ELECTROSTATICS

Electric charges & Fields

Electrostatics is a branch of physics dealing with the study of nature of electric charges at rest.

STATIC ELECTRICITY (FRICTIONAL ELECTRICITY)

The electricity (charges at rest) develops on object when they're rubbed with each other. The electric charges produced on the surface of a body by friction can't move from one part of the object ~~from~~ to other part. For this reason, frictional electricity is also known as static electricity.

KINDS OF CHARGES

There're 2 kinds of charges,

i). positive charge

ii). negative charge

NOTE : i). Charge is a scalar quantity

ii) Unit of charge is Coulomb (C).

iii) Charge on a proton = 1.6×10^{-19} C.

iv) Charge on electron = -1.6×10^{-19} C.

Origin of electric charge in frictional electricity

i) When 2 objects (glass rod & silk) are rubbed together, they get charged (electrified)

ii) This is due to transfer of electrons from one object to the other.

iii) The object which gains electrons become negatively charged while the object which looses e^- become positively charged.

iv) In a glass rod electrons are less tightly bond as compared to those in silk cloth. As a result, electrons get removed from glass rod & transfer to silk.

v) Hence glass rod becomes positively charged & silk cloth negatively charged.

ELECTRON NO. DENSITY

$$\text{Electron no. density} = \frac{\text{No. of } \bar{e}}{\text{volume}}$$

METHODS FOR CHARGING A BODY

* Charging by conduction
* " by friction.
* " by induction

PROPERTIES OF CHARGES

* Like charges repel & unlike charges attract each other.

* Charges are conserved i.e electric charges obey the law of conservation of charges (For an isolated system, the net charge is ~~zero~~ constant). In other words, charges can neither be created nor be destroyed.

* Charges are additive in nature. i.e the total charge on a body is equal to the algebraic sum of all electric charges distributed on different parts of the objec

* Unlike mass, the total charge of an object isn't affected by its motion.

* Charges are quantised.

QUANTISATION OF CHARGE (discrete nature of charge)

The total charge on a body is alway some integral multiple of elementary charge (either charge of proton or $\bar{e}$) is called quantisation of charge.

i,e $\boxed{Q = \pm ne}$

where $n = 0, 1, 2, 3, \ldots\ldots$

$\therefore$ $Q = e, \pm 2e, \pm 3e, \ldots\ldots$

are net charge of a body.

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2. Find the no. of $\bar{e}$ transferred to cell if one C of charge is produced on a glass rod.

Given, $q = 1C$ $n = ?$

$e = 1.6 \times 10^{-19}$ C

we've

$Q = ne$

$1 = n \times 1.6 \times 10^{-19}$ C

$\therefore n = \frac{10^{19}}{1.6}$

$= \frac{\cancel{16}^{5} \times 10^{19}}{\cancel{168}}$

$= 6.2 \times 10^{18}$

$$\begin{array}{r} 0.62 \\ 8\overline{)5} \\ 0 \\ \hline 50 \\ 48 \\ \hline 20 \end{array}$$

3. A polythene piece rubbed with wool is formed to ~~ba~~ have a negative charge of 3×10^{-7} C. Estimate the no. of $\bar{e}$ transfered from wool to polythene.

$q = -3 \times 10^{-7}$ C $e = -1.6 \times 10^{-19}$ C

$n = ?$

$q = ne$

$n = \frac{q}{}$

$$= \frac{3 \times 10^{-7}}{1.6 \times 10^{-19}}$$

$$= \frac{3 \times 10^{13}}{16}$$

$$= 1.9 \times 10^{12} \; //$$

COULOMB'S INVERSE SQUARE LAW

The Force of attraction or repulsion between 2 point charges is directly proportional to the product of the 2 charges & inversely proportional to the square of the distance between them.

If q_1 & q_2 are the 2 charges separated by distance 'r' then,

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = k \frac{q_1 q_2}{r^2}$$

where $k = \frac{1}{4\pi \varepsilon_0}$

$\varepsilon_0 \rightarrow$ absolute permittivity of free space

$[\varepsilon_0 = 8.854 \times 10^{-12} \; C^2 N^{-1} m^{-2}]$

$$\therefore \boxed{F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2}}$$

but $k = \frac{1}{4\pi \varepsilon_0} = 9 \times 10^{9} \; Nm^2 C^{-2} \; //$

i) Value of k depends on the nature of medium in which the charges are kept.

ii) $k = \dfrac{1}{4\pi \varepsilon}$ in a medium having permittivity ε.

