Example:

- $u(c_0, c_1) = \ln(c_0) + \beta \ln(c_1)$
- $t_0 = t_1 = 0$
- two consumers (A, B) with different income

	y_0	y_1
A	0	Y_1
B	Y_0	0

↳ incentives to trade

ie: $y_0^A + y_0^B = Y_0$

$y_1^A + y_1^B = \underbrace{Y_1}_{\text{aggregat income}}$

Competitive Equilibrium

A competitive equ. is an interest rate "r"

1) given "r", each consumer chooses c_0 and c_1 to max. utility subject to his lifetime budget constraint

2) r is such that the credit market is in equilibrium [two markets, only need to clear one]

ie: $\underbrace{s_0^A + s_0^B}_{\text{net demand for funds}} = 0$ → no aggregate supply of funds

- no government
- no foreign sector

↳ net demand for funds (ie: $s_0 < 0$ demand → borrow; $s_0 > 0$ supply ↳ lend)

any funds demanded by one agent must be supplied by the other one

note: $s_0^A + s_0^B = 0$

$y_0^A - c_0^A + y_0^B - c_0^B = 0$

ie: $c_0^A + c_0^B = y_0^A + y_0^B$

$C_0 = Y_0$

sum of all expenditures = GDP (income approach)

↳ this is the equ. cond. in the goods market

The solution of 1) is

$$\left.\begin{aligned} MRS^A_{c_0c_1} &= 1+r \\ c_0^A + \frac{1}{1+r}c_1^A &= W^A \end{aligned}\right\} \rightarrow S_0^A(r)$$

$$\left.\begin{aligned} MRS^B_{c_0c_1} &= 1+r \\ c_0^B + \frac{1}{1+r}c_0^B &= W^B \end{aligned}\right\} \rightarrow S_0^B(r)$$

$\rightarrow S_0^A(r) + S_0^B(r) = 0$

valid for any r

find r such that this is true

A:

$$W_0^A = 0 + \frac{1}{1+r}Y_1$$

$$\boxed{W_0^A = \frac{Y_1}{1+r}}$$

$$MRS^A_{c_0c_1} = \frac{\frac{1}{c_0^A}}{\beta\frac{1}{c_1^A}} = \frac{c_1^A}{\beta c_0^A} = 1+r$$

$$\therefore \quad c_1^A = \beta(1+r)c_0^A$$

$$c_0^A + \frac{1}{1+r}c_1^A = W_0^A$$

$$c_0^A + \beta c_0^A = \frac{Y_1}{1+r}$$

$$\boxed{c_0^{A*} = \frac{1}{1+\beta}\frac{Y_1}{1+r}}$$

$$S_0^A = y_0^A - c_0^A$$
$$= 0 - \frac{1}{1+\beta}\frac{Y_1}{1+r}$$

$$\therefore \quad \boxed{S_0^A(r) = -\frac{1}{1+\beta}\frac{Y_1}{1+r}}$$

$\hookrightarrow$ borrower ($S_0^A < 0 \ \forall r$)

B

$$w_0^B = Y_0 + \frac{1}{1+r}\cdot 0$$

$$\boxed{w_0^B = Y_0}$$

$$MRS^B_{c_0c_1} = \frac{c_1^B}{\beta c_0^B} = 1+r$$

$$\therefore c_1^B = \beta(1+r)c_0^B$$

$$\Rightarrow c_0^B + \beta c_0^B = Y_0$$

$$\boxed{c_0^{B*} = \frac{1}{1+\beta} Y_0}$$

$$\therefore S_0^B = y_0^B - c_0^B$$

$$= Y_0 - \frac{1}{1+\beta}Y_0$$

$$= \frac{(1+\beta)Y_0 - Y_0}{1+\beta}$$

$$\therefore \boxed{S_0^B(r) = \frac{\beta}{1+\beta} Y_0} \rightarrow \text{lender}$$

$(S_0^B > 0)$
regardless of r

2) $\Rightarrow$ $S_0^A(r) + S_0^B(r) = 0$

$$\boxed{-\frac{1}{1+\beta}\frac{Y_1}{1+r} + \frac{\beta}{1+\beta} Y_0 = 0}$$

↳ solve this for r

$$\beta Y_0 = \frac{Y_1}{1+r}$$

$$1+r = \frac{Y_1}{\beta Y_0} \qquad \therefore \boxed{r^* = \frac{1}{\beta}\frac{Y_1}{Y_0} - 1}$$

note:

if consumers expect $Y_1 = Y_0$, then $r^* = \frac{1}{\beta} - 1$

ie: interest rate depends on how much consumers care about the future

if consumers expect $\uparrow Y_1$ (ie expansion)

↳ $\Rightarrow \uparrow r$ why?

if people expect higher future income, they will borrow against that future income [consumption smoothing] to $\uparrow C_0$ as well as C_1

↳ this cannot happen in equ.

