DEPARTMENT OF MATHEMATICS

MIDTERM TEST #1

MTH 140 – Calculus I

Last Name (Print): [redacted] First Name: [redacted] Student Number: [redacted]

Signature [redacted]

Date: Sept. 28, 2012, 4:00 pm

Duration: 1.5 hours

Section (circle one)

Dr. Fisseha :	1 2 3 4 5
Dr. Ferrando :	6 7 8 9 10
Dr. Alvarez :	11 12 13 14 15 16 17 18 19 20
Dr. Ha :	21 22 23 24 25

Instructions:

1. This is a closed-book test. **Notes, calculators and other aids are not permitted.**
2. Verify that your test has pages 1-8.
3. (a) Unless otherwise instructed, **make sure you include all significant steps in your solution, presented in the correct order. Unjustified answers will be given little or no credit. Cross out or erase all rough work not relevant to your solution.** Put a box around your final **answer.**

   (b) For multiple choice questions make sure to write your answers in the box at the end of each question **carefully**. There are no part marks in the multiple-choice section and **only** the answer in the box will be marked. The correct response gets full marks, an incorrect response or no response gets no marks.

   (c) Write your solutions in the space provided. If you need more space, use the back of the page. Indicate this fact on the original page, making sure that your solution cannot be confused with any rough work which may be there. Marks (out of 50) are shown in brackets.
4. Do not separate the sheets.
5. Have your student card available on your desk.

For Instructor's use only.

Page	Mark
MC	3 / 9
3	5 /5
4	6 /6
5	2 /7
6	4 /8
7	1 /9
8	1 /6
Total	22 /50

$x - 2 \geq 0$

$x \geq 2$

$1 - \sqrt{x-2} \neq 0$ $\quad x \geq 2 \quad x \in [2,3)\,(3,\infty)$

$\sqrt{x-2} \neq 1$ $\quad x \neq 3, 1$

$x - 2 \neq \pm 1$

$x \neq 3, 1$

1. [3 marks] (Multiple choice question). The domain of the function $f(x) = \dfrac{1}{1 - \sqrt{x-2}}$ is the set:

   A) $(2, \infty)$ B) $[2, \infty)$ C) $(2,3) \cup (3, \infty)$ D) $[2,3) \cup (3, \infty)$ E) None of these

   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. $\boxed{D}$

2. [3 marks] (Multiple choice question). If $\cot \beta = 3$ and $\pi < \beta < 2\pi$, then $\cos \beta$ is equal to:

   A) $1/3$ B) $3/\sqrt{10}$ C) $-1/\sqrt{10}$ D) $-\sqrt{10}/3$ E) None of these

   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. $\boxed{B}$

$\cot \beta = 3$

$\dfrac{\sin}{\cos} = \dfrac{1}{3}$ $\quad \dfrac{\cos \beta}{\sin \beta} = 3$

$\sin \beta = \dfrac{\cos \beta}{3}$ $\quad \cos \beta = 3 \sin \beta$

$\cos^2 \beta + \sin^2 \beta = 1$

$\cos^2 \beta + \dfrac{\cos^2 \beta}{9}$

$9 \cos^2 \beta + \cos^2 \beta = 9$

$10 \cos^2 \beta = \dfrac{\sqrt{9}}{\sqrt{10}} = \dfrac{3}{\sqrt{10}} = \cos \beta$

3. [3 marks] (Multiple choice question). Let $f(x) = |(x+1)(x+2)| + |x-3|$ and consider a number x belonging to the interval $(-2, -1)$. Then $f(x)$ is equal to:

   A) $x^2 + 2x + 5$ B) $x^2 + 4x - 1$ C) $-x^2 - 2x - 5$ D) $-x^2 - 2x - 1$

   E) None of these

   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. $\boxed{D}$

$\cancel{(-x+1)}(-x+2)$

$x^2 + 3x + 2 \,| + |x - 3|$

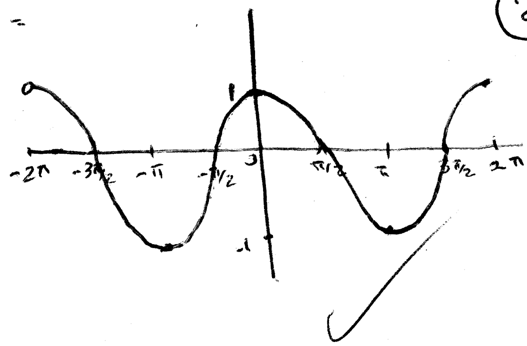
$|x^2 + 3x + 2|\,|$

$-x^2 + 3x - 2 \quad -x + 3$

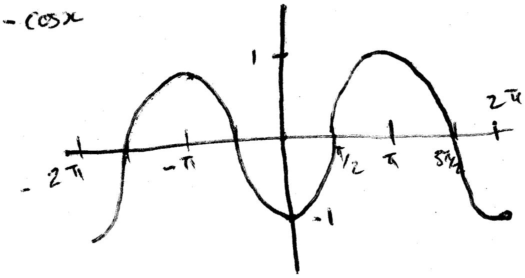
$-x^2 + 2x + 1$

4. [5 marks] Starting with the graph of $g(x) = \cos x$, apply the appropriate transformations to sketch the graph of $f(x) = |2 - 3\cos x|$. Show clearly all your steps.

