

# RYERSON UNIVERSITY

Department of Electrical and Computer Engineering

## FINAL EXAMINATION ELE754/EE8410 Power Electronics

Duration: 3 hours  
Date: December 9<sup>th</sup>, 2013

Instructor:  
Dr. D. Xu

### INSTRUCTIONS:

1. Close-book test. You are allowed to bring one page letter size (double sides on each page) assistant sheet.
2. Show details of calculation steps. Marks will be lost if there is no appropriate units.
3. If doubt exists as to the **interpretation** of any question, you are urged to submit with the answer paper a clear statement of any assumptions made.
4. All the **ac** voltages and currents are **rms** values unless noted otherwise. For three-phase circuits, all the voltages are **line-to-line** voltages unless noted otherwise.
5. Read the question carefully! Part of questions may or may not be related.
6. Table 8-1 and 8-2 are provided on the last page.

Marks:

Q1. 20% \_\_\_\_\_

Q2. 20% \_\_\_\_\_

Q3. 15% \_\_\_\_\_

Q4. 20% \_\_\_\_\_

Q5. 25% \_\_\_\_\_

(Solution)

Student Name: \_\_\_\_\_

Student Number: \_\_\_\_\_ Section Number: \_\_\_\_\_

Program:  Undergraduate

Graduate

**Questions 1 (20%)**

A three-phase full-bridge SCR rectifier shown in Figure 1 is used to drive the dc motor. Assume that the total inductance  $L$  is very large (with a very large external inductor). The ac input voltage is 480V(60Hz). The winding resistance is  $0.25\Omega$  and the back EMF  $E_a=300V$ . The rectifier output power is 10kW. Answer following questions.

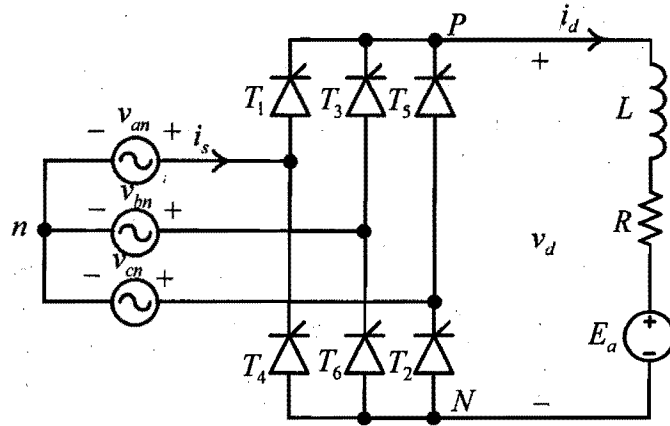


Figure 1 A three-phase full-bridge SCR rectifier with motor load.

1.1) Find the firing angle.

Hint: find the armature current using dc motor armature circuit.

5%

$$I_a^2 R_a + I_a E_a = 10 \text{ kW} = P$$

$$0.25 I_a^2 + 300 I_a - 10^4 = 0 \quad \therefore I_a = 32.46 \text{ A}$$

$$V_d = E_a + I_a R_a = 308.1139 \text{ V}$$

$$\alpha = \cos^{-1} \left( \frac{V_d}{1.35 V_{LL}} \right) = \cos^{-1} (0.4755) = 1.0753 \text{ rad}$$

or  $61.06091^\circ$

1.2) Calculate the RMS value of the ac input current.

5%

$$I_s = \sqrt{\frac{2}{3}} I_d = 26.5 \text{ A}$$

1.3) Find the ac side power factor and current THD

5%

$$PF = DPF \cdot PF = \cos \alpha \cdot 0.955 = 0.955 \cdot \cos 61.6291 = 0.4541$$