Unit of charge :

we've $F = k \dfrac{q_1 q_2}{r^2}$

$F = \dfrac{9\times10^9 \times 1C \times 1C}{1 m^2}$

where $F = 9\times10^9 N$

Hence, 1 coulomb is the charge which repels an equal & similar charge by a force of $9\times10^9 N$ separated by a dist of 1 m in air or vaccum.

Relative Permittivity (Dielectric constant)

$F_{vac} = \dfrac{1}{4\pi \varepsilon_0} \dfrac{q_1 q_2}{r^2}$ —— ①

$F_{med} = \dfrac{1}{4\pi \varepsilon} \dfrac{q_1 q_2}{r^2}$ —— ②

$\dfrac{①}{②} = \dfrac{F_{vac}}{\;} = \dfrac{1/\varepsilon_0}{\;} = \dfrac{\varepsilon}{\;} = \varepsilon_r = K$

Hence dielectric constant are of a medium is defined as the ratio of force between 2 pt charges seperated by a certain distance apart in air or vaccum to the force between same 2 charges separated by the same distance in the medium.

Note: i) It's also defined as the ratio of permitivity of medium to the absolute permitivity.

ii) Dielectric constant of a medium is also known as <u>specific inductive capacity</u>.

<u>Coulomb's Law in vector form</u>

Force acting on q_1 due to q_2

$$\vec{F}_{12} = \frac{k\, q_1 q_2}{r^2}\, \hat{r}_{21}$$

Force acting on q_2 due to q_1

$$\vec{F}_{21} = \frac{k\, q_1 q_2}{r^2}\, \hat{r}_{12}$$

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2} \times \frac{r}{r} \times \hat{r}_{21}$$

$$= k\frac{q_1q_2}{r^3} \times \vec{r}_{21} \quad \text{where} \quad r \times \hat{r}_{21} = \vec{r}_{21}$$

|||ly

$$\vec{F}_{21} = \frac{kq_1q_2}{r^3} \times \vec{r}_{12} \quad \text{where} \quad r \times \hat{r}_{12} = \vec{r}_{12}$$

Q. What's the force between 2 small charged spheres having charges 2×10^{-7} C & 3×10^{-7} C placed 30 cm apart in air?

$r = 30 \times 10^{-2}$ m $\quad q_1 = 2 \times 10^{-7}$ C $\quad q_2 = 3 \times 10^{-7}$ C

$\therefore$

$$F = k\frac{q_1q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times \cancel{2} \times 10^{-7} \times \cancel{3} \times 10^{-7}}{\cancel{30}_{\,5} \times 10^{-2} \times 30 \times 10^{-2}}$$

$$= \frac{\cancel{18}^{\,0.6} \times 10^9 \times 10^4 \times 10^{-14}}{\cancel{30}_{\,10}}$$

$$= 0.6 \times 10^{-2} \text{ N} \quad \text{(repulsive)}$$

Q. Estimate the force of repulsion between a proton & alpha particle separated by a distance of 1 mm in vacuum.

$q_1 = 1.6 \times 10^{-19}\ C$ $\qquad r = 1 \times 10^{-3}\ m$

$q_2 = 2 \times 1.6 \times 10^{-19}\ C$

$$F = \frac{k q_1 q_2}{r}$$

$$= \frac{9 \times 10^{9} \times 1.6 \times 10^{-19} \times 2 \times 1.6 \times 10^{-19}}{10^{-3} \times 10^{-3}}$$