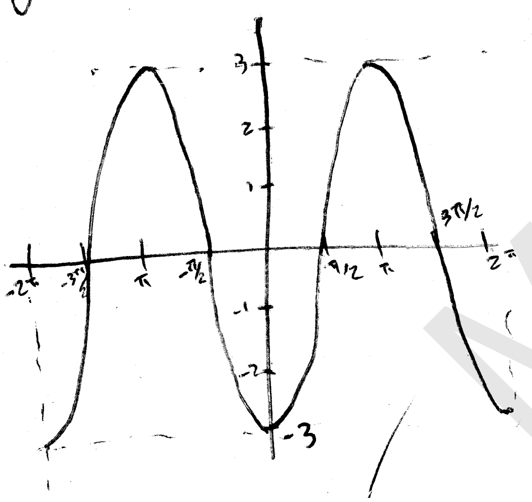
1) $f(x) = \cos x$



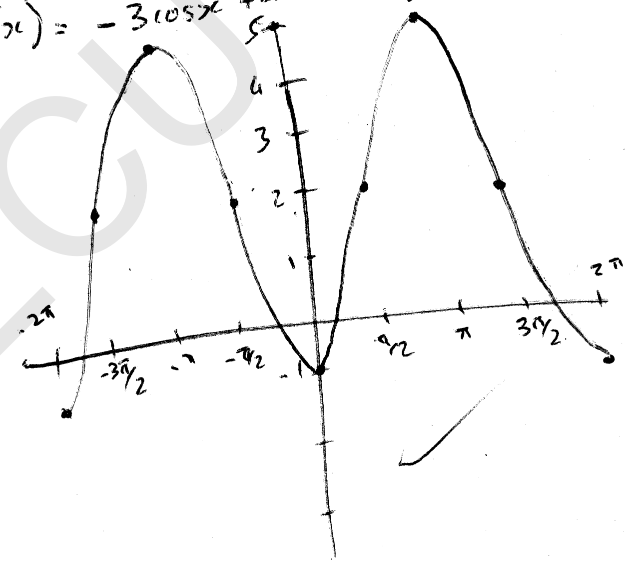
2) $f(x) = -\cos x$



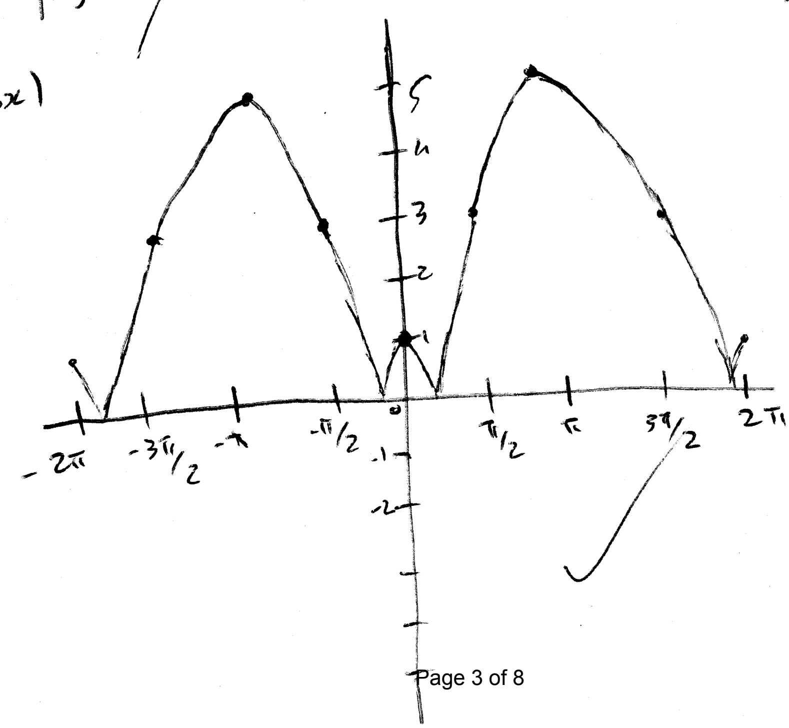
3) $f(x) = -3\cos x$



4) $f(x) = -3\cos x + 2$ or $2 - 3\cos x$



5) $f(x) = |2 - 3\cos x|$



5/5

5. [6 marks] Prove the trigonometric identity

$$\frac{\sin\theta}{1-\cos\theta} = \csc\theta + \cot\theta$$

Show clearly all your steps.

$\frac{\sin\theta}{1-\cos\theta} = \csc\theta + \cot\theta$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{1+\cos\theta}{\sin\theta}$ using trig identities ; $\sin^2\theta + \cos^2\theta = 1$

$\sin\theta + \cos\theta = 1$

~~$\cos\theta$~~ $+$

$\sin\theta = 1 - \cos\theta$

$=$ ~~$1+\cos\theta$~~

$\frac{\sin\theta}{1-\cos\theta} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}$

$= \frac{\sin\theta + \sin\theta\cos\theta}{1-\cos^2\theta}$ $\therefore 1-\cos^2\theta = \sin^2\theta$

$= \frac{\sin\theta + \sin\theta\cos\theta}{\sin^2\theta}$

$= \frac{\cancel{\sin\theta}}{\sin^2\theta} + \frac{\cancel{\sin\theta}\cos\theta}{\cancel{\sin^2\theta}}$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

$= \csc\theta + \cot\theta$ $\left(\therefore \frac{1}{\sin\theta} = \csc\theta \text{ and } \frac{\cos\theta}{\sin\theta} = \cot\theta\right) = \text{R.H.S} \therefore \text{proved}$

6

6. [7 marks] Let $f(x) = \sin^{-1}(x)$ and $g(x) = 2x^2 - \frac{1}{2}$.

   a) Evaluate $(f \circ g)(0)$

   b) Evaluate $(g \circ f)(0)$

   c) Find the domain of the composite function $(f \circ g)(x)$

$\sin^{-1}(0) = 0, \pi, 2\pi$

b) $g \circ f = g(f(x)) = g(\overset{\sin^{-1}(x)}{\cancel{2x^2 - 1/2}}) = 2[\sin^{-1}(x)]^2 - 1/2$