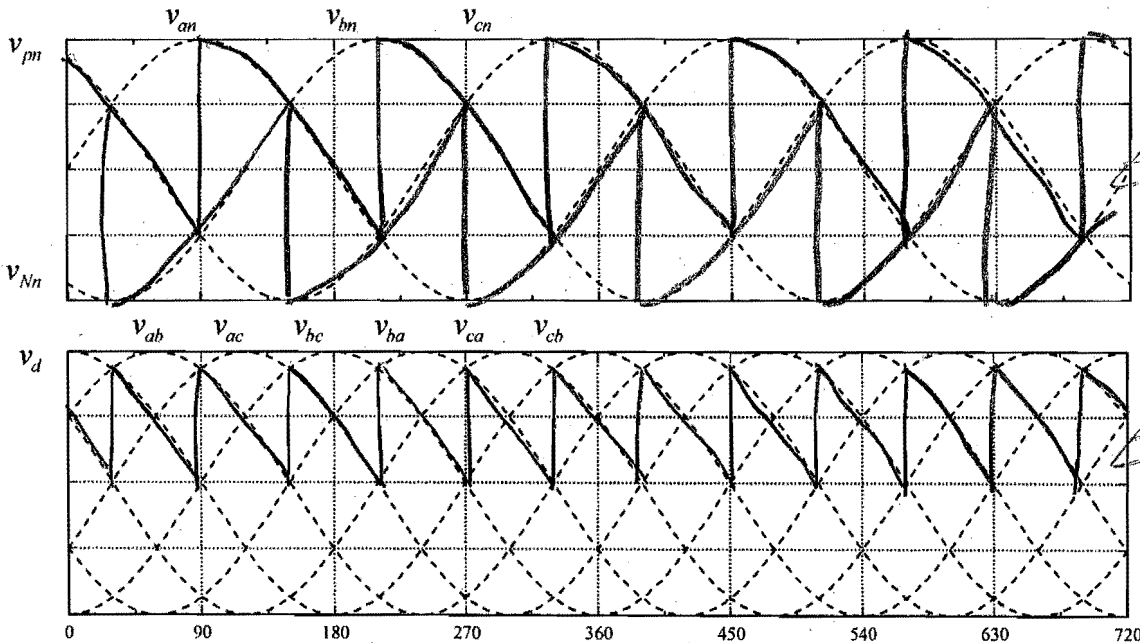
$$THD = 0.3108$$

2.5

2.5

1.4) Using the waveform template below, draw the voltage waveform of  $v_{pn}$ ,  $v_{Nn}$  and  $v_d$  assuming the firing angle is 60 degree.

5%



**Questions 2 (20%)**

Consider the single-phase diode rectifier fed by a square wave inverter as shown in Figure 2. The input voltage  $V_s$  is a 60Hz square wave with peak voltages at  $\pm 120V$ . A very large filter capacitor is connected at DC output. The line inductance  $L_s = 1mH$ . The load resistance is  $20\Omega$ . Answer following questions when the rectifier is in steady state.

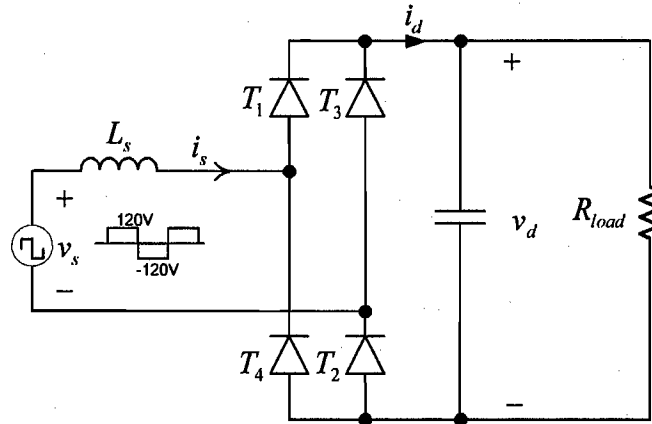
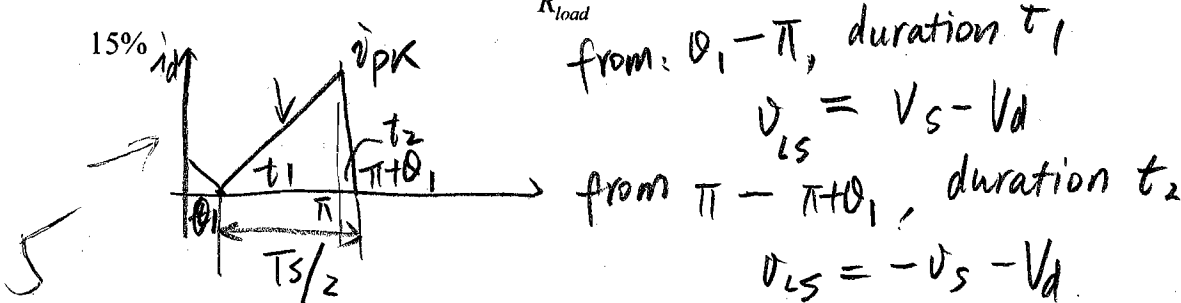