$$= 46.08 \times 10^{6} \times 10^{9} \times 10^{-19} \times 10^{-19}$$

$$= 46.08 \times 10^{-23}\ N$$

Q. Find the force of attraction between nucleus electron in a H_2 atom. Radius of orbit o electron is 1.2 Fermi.

$q_1 = 1.6 \times 10^{-19}\ C$ $\qquad q_2 = -1.6 \times 10^{-19}\ C$

$r = 1.2 \times 10^{-15}\ m$

$$F = \frac{k q_1 q_2}{r}$$

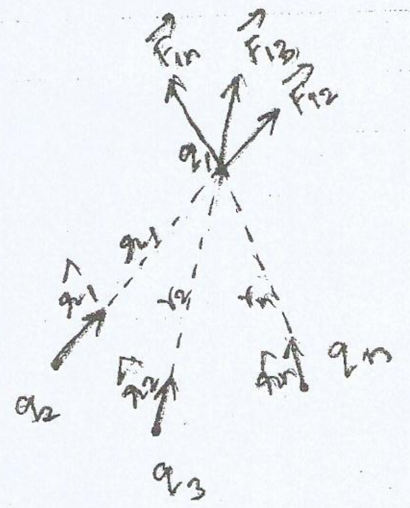
$$= \frac{\cancel{9} \times 10^{9} \times \overset{4}{\cancel{1.6}} \times 10^{-19} \times -\overset{4}{\cancel{1.6}} \times 10^{-19}}{\underset{3}{\cancel{1.2}} \times 10^{-15} \times \underset{3}{\cancel{1.2}} \times 10^{-15}}$$

$$= -16 \times 10^{9} \times 10^{-19} \times 10^{-19} \times 10^{+30}$$

$$= -16 \times 10$$

$$= -160\ N$$

Force between multiple charges (Superposition principle)



When a no. of charges are interacting, the total force on a given charge is the vector sum of the individual forces exerted by each charge on it.

Let $q_1, q_2 \ldots q_n$ be the point charges distributed randomly in free space as shown in fig.

Force acting on q_1 due to all the other charges

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \ldots\ldots + \vec{F_{1n}}$$

$$\therefore \boxed{\vec{F_1} = \frac{k q_1 q_2}{r_1^2}\hat{r_1} + \frac{k q_1 q_3}{r_2^2}\hat{r_2} + \ldots + \frac{k q_1 q_n}{r_3^2}\hat{r_3}}$$

i.e $\vec{F_1} = k q_1 \left[\frac{q_2}{r_1^2}\hat{r_1} + \frac{q_3}{r_2^2}\hat{r_2} + \ldots + \frac{q_n}{r_3^2}\hat{r_3} \right]$

Q. The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge −0.8 μC in air is 0.2 N.

a) What's the distance between the 2 spheres?

b) What's the force on the second sphere due to the first?

a) $F = \dfrac{k\, q_1 q_2}{r^2}$

$$-0.2 = \dfrac{-9\times10^{9} \times 0.4\times10^{-6} \times 0.8\times10^{-6}}{r^2}$$

$$r^2 = \dfrac{+9 \times 0.4 \times \overset{4}{\cancel{0.8}} \times 10^{-3}}{\cancel{0.2}}$$

$$= +9 \times 1.6 \times 10^{-3} = 9\times16\times10^{-4}$$

$$\therefore r = 3\times4\times10^{-2}$$

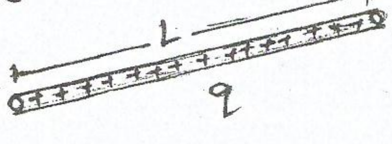
$$= 12\times10^{-2}\ m$$

b) $+0.2\ N$ (opposite in direction)

Continuos Charge Distribution

i) Linear charge distribution

If charge is distributed uniformly over a length of wire, the distribution is called Linear charge distribution.

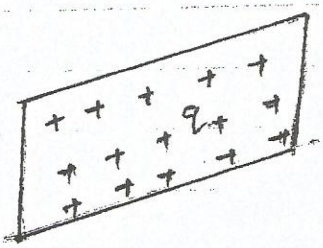


$\lambda = \dfrac{q}{L}$ = Linear charge density

Unit : Cm^{-1}

ii) Surface charge distribution

If charge is distributed uniformly over a surface of a body, the distribution is called surface charge distribution.



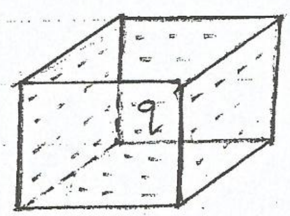
Area = A

$\sigma = \frac{q}{A}$ = surface charge density

Unit : Cm^{-2}

iii) Volume charge distribution

If a charge is distributed over a volume of a body, the distribution is called volume charge distribution.



