(8.314\ m^3 kPa/kmol K)(296.9K)}{1976\ kPa}$

$= 468.42\ m^3$

$= \boxed{468\ m^3}$

Z^1	0.2	0.33	0.4
T_r 1.20	0.009	y	0.018

$\frac{0.4 - 0.2}{0.018 - 0.009} = \frac{0.33 - 0.2}{y - 0.009}$

$Z^1 = y = 0.01485$

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Question II *(Contd.)*

(e) Using the mixing rules method, determine the value of $\bar{a}$.

3 $\bar{a} = \left[\sum \left(y_i \sqrt{a_i}\right)\right]^2$

A:

$y_i = 0.5$

$a = \frac{27}{64} \frac{R^2 T_c^2}{P_c^2}$

$= \frac{27}{64} \frac{(8.314\ m^3 \cdot kPa/kmol \cdot K)^2 (190.6\ K)^2}{(4600\ kPa)^2}$

$a_i = 0.05006$

$\sqrt{a_i} = 0.22375$

$\times\ y_i \to (0.5)$

$= 0.111876$

B

$y_i = 0.5$

$a = \frac{27}{64} \frac{R^2 T_c^2}{P_c^2}$

$= \frac{27}{64} \frac{(8.314\ m^3\ kPa/kmol \cdot K)^2 (304.2\ K)^2}{(7376.46\ kPa)^2}$

$a_i = 0.04959$ unit?

$\sqrt{a_i} = 0.222696$

$\times\ y_i \to 0.5$

$= 0.111348183$

$0.111876 + 0.111348 = (0.22322418 3$

$(0.22)^2 = \boxed{0.04983}$

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[redacted]

Question II ***(Contd.)***

(f) Calculate the mean speed (m/s) of gas C molecules in Tank 2. $\rightarrow$ P = 0.100 Pa

T = 0°C → 273.15 K

σ = 5.22 × 10⁻¹⁰ m

M = 30.07 kg/kmol

2 mean speed : $\bar{c} = \sqrt{\frac{8RT}{\pi M}}$

$= \sqrt{\frac{8(8.314 \text{ m}^3\cdot\text{kPa/kmol}\cdot\text{K})(273.15\text{ K})}{\pi(30.07 \text{ kg/kmol})}}$

~~13.8678~~ $= \boxed{438.54 \text{ m/s}}$ ✓

~~13.87 m/s~~

(g) You start to work with a new type of ideal gas. At 0 °C and 0.100 Pa, the thermal conductivity of this new gas is 0.100 W/m K.

(i) Estimate the thermal conductivity of this new gas at 600 K and 0.100 Pa.

2 $\frac{M}{\mu} \times \frac{C_v}{11}$

$\mu = \frac{M}{N_A \pi \sigma^2}\sqrt{\frac{RT}{\pi M}}$

= 273.15 K

= 0.1 Pa

k = 0.100 W/mK

$C_v = \frac{(0.1)(30.07)}{2.86\times10^{-7}}$

$= 1.05\times10^{7}$

$= \frac{(30.07 \text{ kg/kmol})}{(6.022\times10^{26}\text{ kmol}^{-1})\pi(5.22\times10^{-10})^2}\sqrt{\frac{8.314\times273}{\pi(30.07)}}$

$= 5.833\times10^{-8}\sqrt{28}$

$\mu = 2.8599\times10^{-7}$ Pa·s

$k = \frac{\mu C_v}{M}$

$= \frac{(4.239\times10^{-7}\text{ Pa s})\times(1.05\times10^{7})}{30.07}$

$= \boxed{0.148}$

$\mu = \frac{30.07}{(6.022\times10^{26})\pi(5.22\times10^{-10})^2}\sqrt{\frac{8.314\times600}{\pi(30.07)}}$

$5.833\times10^{-8}\sqrt{52.8}$

$= 4.239\times10^{-7}$

(ii) Estimate the thermal conductivity of this new gas at 0 °C and 100 kPa.