Figure 2 Single-phase diode rectifier with line inductance

2.1) Find the DC output voltage  $V_d$ .

Hint: draw  $i_d$  waveform, use  $I_d = \frac{V_d}{R_{load}}$



from:  $\theta_1 - \pi$ , duration  $t_1$

$$v_{Ls} = v_s - v_d$$

from  $\pi - \pi + \theta_1$ , duration  $t_2$

$$v_{Ls} = -v_s - v_d$$

$$\therefore \frac{v_s - v_d}{L_s} \cdot t_1 = \frac{v_s + v_d}{L_s} \cdot t_2, \text{ where } v_s = 120V$$

$$\frac{t_1}{t_2} = \frac{v_s + v_d}{v_s - v_d}$$

$$t_1 + t_2 = \frac{T_s}{2} \implies t_1 = \frac{T_s}{2} \cdot \frac{v_s + v_d}{2v_s}$$

$$I_0 = \frac{v_d}{R} = \frac{1V_{pk}}{2} \cdot \frac{T_s}{2} \cdot \frac{v_s + v_d}{2v_s}$$

2.2) Find the peak-to-peak current through the inductor  
5%

or  $i_{pk} = 2I_0 = \frac{119.2928}{20} \times 2 \implies i_{pk} = 11.1293 A$

$$v_s^2 - v_d^2 - \frac{v_d}{R} \cdot (8W_s L_s \cdot v_s) = 0$$

$$v_d^2 + v_d \cdot v_s \cdot \frac{8W_s L_s}{R} - v_s^2 = 0$$

$$i_{pk} = \frac{v_s - v_d}{L_s} \cdot \frac{T_s}{2} \cdot \frac{v_s + v_d}{2v_s} = 11.1293$$

$\therefore v_d = 111.2928 V$

**Questions 3 (15%)**

If a single phase full-bridge inverter is designed to output the voltage waveform as shown in Figure 3. Answer following questions when the RMS value of the output voltage is 120V.