Volume = V

$\rho = \frac{q}{V}$ = vol charge distribution

Unit : Cm^{-3}

Electric Field.

The region around a charge where another charge experiences a force of attraction or repulsion is called electric field.

Physical significance of electric field :

* The concept of electricfield helps us to understand the mechanism by which 2 charges placed certain distance apart exert force on each other.

* Electric field at a point depends of 2 factors :
  - Magnitude of the source charge
  - Distance of the test charge from the source charge.

Note: A test charge is a charge of vanishingly small magnitude, so that its presence willn't affect the source charge.

Electric field Intensity (E) :

Electric field intensity at a pt is defined as the force experienced by an unit +ve test charge placed at that point.

i,e $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

or $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$ SI unit : NC^{-1} or Vm^{-1}

Electric field at a pt due to a pt charge

Let P be a point at a distance r from a source charge q.

Inorder to find electric field at P, place a test charge q_0 at that pt. Then, force between q & q_0 is

$$F = \frac{k q q_0}{r^2}$$

Acc. to the definition of electric field

$$E = \frac{F}{q_0}$$

$$= \frac{k q \cancel{q_0}}{\cancel{q_0}\, r^2}$$

$$\therefore \boxed{E = \frac{kq}{r^2}}$$

In vector form,

$$\boxed{\vec{E} = \frac{kq}{r^2} \cdot \hat{r}}$$

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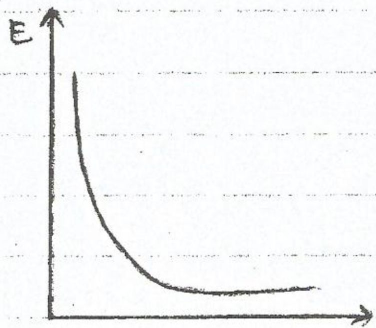
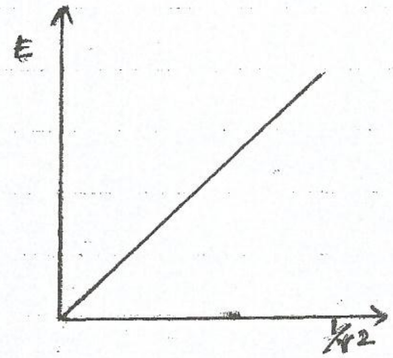
te: i) Plot of E vs r



$E \propto \frac{1}{r^2}$

ii) Plot of E vs $\frac{1}{r^2}$



Electric Field Lines :

Electric Field line is a path through which an unit +ve charge would move if free to do so.

Properties :

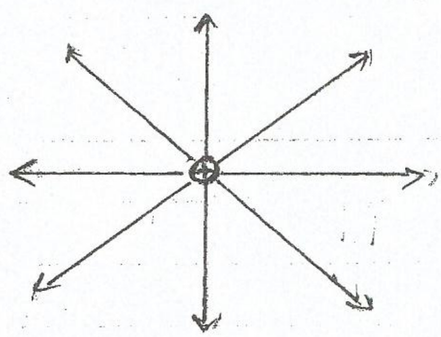
- Electric field lines starts from +ve charge & ends at -ve charge.
- Electric field lines originate or terminate always at right angles from the surface.
- Electric field lines don't pass through a conductor
- A tangent drawn to the line of force at any point shows the direction of electric field at that point.
- The relative closness of electric field lines gives an idea about the relative strength of electric field intensity at

different regions.

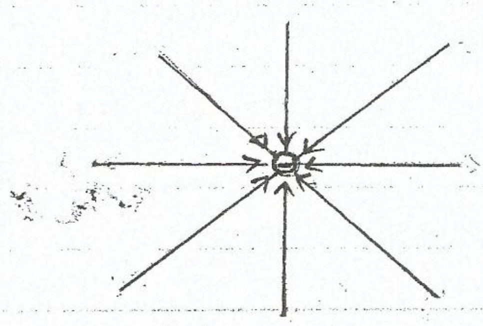
- 2 electric field lines never intersect each other.
  - In case they intersect each other, at the pt of intersection, the electric field has 2 directions which is not possible.

