

Student Name: _____



McGill ID: _____

Course Number: AEMA 101

Course Name: Calculus 1

Examination Date: December 8, 2011

Examination Time: 14:00-17:00

Value of Exam: 40%

Name of Examiner: J.F. Hayes

Name of Co-examiner: M. Knutt

Type of Exam: Final Take Home Multiple Choice Deferral Special Arrangement

Answer Directly on Exam Paper

Answer in Exam Booklet(s)

Calculators are permitted: Yes No If yes, specify i.e. (Graphical, Statistical): _____

Definition Dictionaries are allowed: Yes No

Answer of Scantron: indicate short (1-120 responses)

or long (121-240 responses)

**Electronic devices (i.e. beepers, cellular phones)
are not permitted in the examination room.**

AEMA 101, Calculus I
FINAL EXAMINATION

Examiners: J.F. Hayes and M. Knutt

Date: December 8, 2011
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Use of calculators is not permitted

1. A paper cup containing water has the shape of a frustrum of a right circular cone of height 6 inches, and lower and upper base radii of 1 and 2 inches, respectively. If water is leaking out of the cup at a rate of 3 inches² per hour, at what rate is the water decreasing when the depth of water is 4 inches?
[The volume V of a frustrum of a right circular cone of height h and lower and upper base radii a and b , respectively is $V = \frac{1}{3}\pi h(a^2 + b^2 + ab)$]. 12 marks

2. An industrial tank is to be constructed by adjoining two hemispheres to the ends of a right circular cylinder. The volume of the tank is to be 10π ft³. The material for the hemispherical ends costs \$2 per ft², and that of the cylindrical part costs \$1 per ft². Find the dimensions of the tank that will minimize the cost of constructing the tank. 12 marks

3. Analyse and sketch the graph of $f(x) = \frac{x^2}{x+1}$ paying particular attention to (a) domain, (b) symmetry, (c) asymptotes, (d) intervals of increase and decrease, (e) local extrema, (f) concavity, and (g) points of inflection. 12 marks

- 4(a) Animal physiologists use the formula $S = 0.1w^{2/3}$ to calculate the surface area (in metres²) of a horse from its mass w (in kilograms). A physiologist estimates the mass of a horse to be 216 kg with a possible maximum error of measurement of ± 0.5 kg. Using differentials, determine the maximum percentage error in the calculation of the horse's surface area. 2 marks

- (b) The dosage D in mg of diphenhydramine for a dog of body mass w kg is $D = 4.7w^{2/3}$. Using differentials, estimate the maximum allowable error in w for a cocker spaniel of mass 10 kg if the percentage error in D must be less than 3%. 4 marks

(c) A silo has the shape of a right circular cylinder surmounted by a hemisphere. The altitude of the cylinder is 100 feet and its radius is 10 ft. Use differentials to approximate the amount of paint required to apply a coat of paint 0.001 feet thick to the outside of the silo. 4 marks

(d) Use differentials to approximate $\cos 92^\circ$. 2 marks

5. Find the absolute extrema of $f(x) = 2x\sqrt{16 - 2x^2}$. 10 marks

6. Let $y = f(x)$ be defined implicitly by $\sqrt{x+y} = 1 + x^2y^2$

(a) Find $\frac{dy}{dx}$ (i.e. $f'(x)$) 4 marks

(b) Find $f(0)$ and $f'(0)$ 4 marks

(c) Use differentials to approximate y at $x = 0.01$ 4 marks

7. Using the limit definition of the derivative, $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$, find 10 marks
the derivative of $f(x) = \frac{1}{\sqrt{2x+5}}$ at $x = -2$.

8. Evaluate the following limits if they exist:

(a) $\lim_{x \rightarrow 0} \frac{\tan 7x}{2x}$ 5 marks

(b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{3+7x^2}}{x-1}$ 5 marks

- 9(a) State the Intermediate Value Theorem (IVT) 2 marks
- (b) At 8:00 AM on Saturday morning, a man begins to run up a path on the side of a mountain to his campsite and it takes him 40 minutes to get there. At 8:00 AM the following morning he runs back down the path and it takes him 20 minutes to get down. At some point on the path on the way down he claims he passed the same point on the way up at exactly the same time. Use the IVT to show that this is possible. 8 marks