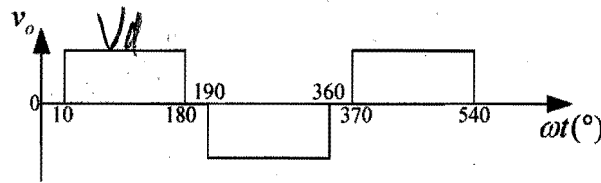


Figure 3 Inverter output voltage waveform

3.1) Find the DC link voltage

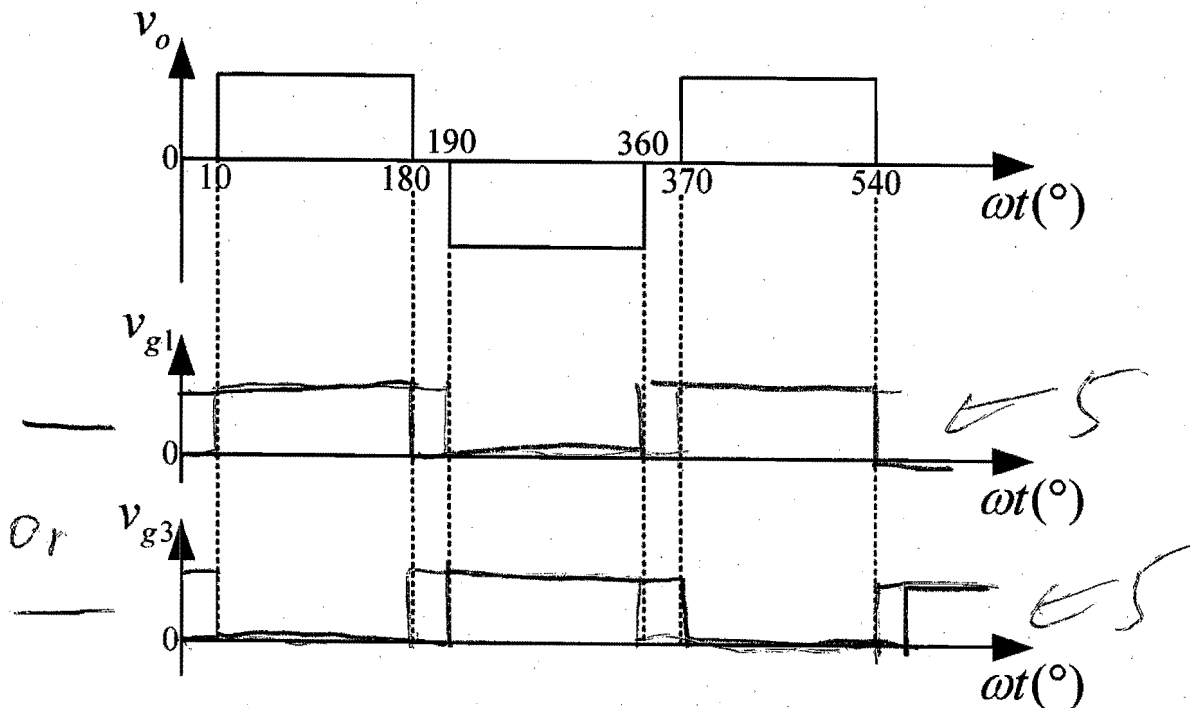
5%

$$120 = \sqrt{\frac{1}{\pi} V_d^2 \left(\frac{170}{180} \times \pi\right)} = \sqrt{\frac{170}{180}} \cdot V_d$$

$$\therefore V_d = \sqrt{\frac{180}{170}} \times 120 = 123.48V$$

3.2) Draw the gating signals for top two devices T1 and T3, using following template

10%



**Questions 4 (20%)**

A 5kW single phase single stage full-bridge inverter is used to bridge the photo-voltaic panels to the 240V, 60Hz grid through an 120V/240V transformer, as shown in Figure 4. The PWM gating signals are generated by a synchronous sinusoidal unipolar PWM generator with  $m_f = 81$ . The output voltage of the PV panel is variable depending on the weather condition. Assume that the inductance is 1mH and the transformer is ideal transformer. Answer following questions when the inverter is operating in steady state with no over modulation.

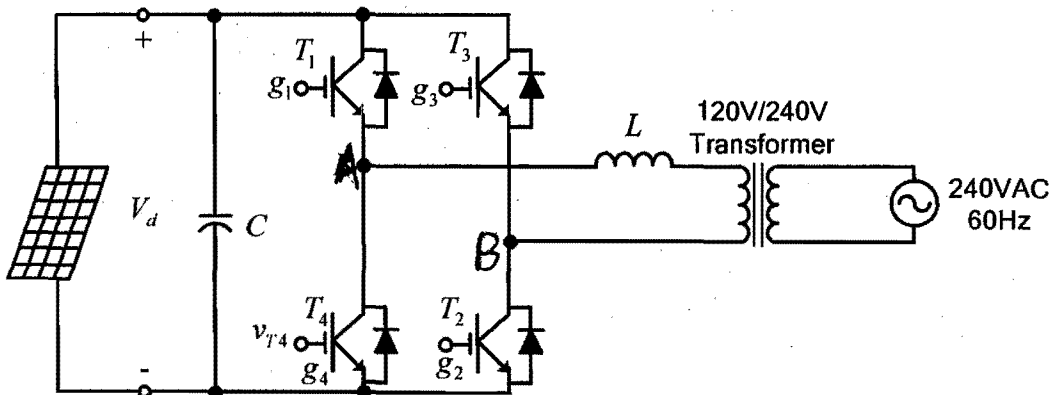
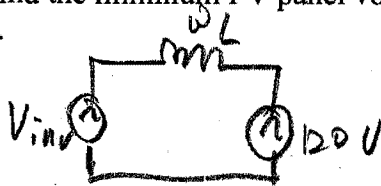