Plotting of Electric field lines

* For a +ve charge $(q > 0)$

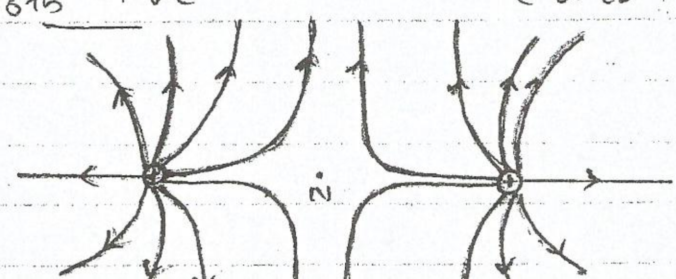


* For a -ve charge $(q < 0)$

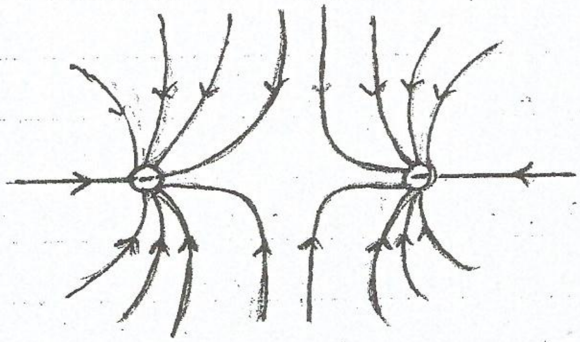


like charge

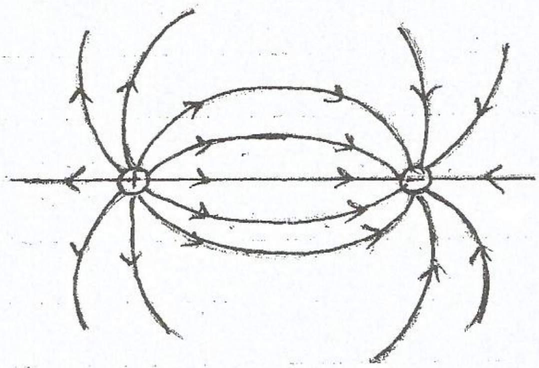
* Both +ve $(q_1 q_2 > 0)$



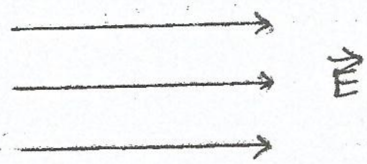
* both -ve



* unlike charges ($q_1 q_2 < 0$)

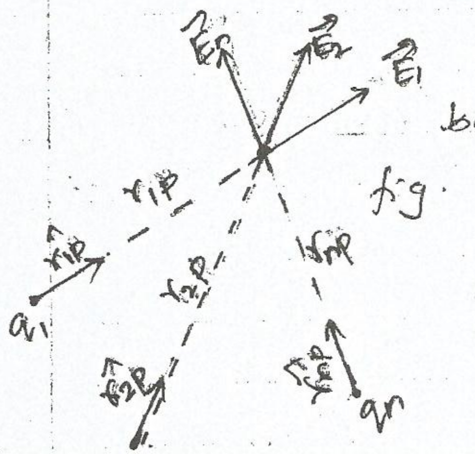


* Uniform electric field



Electric Field due to systems of charges

Consider pt charges $q_1, q_2 ---- q_n$ be distributed in free space as shown fig.



Acc. to superposition principle,

The elec. field at p

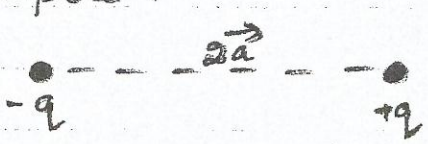
$$\vec{E} = \vec{E_1} + \vec{E_2} + \ldots\ldots + \vec{E_n}$$

$$= \frac{kq_1}{r_{1p}^2}\hat{r_{1p}} + \frac{kq_2}{r_{2p}^2}\hat{r_{2p}} + \ldots\ldots + \frac{kq_n}{r_{np}^2}\hat{r_{np}}$$

$$\therefore \boxed{\vec{E} = k\left[\frac{q_1}{r_{1p}^2}\hat{r_{1p}} + \frac{q_2}{r_{2p}^2}\hat{r_{2p}} + \ldots + \frac{q_n}{r_{np}^2}\hat{r_{np}}\right]}$$

Electric Dipole

A system of 2 equal & oppo charges separated by a small vector distance is called a dipole.



te: The term '$2\vec{a}$' is called dipole length, pointing from $-q$ to $+q$.