1 $k = \mu C_v$

μ :

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Question III ***(16 marks)***

An isoteniscope with a pressure transducer is used to determine the vapour pressure (P_v) of an unknown substance. The transducer displays ΔP readings which represent the difference between the pressure in the room and the pressure in the exit arm of the isoteniscope. ΔP readings for the unknown substance at two different temperatures are provided in the table. You can use the following formula to determine the P_v of the pure substance:

Temperature (K)	**ΔP (mmHg)**
298.15	423.51
323.15	48.89

$$P_v = P_b - \Delta P$$

where P_b is the corrected barometric pressure. From a barometer you read the room pressure as 660 mmHg. The substance has a molar mass of 58 kg/kmol and is completely miscible in water.

(a) Using the data provided here and on page 8 of the formula sheet, determine the corrected barometric pressure (mmHg).

1 room = 660 mmHg

− Temp. Corre = −2.15

+ Lat. Corre = +0.34

660
− 2.15
+ 0.34

$\boxed{658.19 \text{ mmHg}}$

(b) Determine the vapour pressure (kPa) of the unknown substance at 298.15 K and 323.15 K, and use these values to develop a correlation relating the vapour pressure (in kPa) to the absolute temperature.

$P_v = P_b - \Delta P$

5 $P_b = 658.19$ mmHg

298.15 K → 423.51 mmHg

323.15 K → 48.89 mmHg

298.15 K:

$P_v = (658.19 - 423.51)$ mmHg

$= 234.68$ mmHg

$\times \frac{101.325 \text{ kPa}}{\text{atm}} \times \frac{1 \text{ atm}}{760 \text{ mmHg}}$

$= 31.29$ kPa

$\ln P = -\frac{\Delta H_v}{RT} + C_0$

$\ln(31.29 \text{ kPa}) = \frac{-\Delta H_v}{R(298.15)} + C_0$

323.15 K:

$P_v = 658.19 \text{ mmHg} - 48.89 \text{ mmHg}$

$= 609.3$ mmHg

$\times \frac{101.325}{760}$

$= 81.23$ kPa

$\ln(81.23) = \frac{-\Delta H_v}{R(323.15)}$

next page

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[redacted]

Question III *(Contd.)*

(c) In your next experiment, you add the unknown substance to water in a vessel, mix the contents, and then seal the vessel. At equilibrium, the liquid phase is comprised of 10 mol% of the unknown substance and 90 mol% water. The vapour pressure for water is related to the absolute temperature as per the following correlation:

$$\ln P_v = \frac{-4891.75}{T} + 17.728 \qquad \text{where } P_v \text{ is in kPa and T is in K}$$

Assuming that this mixture obeys Raoult's Law, determine the mol fraction of water in the vapour phase at 323.15 K.

5

$P_i = y_i P = P_{v_i} x_i$ $\qquad Y_1 = \frac{x_1}{x_1 + (x_2) x_2}$ $\qquad$ 10 mol % X

90 mol % water

$\ln P_v = \frac{-4891.75}{323.15 K} + 17.728$

(water) $P_v = 13.33$ kPa $(0.9) \rightarrow 11.997$ kPa

(X) $P_v = (81.23$ kPa$)(0.1) \rightarrow 8.123$ kPa

$\boxed{20.12 \text{ kPa}}$ of the mixture

Raoult's Law

$\frac{P_{water}}{P_{Total}}$ = mol fraction of water

$\frac{13.33 \text{ kPa}}{20.12 \text{ kPa}} = \boxed{0.66}$

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Question III ***(Contd.)***

2 **(d)** Using the correlation developed in part (b) determine the most likely identity of the unknown substance (*note: the identity is included in Table 7-5 in the formula sheet*)

30.78 − 30.57 = 0.21

$\Delta H_v = 30.57$ kJ/mol

ΔH_v of Benzene = 30.78 kJ/mol

most likely benzene

(−1)

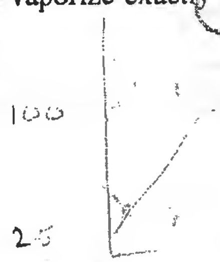
3 **(e)** If you had 10 kmol of pure liquid water (M_{water} = 18 kg/kmol) at 25°C, how much power (kW) would it take to heat up and vaporize exactly one-half of the water in 30 minutes at a constant pressure of 1 atm?