if $x = 0$ $g(f(0)) = 2[\sin^{-1}(0)]^2 - 1/2$

$= 2[0]^2 - 1/2$

$= -1/2$

$= 2 - 1/2$

1

$g(f(0)) = \boxed{3/2}$

or if $\sin^{-1}(0) = \pi$ then

$= 2[\pi^2] - 1/2$

$g(f(0)) = \boxed{\frac{4\pi^2 - 1}{2}}$

if $\sin^{-1}(0) = 2\pi$ then

$f(g(0)) = \frac{8\pi^2 - 1}{2}$

$\therefore g(f(0)) = 4(k\pi)^2 - 1$

$\therefore g(f(0)) = \frac{4(k\pi)^2 - 1}{2}$

where $k \in \mathbb{Z}$

a) $f \circ g = f(g(x)) = f(\cancel{\sin^{-1}(x)})(2x^2 - 1/2)$

$f \circ g = f(g(x)) = \sin^{-1}(2x^2 - 1/2)$

1

$\therefore f(g(0)) = \sin^{-1}(2 \cdot 0 - 1/2)$

$= \sin^{-1}(-1/2)$

$= 7\pi/6$ or $11\pi/6$

$(\therefore \sin \pi/6 = 1/2)$

$(\therefore \cancel{\sin 7\pi/6} = 7\pi/6$ or $11\pi/6)$

$= -\frac{\pi}{6}$

$f(g(0)) = 7\pi/6 + k2\pi$ or $11\pi/6 + k2\pi$

where $k \in \mathbb{Z}$

c) Find domain

$(f \circ g)(x) = \sin^{-1}(2x^2 - 1/2)$

Domain of sin is all Real numbers.

Range of sin is $1 \geq y \geq -1$

Domain of $\sin(2x^2 - 1/2)$ is all Real #'s

Range is $\therefore 1 \geq y \geq -1$

$\Rightarrow 2 \geq y \geq 2$

$2x^2 \geq y \geq -2x^2$

$2x^2 - 1/2 \geq y \geq -2x^2 - 1/2$

So Domain of inverse of sin $= \sin^{-1}(2x^2 - 1/2)$. $2x^2 - 1/2 \geq x \geq -2x^2 - 1/2$

is the ~~range~~ of function $\sin(2x^2 - 1/2)$ which is ~~all Real #'s~~.

$\therefore$ ~~Ran~~ Domain of $\sin^{-1}(2x^2 - 1/2)$ of $f \circ g$ is $2x^2 - 1/2 \geq x \geq -2x^2 - 1/2$

$\ln(a\cdot b) = \ln a + \ln b$

7. [8 marks] Consider the function

$$f(x) = 2\ln x - \ln(x^2+1)$$

x^2+1

1

a) Find a formula for the inverse function f^{-1}

b) What is the range of f^{-1}

c) What is the range of f

a) $f(x) = y = 2\ln x - \ln(x^2+1)$

$\frac{y}{2} = \ln\left(\frac{x}{x^2+1}\right)$ 1

$e^{y/2} = \frac{x}{x^2+1}$

$\sqrt{e^y}(x^2+1) = x$ +1

$\sqrt{e^y}x^2 + \sqrt{e^y} = x$

$\sqrt{e^y} = x - \sqrt{e^y}x^2$

$= x(1-\sqrt{e^y})$

+ −2

b) Range of f^{-1} is domain of f

for domain of f x^2 ~~+1>~~ 1 and $x > 0$ 1

∴ Range

$\therefore e^{\ln\left(\frac{x}{x^2+1}\right)} = \frac{x}{x^2+1}$

c) Domain of f.

$x^2+1>0$

$x^2 > -1$

$x > \pm 1$ + −2

~~Range f~~

$\frac{\text{small \#}}{-\text{small \#}}$ $4.01-4 = \frac{4.01}{.01} = 1$ $\frac{+\text{small \#}}{+\text{small \#}}$

8. [9 marks] Consider the function

$$f(x) = \frac{x^2-4}{|x|-2}$$

a) Evaluate $\lim_{x\to -2^-} f(x)$

$\lim_{x\to-2^-} f(x) = \lim_{x\to -2^-} \frac{x^2-2^2}{|x|-2} = \lim_{x\to -2^-} \frac{(x+2)(x-2)}{-(-x)-2} = \frac{-x+2}{-1} = \frac{-2+2}{-1} = \frac{0}{1}$