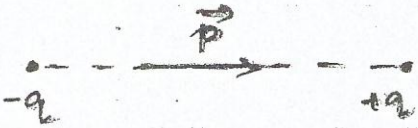
Electric dipolemoment

It's defined as the product of magnitude of either charge & dipole length.

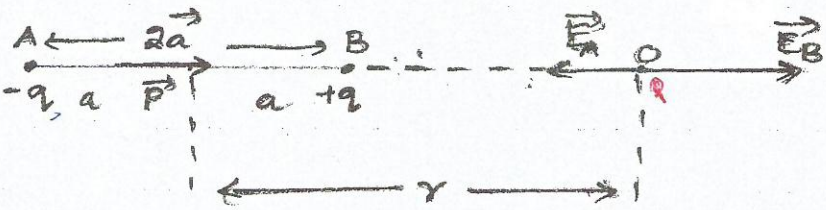
i.e $\boxed{\vec{p} = q \times 2\vec{a}}$

SI unit . Cm

Note: Electric dipole moment is vector quantity whose direction is from $-q$ to $+q$.



Expression for electrifield on axial line of elec. dipole (end-on-position)



Consider an electric dipole having charge as $-q$ & $+q$. Let $2\vec{a}$ be the dipole length & O be a pt at a dist 'r' from the centre of the dipole as shown in fig.

The elec. field at O due to $-q$ at A

$$\vec{E_A} = \frac{kq}{(r+a)^2} \text{ along OA} \quad \text{—— (1)}$$

Elec. field at O due to $+q$ at B

$$\vec{E_B} = \frac{kq}{(r-a)^2} \text{ along OX} \quad \text{—— (2)}$$

$$\because \quad |\vec{E_B}| > |\vec{E_A}|,$$

The magnitude of net elec field at O

$$E = E_B - E_A$$

$$= \frac{kq}{(r-a)^2} - \frac{kq}{(r+a)^2}$$

$$= kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= kq\left[\frac{(r+a)^2 - (r-a)^2}{(r^2-a^2)^2}\right]$$

$$= kq\left[\frac{\cancel{r^2}+\cancel{a^2}+2ar-\cancel{r^2}-\cancel{a^2}+2ar}{(r^2-a^2)^2}\right]$$

$$= kq\,\frac{(4ar)}{(r^2-a^2)^2}$$

$$= \frac{k\,2q\,(2a)\,r}{(r^2-a^2)^2}$$

$$= \frac{k\,2p\,r}{(r^2-a^2)^2}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{2p\,r}{(r^2-a^2)^2}} \quad \text{along } OX$$

or $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\vec{p}\,r}{(r^2-a^2)^2}$

Sp. Cases

If the dipole is of very short length,

i.e $r >> a$, then

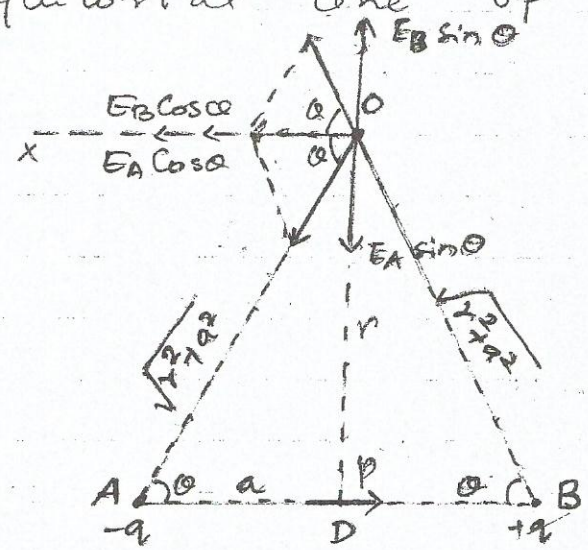
'a' can be neglected as compared to 'r'

ie $E = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{r^{4}}$

$\therefore \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}}$

Expression for electric field at a point on equitorial plane (line) of an electric dipole [⊥ bisector (or) broad side on]

Consider an elec. dipole having charges $-q$ & $+q$ of dipole length $2\vec{a}$. Let O be a pt at a distance 'r' the equitorial line of dipole as shown in fig.