M = 18 kg/kmol $\Delta H_v = 40.67 \times 10^3$

n = 5 kmol

T = 25°C

C_{water} = 4.19 J/mol



t = 30 mins = 30 min · 60 s/min = 1800 s

Q = mcΔt + nΔH_v

= (5 kmol)(18 kg/kmol)(4.19 kJ/kg·°C)(75°C) + (5 kmol)(40.67 × 10³ kJ/kmol)

= 28282.5 kJ + 203350 kJ

= 231632.5 kJ

Power = Q/t = 231632.5 × 10³ J / 1800 s

= 128684.72 W

= $\boxed{128.68 \text{ kW}}$

(−0.5)

$\boxed{\text{Ans. } P = 144.397 \text{ kW}}$

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Question IV ***(16 marks)***

A newly discovered molecule called Enggium has been found to have an ε/k value of 144.85 K, a collision diameter of 4×10^{-10} m, and a molar mass of 81.76 kg/kmol. The potential energy function of this particular substance, which crystallizes in a face centred cubic (FCC) structure, can be described by the equation

$\frac{\varepsilon}{k} = 144.85\ K$

$$\phi(r) = 2\varepsilon\left[\left(\frac{\sigma}{r}\right)^{6} - \left(\frac{\sigma}{r}\right)^{3}\right]$$

$\varepsilon = (144.85)(1.3805\times10^{-23}\ J/K)$
$= 1.99965\times10^{-21}\ J$

(a) Calculate the equilibrium separation distance (angstroms) between two molecules of Enggium.

$\sigma = 4\times10^{-10}\ m$

3

equilibrium ∴ F = 0
F is derivative of $\phi(r)$

$\phi(r) = 2\varepsilon\sigma^6 r^{-6} - 2\varepsilon\sigma^3 r^{-3}$

$d = 2\varepsilon(-6)\sigma^6 r^{-7} - 2\varepsilon(-3)\sigma^3 r^{-4}$

$0 = -12\varepsilon\frac{\sigma^6}{r^7} + 6\varepsilon\frac{\sigma^3}{r^4}$

-1

$\therefore r_0 = 1.122\ \sigma$ x

$= 1.122\ (4\times10^{-10}\ m)$

$= \boxed{4.488\times10^{-10}\ m}$

ANS: 5.03968×10^{-10} m

(b) Determine the value of the potential energy (J) between two molecules of Enggium when they are at equilibrium separation distance. $r_0 = 4.488\times10^{-10}\ m$

$\varepsilon = 1.99965\times10^{-21}\ J$

3

$\phi(r) = 2(1.99965\times10^{-21}\ J)\left[\frac{4^6\times10^{-60}\ m}{4.488^6\times10^{-60}\ m} - \frac{4^3\times10^{-30}\ m}{4.488^3\times10^{-30}\ m}\right]$

$= 2\varepsilon\left[0.501 - 0.708\right]$

method ok.

$= \boxed{-8.268\times10^{-22}\ J}$

$\boxed{\phi r = -10\times10^{-22}\ J}$

12

Question IV ***(Contd.)***

(c) Determine the mass (kg) of 1 m^3 of pure Enggium.

3 Bulk density $FCC = \frac{\sqrt{2}m}{\sigma^3}$ - 1 single molecule $M = 81.76 kg/Kmol$ $\sigma = 4E^{-10} m$

$\frac{M}{N_A}$ = mass of one molecule

$= 1.357688E^{-25} kg/molecule$

$\rho = \frac{\sqrt{2}(1.357688E^{-25} kg)}{(4E^{-10} m)^3}$

$\rho = \frac{1.920060783E^{-25} kg}{6.4E^{-29} m^3}$

$\rho = 3000 kg/m^3$

$\boxed{\text{Mass of } 1m^3 \text{ of Enggium} = 3000.095 kg}$ ✓

3000.094974

(d) Determine the number of molecules that could be put in a volume of 1 m^3 of pure solid Enggium.