Figure 4 single phase single stage full-bridge PV inverter

4.1) Find the minimum PV panel voltage when the inverter outputs 5kW real power at unity power factor.

7%



At  $PF=1$ ,  $I_{inv} = \frac{5 \times 10^3}{120} = 41.67A$

$$\begin{aligned} \bar{V}_{inv} &= 120 \angle 0^\circ + j\omega L \cdot I_{inv} \angle 0^\circ \\ &= 120 + j2\pi \times 60 \times 1 \times 10^{-3} \times 41.67 \angle 0^\circ \\ &= 121.0237 \angle 7.4576^\circ \end{aligned}$$

$\sqrt{2} V_{inv,PK} = 2 \cdot m_a \cdot \frac{V_d}{2} = m_a \cdot V_d \quad \therefore V_{d,min} = V_{inv,PK}$  when  $m_a=1$

4.2) Explain the reason of using unipolar PWM in the PV applications.

5%

$V_{d,min} = \sqrt{2} \times 121.0237 = 171.15V$

The dominant harmonics at  $2m_f \pm 1$ .

$\therefore$  Smaller harmonics current and higher current THD

4.3) Find the two dominant harmonic currents if the modulation index is 0.8 and the inverter output power is 5kW at unity power factor.

8%  
 When  $m_a = 0.8$   $V_d = \frac{\sqrt{2} V_{in} U}{m_a} = \frac{171.15}{0.8} = 213.9417$

The dominant harmonics at  $2m_f \pm 1$ , the coefficient = 0.314

$V_{2m_f \pm 1} = 0.314 \times V_d = 67.18V$  4

$I_{2m_f+1} = \frac{V_{2m_f+1}}{(2m_f+1) \omega_s L} = \frac{67.18}{(2 \times 81 + 1) \times 2\pi \times 60 \times 1 \times 10^{-3}} = 1.0932A, PK, 0.7730A (RMS)$

$I_{2m_f-1} = \frac{V_{2m_f-1}}{(2m_f-1) \omega_s L} = \frac{67.18}{(2 \times 81 - 1) \times 2\pi \times 60 \times 1 \times 10^{-3}} = 1.1068A, PK, 0.7826A (RMS)$

**Questions 5 (25%)**

If a 1.5MW wind turbine is connected to a 13.8kV, 60Hz feeder through an 1.5MW, 3-phase 13.8kV/690V step-down transformer and full power converter. The grid side inverter and transformers are shown in Figure 5. The PWM gating signals are generated by a synchronous sinusoidal PWM generator with switching frequency at 1980Hz. The dc link voltage  $V_d = 1240V$ . The inductance of inductor  $L$  is 0.1pu. Assume the ideal transformer and ideal feeder, answer the following questions when the converter is operating in steady state with no over modulation.