Elec. field at pt O due to $-q$ at A

$$\vec{E_A} = \frac{kq}{r^2+a^2} \text{ along OA} \quad \text{——} \quad (1)$$

Elec. field at O due to $+q$ at B

$$\vec{E_B} = \frac{kq}{r^2+a^2} \text{ along BO} \quad \text{——} \quad (2)$$

$\vec{E_A}$ is resolved into 2 components,
$E_A \cos\theta$ & $E_A \sin\theta$ as shown in fig,

|||ly $\vec{E_B}$ is resolved into
$E_B \cos\theta$ & $E_B \sin\theta$

$\because$ $|\vec{E_A}| = |\vec{E_B}|$, the
components $E_A \sin\theta$ & $E_B \sin\theta$ are equal &
oppo & hence cancels each other.

$\therefore$ The net elec. field at O

$$E = E_A \cos\theta + E_B \cos\theta$$

$$= 2 E_A \cos\theta$$

$$= 2 \frac{kq}{r^2+a^2} \cos\theta$$

$$= 2 \frac{kq}{(r^2+a^2)} \times \frac{a}{\sqrt{r^2+a^2}}$$

$$= \frac{kp}{(r^2+a^2)^{3/2}} \quad [\because p = q(2a)]$$

$$\therefore \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2+a^2)^{3/2}}} \text{ along OX}$$

$$\therefore \vec{E} = -\frac{1}{4\pi\varepsilon_0} \frac{\vec{p}}{(r^2+a^2)^{3/2}}$$

$-$ve sign shows that $\vec{E}$ is oppo to $\vec{p}$.

Sp. Case : If the dipole is of very short length

$a << r$

$\therefore$ 'a' is negligible compared to 'r'

$$\boxed{E = \frac{1}{4\pi \varepsilon_0} \frac{p}{r^3}}$$

Note : For a short dipole

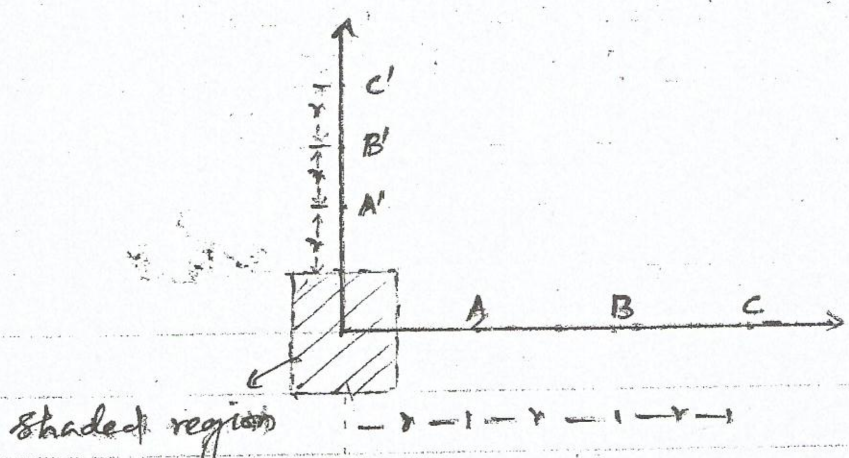
$$E_{axial} = k \frac{2p}{r^3} \quad \text{--- (1)}$$

$$E_{equitorial} = k \frac{p}{r^3} \quad \text{--- (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{E_{axial}}{E_{equi}} = \frac{2}{1}$$

$$\therefore \boxed{E_{axial} = 2 E_{equitorial}}$$

Q.



Point	A	B	C	A'	B'	C'
Electric Field	E	E/8	E/27	E/2	X	Y

Ans e.f at diff. points due to a charge distribution in the shaded region is given in the table.

i) Name the systems of charges present in the shaded region

ii) Write the value of Electricfield in the boxes named X & Y.