2 Mass $1m^3 = 3000.095 kg$

Mass 1 molecule $= 1.357688E^{-25} kg/molecule$

$\boxed{\#\text{ of molecules in } 1m^3 = 2.2097E^{28} \text{ molecules}}$ ✓

(e) Determine the length (m) of one side of a unit cell of Enggium

2 4 molecules / FCC unit cell

$2.2097E^{28}$ molecules$/m^3$

$\frac{2.2097E^{28}}{4} = 5.52427E^{27}$ unitcells$/m^3$ $\xrightarrow{\text{Cube root}}$ 1767766953 unitcells$/m$

$\boxed{5.65685E^{-10} m \text{ for one side of a unitcell}}$ ✓

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Question IV *(Contd.)*

(f) Determine the void fraction for Enggium.

3

$$\frac{\text{Volume of unit cell} - \text{volume of molecules in unit cell}}{\text{volume of unit cell}}$$

$V_{unit cell} = (5.65685E^{-10} m)^3$

$= 1.8102 \times 10^{-28} m^3$

$V_{molecules}$: 4 molecules, $\sigma = 4 \times 10^{-10} m$

$\cancel{4\left(\frac{\pi D^3}{6}\right)} = \cancel{4\left(\frac{\pi (4E^{-10} m)^3}{6}\right)}$

$V_{molecules} = 1.3404E^{-28} m^3$

$$\frac{1.8102E^{-28} m^3 - 1.3404E^{-28} m^3}{1.8102E^{-28} m^3}$$

$\boxed{\text{Void fraction} = 0.2595293}$

$4\left(\pi\left(\frac{d}{2}\right)^3\right)$

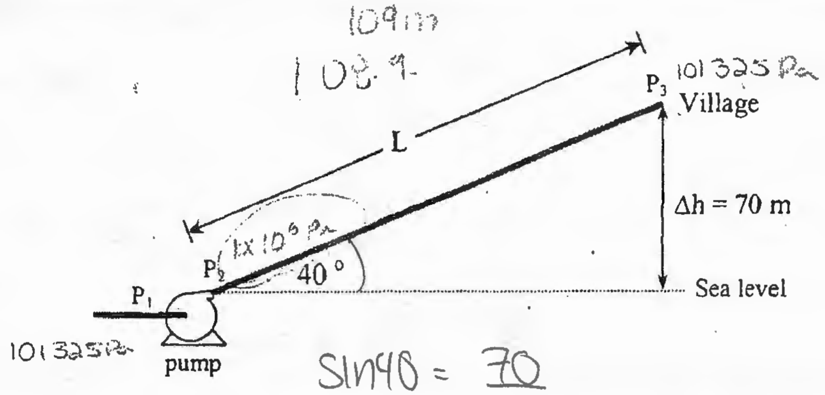
$\frac{4\pi d^3}{8}$

12

Use the other side of this page if you run out of space.

Question V *(16 marks)*

You have been hired to design a commercial steel pipeline that will be used to transport a special winter heating fuel ($\rho = 950$ kg/m^3) from sea level to a village located on top of a hill.



$\sin 40 = \frac{\Delta h}{L}$ $\frac{70}{L}$

$L = \frac{70}{\sin 40}$

$\sin 40 = \frac{70}{W}$

You choose a pump that can generate 1.0 MPa of pressure at the outlet (P_2) knowing that fuel has to be delivered to the village at atmospheric pressure (P_3). The fuel also enters the pump at atmospheric pressure (P_1). The fuel must be pumped at a rate of 3.6 m^3/h. You do some research and find the following information about the fuel:

Temperature (°C)	τ (Pa)	du/dy (s^{-1})
-5.0	50.0	10000
~~30.0~~	~~60.0~~	~~13333~~
-5.0	75.0	15000

(a) Using the data provided, determine the viscosity of the fluid under working conditions.

2

$\mu_{app} = \frac{\tau}{-\left(\frac{du}{dy}\right)}$ ✓

winter ∴ temp = -5 °C

$= \frac{75\ \text{Pa}}{15000\ \text{s}^{-1}}$

$= \frac{50.0\ \text{Pa}}{10000\ \text{s}^{-1}}$

$= 5 \times 10^{-3}$ Pa·s

$\boxed{\mu_{app} = 5 \times 10^{-3}\ \text{Pa s}}$ ✓

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$\Delta P = 101325\,Pa - 10000\,Pa = -89867.5\,Pa$

Question VI *(Contd)*

(b) Determine the diameter of the pipe needed for this application. Remember to state all assumptions (*Note: if you need to iterate you can start with a diameter of D=0.06 m)*