↳ r must $\uparrow$ to discourage borrowing [recall supply is fixed at $\frac{\beta}{1+\beta} Y_0$]

similarly, $\downarrow Y_1^e$ (recession) $\rightarrow \downarrow r$

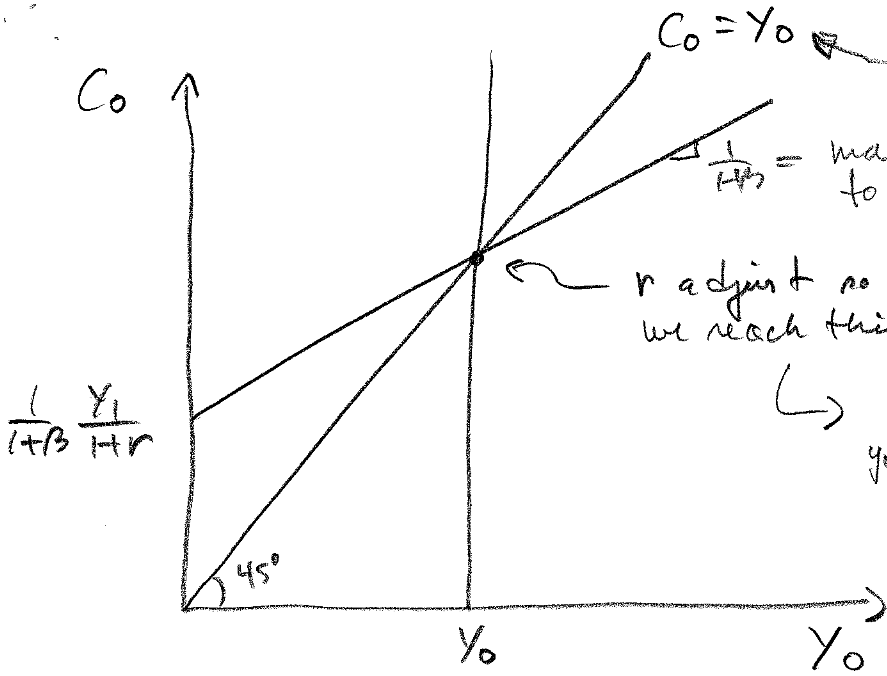
We will not be interested in individual C_0^A or C_0^B

