MATH 1007E – Test 2

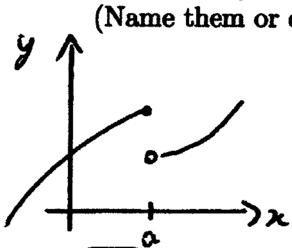
Thursday, Oct. 15th 2015

NAME: $\boxed{\text{Solutions}}$

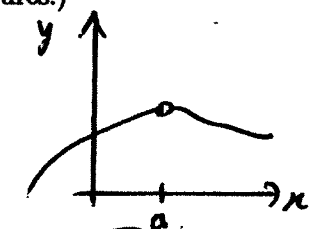
STUDENT NUMBER: ______________________

This test has 5 questions (worth a total of 20 marks). Calculators are not allowed. You have 50 minutes. Write your answers in the spaces provided and put a box around the final answers where appropriate.

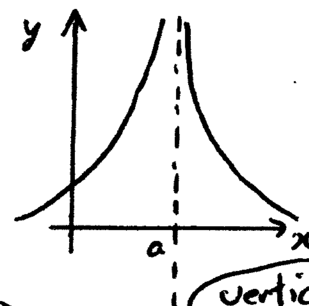
1. (3 points) What are the three common types of discontinuities? (Name them or draw pictures.)



$\boxed{\text{jump}}$

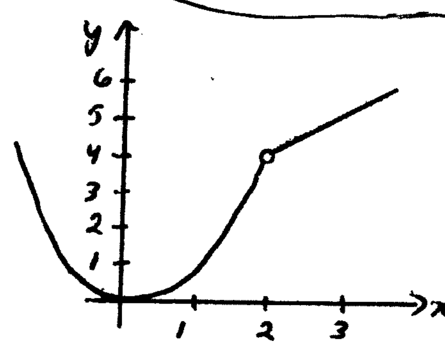


$\boxed{\text{hole}}$



$\boxed{\text{vertical asymptote}}$

2. (4 points) Consider the function $f(x) = \begin{cases} x^2 & x < 2 \\ x+2 & x > 2 \end{cases}$.



(a) Find $\lim\limits_{x\to 2^-} f(x)$.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 = \boxed{4}$$

(b) Find $\lim\limits_{x\to 2^+} f(x)$.

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x+2) = \boxed{4}$$

(c) Does $\lim\limits_{x\to 2} f(x)$ exist? Why or why not?

$\boxed{\text{Yes}}$, since $\boxed{\lim\limits_{x\to 2^-} f(x) = 4 = \lim\limits_{x\to 2^+} f(x)}$

(d) Is $f(x)$ continuous at $x = 2$? Why or why not?

$\boxed{\text{no}}$, $\boxed{f(2) \text{ is not defined}}$

or $\boxed{\text{there is a hole in graph}}$

3. (4 points) Find the vertical and horizontal asymptotes of $y = f(x) = \dfrac{2x^2+1}{x^2-5x+6} = \dfrac{2x^2+1}{(x-2)(x-3)}$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{2x^2+1}{x^2-5x+6} = \lim_{x\to\infty} \frac{2+1/x^2}{1-5/x+6/x^2} = 2$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x^2+1}{x^2-5x+6} = \lim_{x\to\infty} \frac{2+1/x^2}{1-5/x+6/x^2} = 2$$

$\boxed{y=2 \text{ is a horizontal asymptote}}$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{2x^2+1}{(x-2)(x-3)} = \infty, \quad \lim_{x\to 2^+} \frac{2x^2+1}{(x-2)(x-3)} = -\infty$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{2x^2+1}{(x-2)(x-3)} = -\infty, \quad \lim_{x\to 3^+} \frac{2x^2+1}{(x-2)(x-3)} = \infty$$

So $\boxed{x=2 \text{ and } x=3 \text{ are vertical asymptotes}}$

4. (6 points) Find the following limits.

(a) $\displaystyle\lim_{x\to 2^-} \frac{x^2-4}{(x-2)^2}$ (b) $\displaystyle\lim_{x\to\infty} \frac{3x+5}{4x^2+3x+2}$ (c) $\displaystyle\lim_{x\to\infty} \frac{3x^2+x+2}{9x^2+8}$

(a)
$$= \lim_{x\to 2^-} \frac{(x-2)(x+2)}{(x-2)^2} = \lim_{x\to 2^-} \frac{x+2}{x-2} = \boxed{-\infty}$$

(b)
$$= \lim_{x\to\infty} \frac{\frac{3}{x}+\frac{5}{x^2}}{4+\frac{3}{x}+\frac{2}{x^2}} = \boxed{0}$$

(c)
$$= \lim_{x\to\infty} \frac{3+\frac{1}{x}+\frac{2}{x^2}}{9+\frac{8}{x^2}} = \frac{3}{9} = \boxed{1/3}$$

5. (3 points) Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{2}{x+2}$.

$$f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to0} \frac{1}{h}\left(\frac{2}{x+h+2} - \frac{2}{x+2}\right)$$

$$= \lim_{h\to0} \frac{1}{h}\left(\frac{2(x+2)-2(x+h+2)}{(x+h+2)(x+2)}\right) = \lim_{h\to0} \frac{1}{h}\left(\frac{\cancel{2x}+\cancel{4}-\cancel{2x}-2h-\cancel{4}}{(x+h+2)(x+2)}\right)$$

$$= \lim_{h\to0} \frac{1}{\cancel{h}}\left(\frac{-2\cancel{h}}{(x+h+2)(x+2)}\right) = \lim_{h\to0} \frac{-2}{(x+h+2)(x+2)} = \boxed{\frac{-2}{(x+2)^2}}$$