Kinetic Energy
Work
Potential Energy

Tutorial #1 PHYS1004
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1 Kinematics Constant Acceleration

- Equations of motion can be obtained by integrating a constant acceleration
- Enough to solve any constant acceleration problem
  - Solve as simultaneous equations
- Additional useful forms (review from high school):

$$a = \frac{dv}{dt} \Rightarrow v = v_0 + at \Rightarrow x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Learning Objectives

- Apply the relationship between a particle's kinetic energy, mass, and speed.
- Identify that kinetic energy is a scalar quantity.

2 Kinetic Energy

- **Kinetic energy**:
  - The faster an object moves, the greater its kinetic energy
  - Kinetic energy is zero for a stationary object
- For an object with velocity "*v*":

$$K = \frac{1}{2}mv^2$$

- The unit of kinetic energy is a **joule** (J)

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2.$$

Learning Objectives

- Apply the relationship between a force (magnitude and direction) and the work done on a particle by the force when the particle undergoes a displacement.
- Calculate work by taking a dot product of the force vector and the displacement vector, in either magnitude-angle or unit-vector notation.
- Apply the work-kinetic energy theorem to relate the work done by a force (or the net work done by multiple forces) and the resulting change in kinetic energy.

3 Work and Kinetic Energy

- Account for changes in kinetic energy by saying energy has been transferred *to* or *from* the object
- In a transfer of energy via a force, **work** is:
  - *Done on the object by the force*

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

- This is not the common meaning of the word “work”
  - To do work on an object, energy must be transferred
  - Throwing a baseball does work
  - Pushing an immovable wall does not do work

3 Work and Kinetic Energy

- Start from force equation and 1-dimensional velocity:

$$F_x = ma_x, \quad \Rightarrow \quad v^2 = v_0^2 + 2a_x d.$$

- Rearrange into kinetic energies:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d.$$

- The left side is now the change in energy

- Therefore work is:

$$W = F_x d.$$

3 Work and Kinetic Energy

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

- For an angle φ between force and displacement:

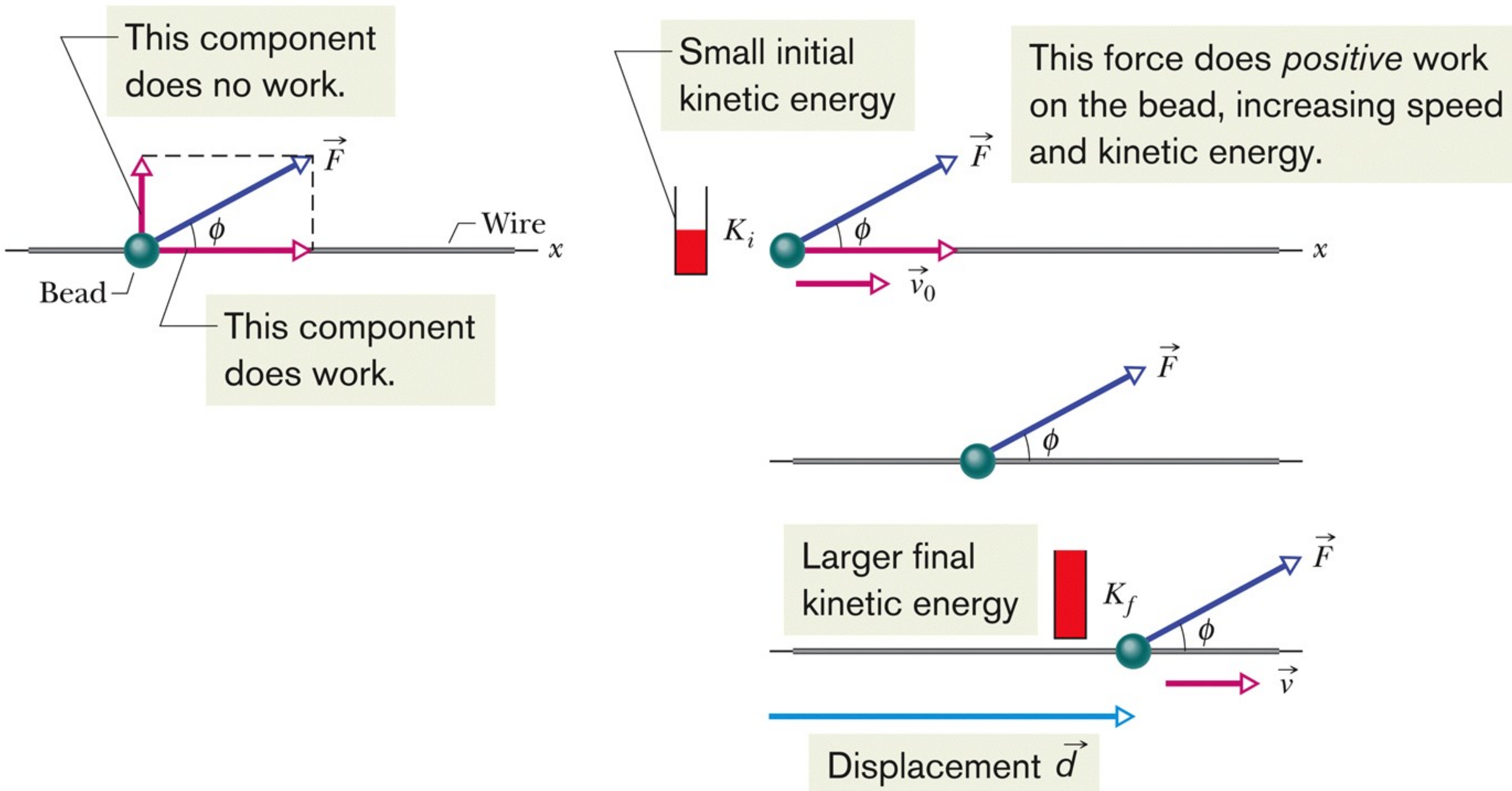
$$W = Fd\cos\phi$$

- As vectors we can write:

$$W = \vec{F} \cdot \vec{d}$$

- Notes on these equations:
  - Force is constant
  - Object is particle-like (rigid)
  - Work can be positive or negative

3 Work and Kinetic Energy



- Work has the SI unit of joules (J), the same as energy
- Work transferred into kinetic energy

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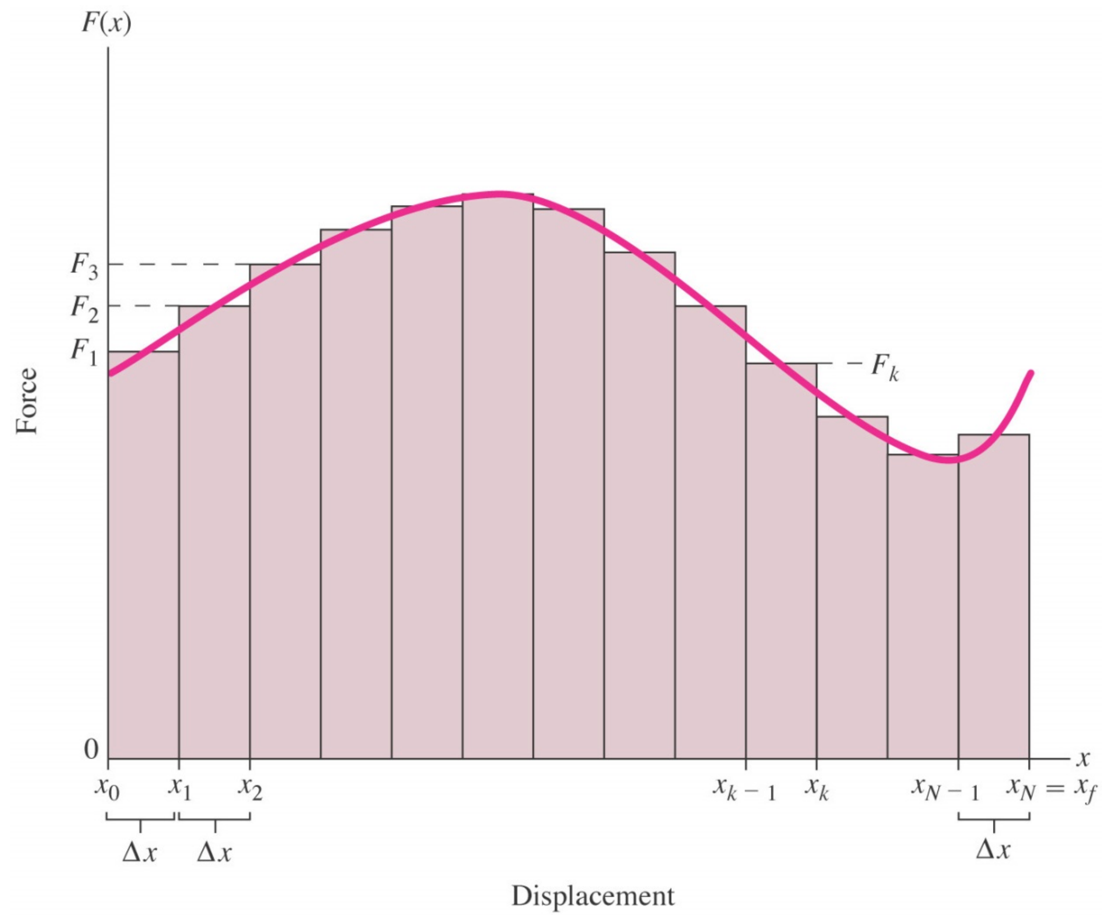
4 Work Done by a General Variable Force

