Last name_____ First name_____ Student # _____

Instructor: Dr. Hua
Closed book, no calculators!
Total = 20 marks

I. Multiple-choice Questions (2 × 5 = 10 marks)

Write your answer to questions in this part in the following boxes:

1. ☐ 2. ☐ 3. ☐ 4. ☐ 5. ☐

AACED

1. $\lim_{x\to+\infty}\frac{\sqrt{4x^2+1}}{x}=$

(A) 2; (B) −2; (C) 4; (D) −4; (E) ∞.

Answer. (A)

2. Suppose an amount $10,000 is deposit in a bank account that offers annual interest rate 3% compounded continuously. Then the balance after t years is

(A) $10000\times e^{0.03\,t}$; (B) $10000\times 0.03e^{t}$; (C) $10000\times e^{0.03t/12}$;
(D) $10000\times(1+0.03)^{t}$; (E) $10000\times(1+0.03/12)^{t}$.

Answer. (A)

3. The derivative of the function $y=\frac{x^{1/3}}{x-1}$ is $y'=-\frac{2x+1}{3x^{2/3}(x-1)^2}$. The critical number(s) of this function is/are

(A) −1 / 2 only; (B) −1 / 2 and 1 only; (C) −1 / 2 and 0 only;
(D) 0 and 1 only; (E) −1 / 2, 0 and 1 only.

Answer. (C) At $x = 0$, the derivative does not exist. At $x = -1/2$, the derivative is zero. They are critical numbers. The derivative does not exist at $x = 1$, but $x = 1$ is not in the domain of the function. It is not a critical number.

4. If $2x + 3y = 24$, $x \ge 0$, $y \ge 0$, then the maximum value of $V = x^2y$ is

(A) 144; (B) 1024 / 9; (C) 216; (D) 256;
(E) 512 / 3.

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Solution. (E) Since $y = \frac{1}{3}(24 - 2x)$, $V = \frac{1}{3}x^2(24 - 2x) = \frac{1}{3}(24x^2 - 2x^3)$, $0 \le x \le 12$, and $V' = \frac{1}{3}(48x - 6x^2)$. Let $V' = 0$. $x = 0, 8$. When $x = 0$, $y = 8$, and $z = 0$; when $x = 8$, $y = 8 / 3$, and $x^2y = 8^3 / 3$. Since $V' > 0$ when $0 < x < 8$, and $V' < 0$ when $x > 8$, V attains a relative maximum at $x = 8$. Since $V(0) = 0$, and $V(12) = 0$, this relative maximum is an absolute maximum.

5. Let $F(x)$ be an antiderivative of the function $f(x) = 2x^3 - x$ such that $F(0) = 1$. Then $F(2) =$

(A) 1; (B) 3; (C) 5; (D) 7; (E) 9.

Solution. (D) The general antiderivative of $f(x)$ is $\int (2x^3 - x)dx = \frac{1}{2}(x^4 - x^2) + C$. Since $F(0) = C = 1$. $F(x) = \frac{1}{2}(x^4 - x^2) + 1$. Then $F(2) = 7$.

II. Long-answer Questions (10 marks)

1. (8 marks) Consider the function $y = \frac{x-1}{x^2}$. The derivative of this function is $y' = -\frac{x-2}{x^3}$, and the second derivative of this function is $y'' = \frac{2x-6}{x^4}$.

(a) (2 marks) For which values of x, does this function decrease?

(b) (2 marks) For which values of x, is the graph of this function concave down?

(c) (2 marks) Find the horizontal and vertical asymptotes.

(d) (2 marks) Sketch the graph of the function.

Solution. (a) $x < 0$ or $x > 2$.

(b) $x < 0$ or $0 < x < 3$.

(c) HA: y=0; VA: x=0.

(d)

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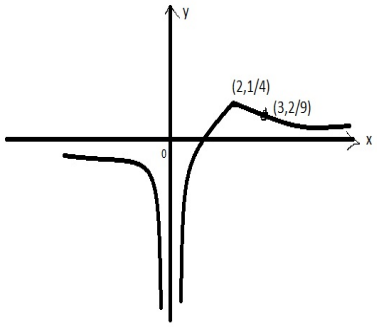

2. (2 marks) Find the following indefinite integral: $\int \frac{\sqrt{\ln x}}{x} dx$.

Solution. Let $u = \ln x$. Then $u' = 1 / x$.

$$\int \frac{\sqrt{\ln x}}{x} dx = \int \sqrt{u} du = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (\ln x)^{3/2} + C .$$