Hint:  $S_{base} = 1.5MW$  and  $V_{base} = 690V$ .  $S_{base} = \sqrt{3} \cdot V_{base} \cdot I_{base}$  and  $Z_{base} = \frac{V_{base}}{\sqrt{3} \cdot I_{base}}$

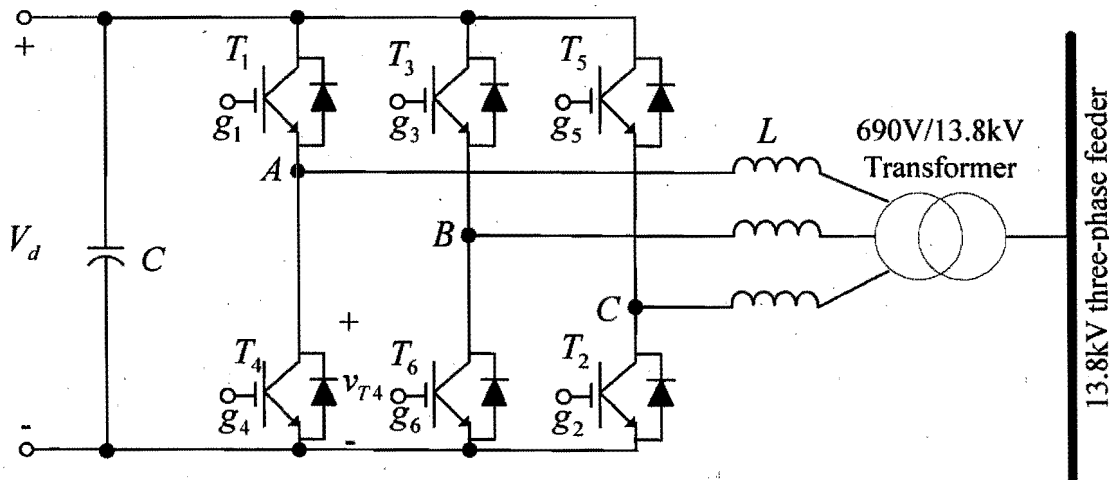


Figure 5 Wind turbine grid side inverter

5.1) Find the amplitude modulation index if the inverter is feeding 1.5MW real power to the feeder at unity power factor.

10%

$$S_b = 1.5 \times 10^6 \text{ W}$$

$$V_b = 690 \text{ V}$$

$$I_b = \frac{S_b}{\sqrt{3}V_b} = 1255.1 \text{ A}$$

$$Z_b = \frac{V_b}{\sqrt{3}I_b} = 0.3174 \Omega$$

$$L_b = \frac{Z_b}{2\pi f_b} = 0.842 \text{ mH}, \quad f_b = 60 \text{ Hz}$$

$$L = 10\% L_b = 0.0842 \text{ mH}$$

$$m_f = \frac{1980}{60} = 33$$

At Unity power factor

$$V_{inV,IP} = \frac{690}{\sqrt{3}} + 1255.1 \times j\omega_s L = 400.3586V \angle 5.71^\circ \frac{690}{\sqrt{3}} = 398.3717V$$

$$or\ V_{inV,IP} = 1 + 1 \times j1 \times 0.1 = 1.005 \angle 5.71^\circ \quad V_{inV,IP} = \frac{690}{\sqrt{3}} \times 1.005 = 400.3586V$$

$$V_{inV,LL} = (m_a \cdot \frac{V_d}{2} \cdot \sqrt{\frac{2}{2}} \cdot \sqrt{3}) = \sqrt{\frac{3}{8}} m_a \cdot V_d$$

$$V_{inV,phase} = m_a \frac{V_d}{2\sqrt{2}} = \sqrt{\frac{1}{8}} m_a \cdot V_d \rightarrow m_a = \frac{\sqrt{8} \times 400.3586}{1240} = 0.9132$$

5.2) Calculate the THD of the output current with only considering the dominant harmonic currents at  $m, \pm 2$  and  $m, \pm 4$ . Assume that the amplitude modulation index is 1.0 and modulating waves are in phase with the grid voltages.