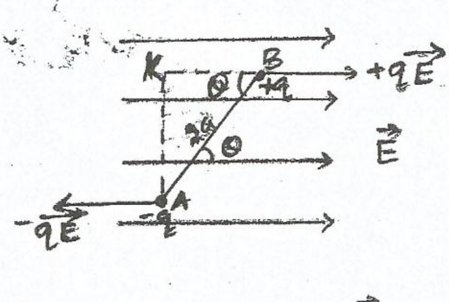
i) Short electric dipole

ii) $\frac{E}{16}$ & $\frac{E}{54}$

Expression for torque acting on an electric dipole based in uniform electric field

________ α ________ α ________

Consider an electric dipole placed in uniform electric field $\vec{E}$ as shown in fig.



The 2 forces $+q\vec{E}$ & $-q\vec{E}$ make the dipole to rotate creating a torque.

Torque, τ = (magnitude of force) $\times \perp^{rd}$ distance

$$\tau = qE\,(AK) \quad — (1)$$

From ΔAKB

$$\sin\theta = \frac{AK}{AB}$$

$$AK = AB \sin\theta$$

$$= 2a\sin\theta \quad — (2)$$

Sub (2) in (1)

$$\tau = 2a\sin\theta\, qE$$

$$\therefore \boxed{\tau = pE\sin\theta}$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Sp. Case : 1) when $\theta = 0$ (ll^{el})

$\therefore \tau = \cancel{\sin}\ pE\sin 0 = 0$ (min

2) when $\theta = 180°$ (anti ll^{el})

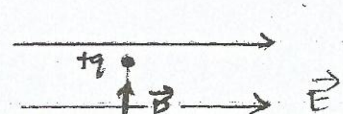
$\therefore \tau = pE\sin 180° = 0$ (min val

3) when $\theta = 90°$ ($\perp$)

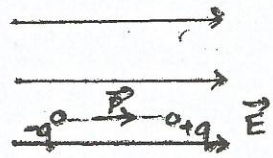
$\therefore \tau = pE\sin 90° = pE$ (max. value)

Note : i) Two pairs of $\perp$ vectors are: $\vec{\tau} \perp \vec{p}$, $\vec{\tau} \perp \vec{E}$

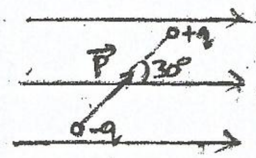
ii) For max. torque



) For minimum torque

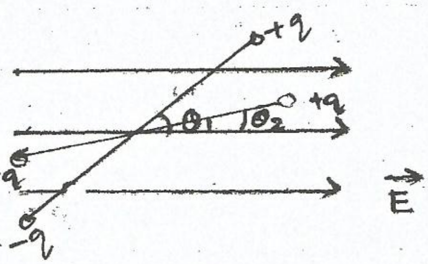


) For half of the max. torque



Work done in rotating a dipole in uniform e.f
(OR) Potential Energy of a dipole " "

Consider an electric dipole having charges $-q$ & $+q$ placed in uniform e.f. as shown in fig. Let $2\vec{a}$ be the dipole length & θ_1 be initial angle.



The amount of WD by the force to rotate the dipole from initial $\angle\ \theta_1$ to final $\angle\ \theta_2$ is equal to the potential energy of the dipole.

Now, the small amnt of WD to rotate the dipole thru a small angle $d\theta$,

$$dW = \tau \, d\theta$$

$$= pE \sin\theta \, d\theta$$

$\therefore$ Total WD to rotate the dipole from θ_1 to θ_2

i,e $$W = U = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

$$\Rightarrow U = pE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= pE \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= pE \left[-\cos\theta_2 + \cos\theta_1\right]$$

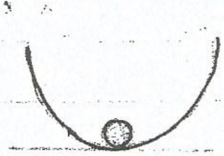
$$\therefore \boxed{U = pE\left[\cos\theta_1 - \cos\theta_2\right]}$$

Sp. Case : If $\theta_1 = 90^\circ$ & $\theta_2 = \theta$

$$U = pE\left[\cos 90 - \cos\theta\right]$$

$$= -pE\cos\theta$$

i,e $$U = -\vec{p}\cdot\vec{E}$$

Note: i)  P.E = 0 [stable equilibrium]

i,e For stable equilibrium, P.E is minimum

$U_{min} = -\vec{p}\cdot\vec{E} = -pE$ where $\theta = 0^\circ$