10

$\tau = 50\,Pa$ $\Delta L = 109\,m$ ✓ ① rate $36\,m^3/h$

$\frac{du}{dy} = 10000\,s^{-1}$ $\Delta h = 70\,m$ $\frac{36\,m^3}{h} \times \frac{1h}{3600\,s} = 0.01\frac{m^3}{s}$

$\rho = 950\,kg/m^3$

② $\frac{\Delta P}{\Delta L} + \rho g \frac{\Delta h}{\Delta L} + \frac{2\tau}{r} = 0$ $\mu_{app} = 5\times10^{-3}\,Pa\cdot s$ $Q = 0.01\,\frac{m^3}{s}$

$Q = \bar{U} A$ (assume D = 0.06m)

$A = \frac{\pi D^2}{4}$

$= \frac{\pi (0.06\,m)^2}{4}$

$A = 2.83\times10^{-3}\,m^2$

$Q = \bar{U} A$

$\bar{U} = \frac{Q}{A} = \frac{0.01\,m^3/s}{2.83\times10^{-3}\,m^2}$

③ $= \underline{3.54}\,m/s$ ✓

$Re = \frac{D\bar{U}\rho}{\mu} = \frac{(0.06\,m)(3.54\,m/s)(950\,kg/m^3)}{5\times10^{-3}\,Pa\cdot s}$

$= 40319$ ✓ > 4000

∴ turbulent flow

commercial steel

Moody chart $Re\ 4.03\times10^4$

$f = 0.0069$ ✓

④ $-\left[\frac{\Delta P}{L} + \rho g \frac{\Delta h}{L}\right] = \frac{2f\rho(\bar{U})^2}{D}$ ✓

$-\left[\frac{-89867.5\,Pa}{109\,m} + (950\,kg/m^3)(9.81\,m/s^2)\left(\frac{70\,m}{109\,m}\right)\right]$

$= 2259.7$

$= \frac{2f\rho(\bar{U})^2}{D}$

$= 2(0.0069)\rho(\bar{U})^2$

$D = \left(\frac{2259.7}{2(0.0069)\rho(\bar{U})^2}\right)^{-1}$

$D = 0.0727\,m$ ✓

⑤ ~~(Repeat steps ② – ④ with new value for D).~~

Iteration #2 : $D = 0.0727$

plug it back in.

$A = 4.15\times10^{-3}$

$\bar{U} = 2.41\,m/s$

$Re = 33289$

∴ $f = 0.007$ → close enough to $f = 0.0069$

∴ $\boxed{D = 0.0727\,m}$ 4

~~??? what happened?~~

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Use the other side of this page if you run out of space.

Question VI ***(Contd)***

(c) Determine the shear stress (Pa) at the wall of the pipe.

2

Assume $D = 0.07\,m$ $\quad -2$

$r_w = 0.035$

$$\frac{\tau_w}{r_w} = \frac{\tau}{r}$$

(d) Determine the power (W) supplied by the pump.

2

$$\text{Power} = Q(\Delta P)\,V$$

$$= \left(0.01\,\frac{m^3}{s}\right)(898675\,Pa)$$

$$= \boxed{8986.75\ W}$$

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Question VI *(16 marks)*

A thick-walled tube of stainless steel (κ = 20W/m K) has a 3 cm inner diameter, a 5 cm outer diameter, and a length of 100 m. The temperature of the fluid inside the tube is constant at 300°C and the temperature of the outside environment is maintained at -20°C. As the lead engineer, you want to reduce the energy loss from the fluid inside the pipe to the environment. You decide to insulate the outer surface of the pipe. Two different insulation materials are available with specifications shown in the table below:

Insulation	Thickness (cm)	κ (W/m K)
A	2	0.1
B	3	0.5

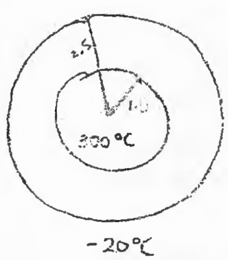
You decide to investigate two possible options:

Option 1: Wrap the tube with one layer of insulator A and then one layer of insulator B.

Option 2: Wrap the tube with one layer of insulator B and then one layer of insulator A.