10% When  $m_a = 1$ .  $V_{inV,p} = \sqrt{\frac{1}{8}} \times 1 \times 1240 = 438.4062A$

or  $V_{inV,LL} = 0.612 \times 1240 = 758.88V \rightarrow V_{inV,p} = 438.14V$

$$I_a = \frac{V_{inV} - \frac{690}{\sqrt{3}}}{\omega_s L} = \frac{438.40 - 398.37}{2\pi \times 60 \times 0.0842 \times 10^{-3}} = 1261.3A$$

$$I_{mf+2} = \frac{0.195 \times 1240 / \sqrt{3}}{(33+2) \times 2\pi \times 60 \times 0.0842 \times 10^{-3}} = 125.67A$$

$$I_{mf-2} = 141.88A$$

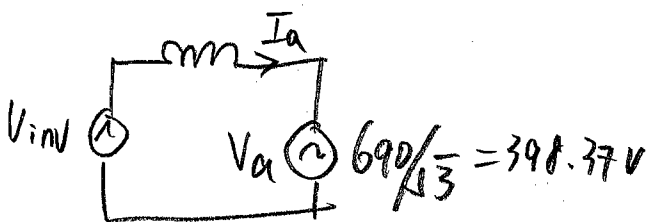
$$I_{mf+4} = \frac{0.011 \times 1240 / \sqrt{3}}{(33+4) \times 2\pi \times 60 \times 0.0842 \times 10^{-3}} = 6.70A$$

$$I_{mf-4} = 8.56A$$

5.3) When the real power is zero (no wind), the inverter is required to supply reactive power to the grid at 1.5MVar. Find the minimum dc link voltage.

5%

$$\therefore THD = \sqrt{I_{mf+2}^2 + I_{mf-2}^2 + I_{mf+4}^2 + I_{mf-4}^2} / I_a = 0.1505 = 15\%$$



$$V_{inV} = V_a + j\omega L_s \cdot I_a$$

$$= 398.37 + j \times 2\pi \times 60 \times 0.0842 \times 10^{-3} \times 1255 \angle 90^\circ = 438.21V$$

$$V_d = \frac{\sqrt{8} \times V_{inV,p}}{m_a}$$

$$\therefore V_{dmin} = \sqrt{8} V_{inV,p} = 1239.4V$$

Table 8-1 Generalized Harmonics of  $v_{AO}$  for a Large  $m_f$ .

$h$ \ $m_a$	0.2	0.4	0.6	0.8	1.0
1 <sup>st</sup>	0.2	0.4	0.6	0.8	1.0
$m_f$	1.242	1.15	1.006	0.818	0.601
$m_f \pm 2$	0.016	0.061	0.131	0.220	0.318
$m_f \pm 4$					0.018
$2m_f \pm 1$	0.190	0.326	0.370	0.314	0.181
$2m_f \pm 3$		0.024	0.071	0.139	0.212
$2m_f \pm 5$				0.013	0.033
$3m_f$	0.335	0.123	0.083	0.171	0.113
$3m_f \pm 2$	0.044	0.139	0.203	0.176	0.062
$3m_f \pm 4$		0.012	0.047	0.104	0.157
$3m_f \pm 6$				0.016	0.044

Note:  $(\hat{V}_{AO})_h / \frac{1}{2} V_d [ = (\hat{V}_{AN})_h / \frac{1}{2} V_d ]$  is tabulated as a function of  $m_a$  where  $(\hat{V}_{AO})_h$  are the peak values of the harmonic voltages

Table 8-2 Generalized Harmonics of  $v_{LL}$  for a Large and Odd  $m_f$ .

$h$ \ $m_a$	0.2	0.4	0.6	0.8	1.0
1 <sup>st</sup>	0.122	0.245	0.367	0.49	0.612
$m_f \pm 2$	0.010	0.037	0.080	0.135	0.195
$m_f \pm 4$				0.005	0.011
$2m_f \pm 1$	0.116	0.2	0.227	0.192	0.111
$2m_f \pm 5$				0.008	0.020
$3m_f \pm 2$	0.027	0.085	0.124	0.108	0.038
$3m_f \pm 4$		0.007	0.029	0.064	0.096
$4m_f \pm 1$	0.100	0.096	0.005	0.064	0.042
$4m_f \pm 5$			0.021	0.051	0.073
$4m_f \pm 7$				0.010	0.030

Note:  $(V_{LL})_h / V_d$  are tabulated as a function of  $m_a$  where  $(V_{LL})_h$  are the rms values of the harmonic voltages