- We take a one-dimensional example
- We need to integrate the work equation (which normally applies only for a constant force) over the change in position
- We can show this process by an approximation with rectangles under the curve



$$W = \int_{x_0}^{x_f} F(x)\, dx$$

Potential Energy and Conservative Forces

Potential Energy = - Work

U(x) = - *W*

Using the force rather than the work:

$$U(x) - U(x_0) = -\int_{x_0}^{x} F(z)\, dz$$

The inverse:

$$F(x) = -\frac{dU(x)}{dx}$$

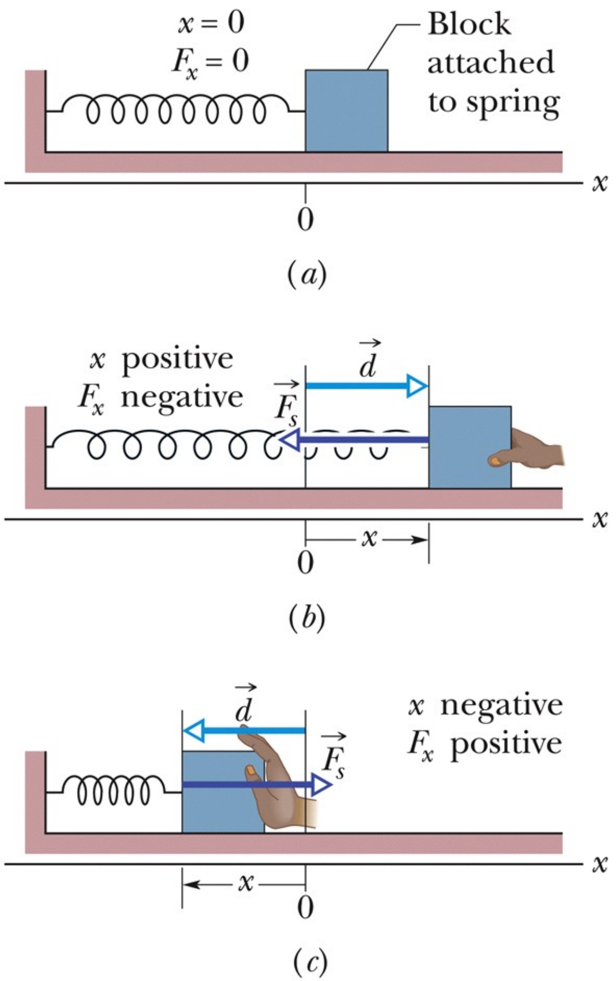
5 Work Done by a Spring Force (an example)

Learning Objectives

- Identify that a spring force is a variable force.
- Calculate the work done on an object by a spring force by integrating the force from the initial position to the final position of the object or by using the known generic result of the integration.
- Calculate work by graphically integrating on a graph of force versus position of the object.

5 Work Done by a Spring Force (an example)

- A **spring force** is the *variable force* from a spring
  - A spring force has a particular mathematical form
  - Many forces in nature have this form
- Figure (a) shows the spring in its **relaxed state**: since it is neither compressed nor extended, no force is applied
- If we stretch or extend the spring it resists, and exerts a *restoring force* that attempts to return the spring to its relaxed state



5 Work Done by a Spring Force (an example)

- The spring force is given by **Hooke's law**:

$$\vec{F}_s = -k\vec{d}$$

- The negative sign represents that the force always opposes the displacement
- The **spring constant** *k* is a is a measure of the stiffness of the spring
- This is a variable force (function of position) and it exhibits a linear relationship between *F* and *d*
- For a spring along the *x*-axis we can write:

$$F_x = -kx$$

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5 Work Done by a Spring Force (an example)

- We can find the work by integrating:

$$W_s = \int_{x_i}^{x_f} -F_x \, dx.$$

- Plug *kx* in for F_x:

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$$

- The work:
  - Can be positive or negative
  - Depends on the *net* energy transfer

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

5 Work Done by a Spring Force (an example)

- For an initial position of $x = 0$:

$$W_s = -\tfrac{1}{2}kx^2$$ **Eq. (7-26)**

- For an applied force where the initial and final kinetic energies are zero:

$$W_a = -W_s.$$ **Eq. (7-28)**

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

6 Summary

Kinetic Energy

- The energy associated with motion

$$K = \frac{1}{2}mv^2$$ **Eq. (7-1)**

Work

- Energy transferred to or from an object via a force
- Can be positive or negative

Work Done by a Constant Force

$$W = Fd\cos\phi$$ **Eq. (7-7)**

$$W = \vec{F}\cdot\vec{d}$$ **Eq. (7-8)**

- The **net work** is the sum of individual works

Work and Kinetic Energy

$$\Delta K = K_f - K_i = W,$$ **Eq. (7-10)**

$$K_f = K_i + W,$$ **Eq. (7-11)**

6 Summary

Work Done by a Variable Force

- Found by integrating the constant-force work equation

$$W = \int_{x_i}^{x_f} F(x)\, dx$$

Potential Energy

- The inverse with *U(x)* = - *W*:

$$F(x) = -\frac{dU(x)}{dx}$$

Forces that Vary with Position

Work Done by a Spring with *F* = -*k x*

Integrating gives the work done to stretch

the spring from 0 to L:

$$W = -k\int_0^L x\, dx = -\tfrac{1}{2}kx^2\Big|_0^L = -\tfrac{1}{2}kL^2$$