(a) Determine the heat energy that is lost for the bare (uninsulated) pipe (W).

4



$r_1 = 0.015$ m

$r_2 = 0.025$ m

$L = 100$ m

$$Q = \frac{-2\pi (L)(T_{\infty,i} - T_i)}{\dfrac{\ln\left(\frac{r_2}{r_1}\right)}{k_1}}$$

$$= \frac{-2\pi(100\,m)(-20 - 300\,°C)}{\dfrac{\ln\left(\frac{0.025}{0.015}\right)}{(20\,W/mK)}}$$

$$= \frac{201061.9298}{0.02554}$$

$$= 7872037.754 \text{ J}$$

$$= \boxed{7.87 \text{ MW}}$$

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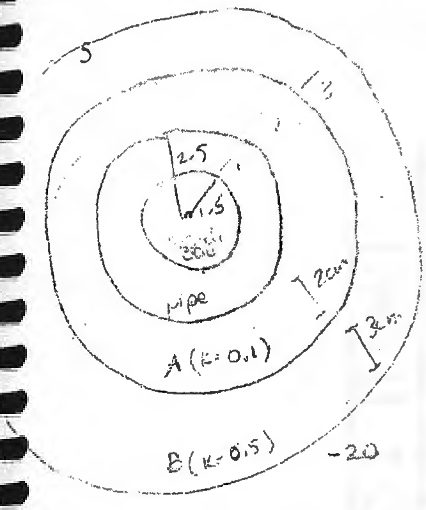
Question VI *(Contd)*

(b) Determine the heat energy that would be lost in Option 1 (W)



$r_1 = 0.015\,m$

$r_2 = 0.025\,m$

$r_3 = 0.045\,m$

$r_4 = 0.075\,m$

$L = 100\,m$

$$Q = \frac{-2\pi(L)(T_{in1} - T_1)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k_3}}$$

$$= \frac{-2\pi(100\,m)(-20 - 300\,°C)}{\frac{\ln\left(\frac{0.025\,m}{0.015\,m}\right)}{20\,W/mK} + \frac{\ln\left(\frac{0.045\,m}{0.025\,m}\right)}{(0.1\,W/mK)} + \frac{\ln\left(\frac{0.075\,m}{0.045\,m}\right)}{(0.5\,W/mK)}}$$

$$= \frac{64000\pi}{6.925059}$$

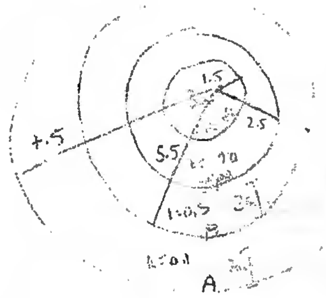
$$= 29{,}033.96558\,W$$

$$= \boxed{29.03\,kW}$$

Question VI *(Contd)*

5 **(c)** Determine the heat energy that would be lost in Option 2 (W)

$$Q = \frac{-2\pi(L)(T_{m1} - T_1)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k_3}}$$



$r_1 = 0.015\,m$

$r_2 = 0.025\,m$

$r_3 = 0.055\,m$

$r_4 = 0.075$

$L = (100\,m)$

$$= \frac{-2\pi(100\,m)(-20 - 300\,°C)}{\frac{\ln\left(\frac{0.025}{0.015}\right)}{20\,W/mK} + \frac{\ln\left(\frac{0.055}{0.025}\right)}{0.5\,W/mK} + \frac{\ln\left(\frac{0.075}{0.055}\right)}{0.1\,W/mK}}$$

$$= \frac{64000\pi}{4.704}$$

$= 42742.7097\,W$

$= \boxed{42.7\,kW}$

2 **(d)** The cost of heat energy is $0.05 per kWh. Calculate the projected annual cost savings if Option 1 is adopted for the pipe.

Uninsulated: 7.87 MW → 7872037.754 W

option 1: 29.03 kW → 29033.96558 W

7872037.754 W − 29033.96558 W = 7843003.788 W difference

$7843.003788\,kW \times \frac{24\,hrs}{day} \times \frac{365\,days}{year}$

$= 68704713$ kWhrs/year

× $0.05/kWh

$= 3435235.657$ \$

$\boxed{\$\ 3435235.66}$

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