↳ we want to look at C_0 - there, C_0 will depend on r, Y_0, and Y_1

$$C_0(r) = C_0^{A*} + C_0^{B*}$$

$$\boxed{C_0(r) = \frac{1}{1+\beta}\frac{Y_1}{1+r} + \frac{Y_0}{1+\beta}}$$

↳ we will graph this as a function of Y_0



$\frac{1}{1+B}$ = marginal propensity to consume out of Y_0 = MPC

r adjust so that we reach this point

$\hookrightarrow$ $S_0^A + S_0^B = 0$

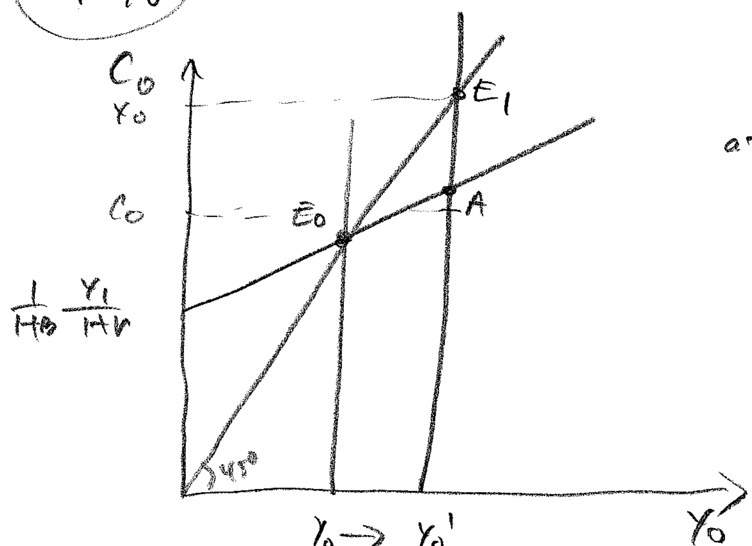
$Y_0^A - C_0^A + Y_0^B - C_0^B = 0$

$C_0^A + C_0^B = Y_0^A + Y_0^B$

$\boxed{C_0(r) = Y_0}$

ie: $AD_0 = AS_0$

↑ Y_0



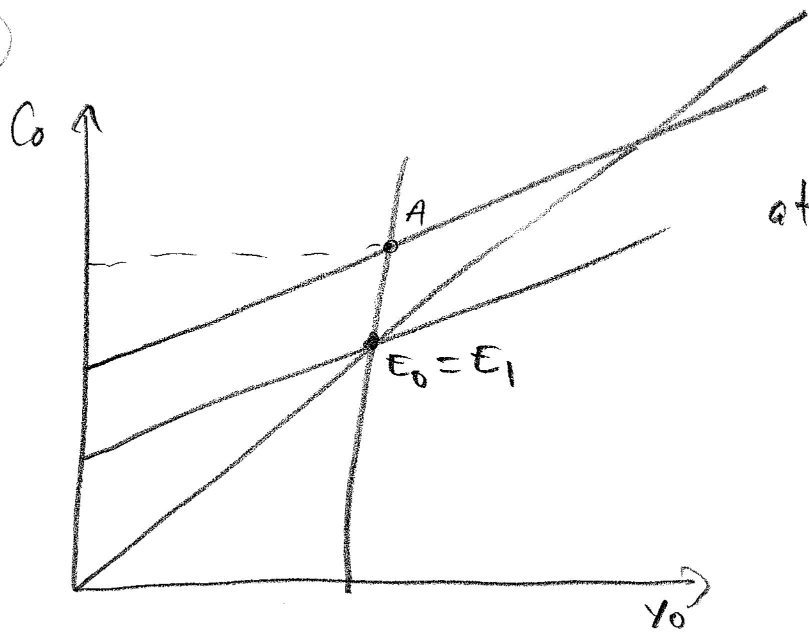
at A, $C_0 < Y_0$ (ie: $S_0^P = Y_0 - C_0 > 0$)

∴ r↓ to discourage saving

ie ↓ $S_0^P > 0$ to $S_0^P = 0$

$E_0 \to E_1$

↑ Y_1



at A, $C_0 > Y_0$

ie! $S_0^P = Y_0 - C_0 < 0$

aggregate borrowing

∴ ↑r to discourage borrowing

$E_0 \to E_1$ $E_1 = E_0$

but with higher r

The government

- The government also exists for 2 periods. Its goal is to spend G_0 in period 0 and G_1 in period 1. G_0 and G_1 are exogenous [ie determined by political considerations]

  ↳ the government needs to choose how to finance these spending levels:
  - → collect taxes?
  - → borrow (issue debt)

- if the government chooses to borrow, it must do so in the credit market at the same rate "r" as consumers trade funds

  ↳ note: in practice, consumers borrow at a higher rate than the government because lending to an individual is more risky than lending to a government [risk premium]
  However, here, consumer's debt is riskless (no default)

- taxes are lump sum and every consumer pays the same taxes → $t_{0i} = t_0 \ \forall i$

  ie: $T_0 = \sum_{i=1}^{M} t_{0i} = M t_0$ or $t_0 = \frac{T_0}{M}$

similarly $T_1 = M t_1$ or $t_1 = \frac{T_1}{M}$

let B_0 = # of 1\$ Bonds that the government issues at time 0 in the credit market (ie: Borrowings)

∴ Budget constraint:

in period 0 → $G_0 = T_0 + B_0$ → debt ⇒ $G_0 > T_0$

in period 1 → $G_1 + \underbrace{(1+r)B_0}_{\text{repay debt and interest (add to expenses)}} = T_1$ (no B_1) ↳ why?

⇒ $G_1 < T_1$

note: $C + S = Y^d$

$$\underbrace{G_0}_{\text{consumption}} - \underbrace{B_0}_{} = \underbrace{T_0}_{\text{income of the government}}$$

→ ∴ $\underbrace{S^G}_{\text{government's savings}} = -B_0$

note: $B_0 < 0$ (ie: $S^G > 0$) is possible

↳ in this case, the government lends (does not borrow) to the private sector [unlikely] in the aggregate

→ as we did for the consumer, we can collapse these 2 constraint into one intertemporal budget constraint

→ $B_0 = G_0 - T_0$

$G_1 + (1+r)(G_0 - T_0) = T_1$

∴ $\boxed{G_0 + \frac{1}{1+r} G_1 = T_0 + \frac{1}{1+r} T_1}$

ie the present value of government spending = present value of taxes [Because debt must be repaid here]

Now, with the government participating in the credit market, the equilibrium condition in this market becomes

$$\boxed{\underbrace{S_0^P + S_0^G}_{\text{national saving} \to S} = 0}$$

$0 \to$ no firms, no foreign sector

or $S_0^P - B_0 = 0$

$\boxed{S_0^P = B_0}$ ie: government's debt must coincide with private savings in a closed economy without firms

note: $S_0^P = B_0$

$$Y_0 - \cancel{T_0} - C_0 = G_0 - \cancel{T_0}$$

$$Y_0 - C_0 = G_0$$

$$\boxed{Y_0 = C_0 + G_0}$$

equivalent [Walras Law)

using income approach: GDP (supply of goods) = Demand for goods $\Leftrightarrow$ market clearing condition for the goods market