$\frac{\text{small \#}}{+\text{small \#}}$

b) Evaluate $\lim_{x\to -2^+} f(x)$

c) Evaluate $\lim_{x\to -2} f(x)$

d) Sketch the graph of $f(x)$

a) $\lim_{x\to -2^-} f(x) = \frac{\text{small +ve \#}}{\text{smaller +ve \#}} = \infty$ 0

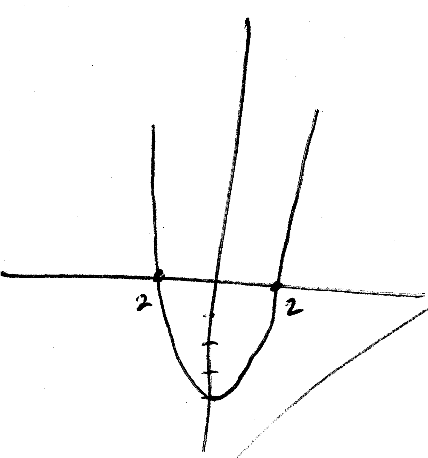
b) $\lim_{x\to -2^+} f(x) = \frac{\text{small +ve \#}}{\text{very small +ve \#}} = \infty$ 0

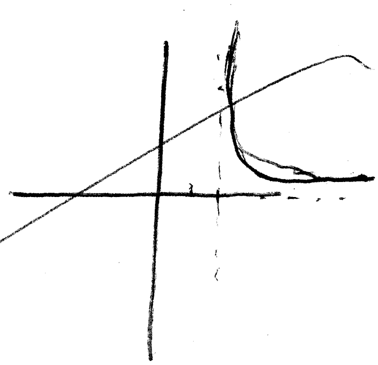
c) $\lim_{x\to 2} \frac{x^2-4}{|x|-2}$ $\frac{(x)^2-2^2}{(-x-2)} = \frac{(-x+2)(-x+2)}{(-x-2)} = \lim_{x\to -2} x+2 = -2+2 = 0$

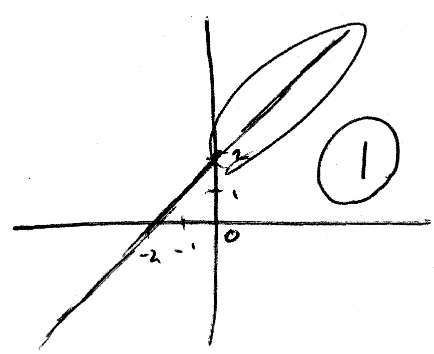
d) $g(x) = x^2-4)$ $h(x) = \frac{1}{|x|-2}$ $f(x) = \frac{x^2-4}{|x|-2} = \frac{x^2-2^2}{x-2} = \frac{(x+2)(x-2)}{(x-2)}$

$f(x) = x+2$

graph of this







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9. [6 marks] Let the function

$$f(t) = \frac{1}{t\sqrt{1+t}} - \frac{1}{t}$$

a) Is $t = 0$ a vertical asymptote of $f(t)$? Show your work. Yes

if $t=0$ $\frac{1}{0\sqrt{1+0}} - \frac{1}{0}$ = undefined as division by zero

b) Is $t = -1$ a vertical asymptote of $f(t)$? Show your work.

$1+t \geq 0$

$t \geq -1$

but

$t\sqrt{1+t} \geq 0$

$\sqrt{1+t} > 0$

$1+t > 0$

$t > -1$

by plugging in $t=-1$ we get

$= \frac{1}{-1\sqrt{1-1}} - \frac{1}{-1}$ ~~$= 0+1 = 1 \therefore f(t)$ at $t=-1$ is defined~~

$= \frac{1}{-1 \cdot 0} + 1$

$= \frac{1}{0} + 1$ Division by zero so function cannot be defined.

$\therefore$ The function has a vertical asymptote at $t=-1$

1