Math 1180 Exam

April 9, 2011

Duration: 3 hours

Family Name:_____

Given Name:_____

Student Number:_____

Instructions: Show and explain all work. Do all questions. Give your answers in the space provided, using the back of the preceding page if necessary. Hand in all pages of this booklet. **No calculators or other aids are permitted.** There are 16 pages in this booklet, including this one.

Question	Mark
1	
2	
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12	
13	
14	
15	
Total	

1. Part A (*3 marks*) Compute the limit:

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

Part B (*3 marks*) Compute the limit:

$$\lim_{x \to 1} \left(\frac{3x}{x-1} + \frac{-3}{x-1} \right)$$

2. Part A (*3 marks*) Find $f'(x)$, where $f(x) = \cos(\sqrt{1-x^2})$.

Part B (*3 marks*) Find $g'(x)$, where $g(x) = \sec(3^x)$.

3. Part A (*3 marks*) Find $f'(x)$, where $f(x) = \left(\frac{x+1}{x^2+3}\right)\tan(1+x^2)$.

Part B (*3 marks*) Find $g'(x)$, where $g(x) = \sqrt{1+x^2}\ln(x+1)$.

4. Part A (*4 marks*) Find the intervals on which $f(x) = x^3 - 3x^2$ increases and the intervals on which it decreases.

Part B (*3 marks*) Show that the equation $x^3 - 4x + 1 = 0$ has a solution in the interval $[0, 1]$.

5. (*6 marks*) Find the critical numbers and classify the extreme values of $f(x) = x^3 - 12x + 5$, $x \in [-3, 3]$.

6. Part A (*2 marks*) Calculate $\int \frac{2x}{1+x^2}\,dx$.

Part B (*3 marks*) Calculate $\int x\sqrt{x-1}\,dx$.

7. Part A (*3 marks*) Calculate $\int x \ln x \, dx$.

Part B (*4 marks*) Calculate $\int x^2 e^x \, dx$.

8. (*5 marks*) Calculate $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\, dx$

9. (*6 marks*) Calculate $\int \frac{x^2+x}{(x-1)(x^2+1)}\,dx$.

10. Part A (*2 marks*) Find the limit of the sequence $\{\frac{\cos n}{n^2}\}$.

Part B (*2 marks*) Find the limit of the sequence $\{\frac{n+1}{n-1}\}$.

11. Part A (*3 marks*) Compute the limit:

$$\lim_{x \to \infty} \frac{x^2}{e^x}$$

Part B (*3 marks*) Compute the limit:

$$\lim_{x \to 0} \frac{x^2}{(\cos x) - 1}$$

12. (*6 marks*) For each of the series below, use an appropriate test to determine whether the series converges. You must justify your answer to receive any credit.

a) $\sum_{k=1}^{\infty} \frac{k^2+3}{4k^2+5}$

b) $\sum_{k=1}^{\infty} \frac{5^k}{k!}$

13. (*6 marks*) For each of the series below, use an appropriate test to determine whether the series converges. You must justify your answer to receive any credit.

a) $\sum_{k=1}^{\infty} \left(\frac{k}{2k+1} \right)^k$

b) $\sum_{k=1}^{\infty} \frac{2}{k^2+5}$

14. Part A (*4 marks*) Find the Taylor polynomials $P_0(x)$, $P_1(x)$, and $P_2(x)$, expanding about $x = 0$, for $f(x) = x + \cos x$.

Part B (*4 marks*) use a Taylor polynomial to approximate e to within 0.01.

15. (*6 marks*) Given $f(x) = \sum_{k=0}^{\infty} \left(\frac{\sin k}{k!}\right) x^k$ for all x, find a power series expression of $f'(x)$, and a power series expression for $F(x) = \int_0^x f(t)\,dt$.