

CLASS: PHY \_\_\_\_\_

STUDENT #: \_\_\_\_\_

NAME: \_\_\_\_\_

ASSIGNMENT 9: Forces. Work.

Assigned: Oct 13 Due Oct 20 Sharp!

1 An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away (Fig. P6.65). The coefficient of static friction between person and wall is  $\mu_s$ , and the radius of the cylinder is  $R$ .

(a) Show that the maximum period of revolution necessary to keep the person from falling is  $T = (4\pi^2 R \mu_s / g)^{1/2}$ .

(b) Obtain a numerical value for  $T$  if  $R = 4.00$  m and  $\mu_s = 0.400$ . How many revolutions per minute does the cylinder make?

$$(a) n = \frac{mv^2}{R} \quad f - mg = 0 \quad f = \mu_s n \quad v = \frac{2\pi R}{T}$$

$$T = \sqrt{\frac{4\pi^2 R \mu_s}{g}}$$

$$(b) T = \boxed{2.54 \text{ s}} \quad \# \frac{\text{rev}}{\text{min}} = \frac{1 \text{ rev}}{2.54 \text{ s}} \left( \frac{60 \text{ s}}{\text{min}} \right) = \boxed{23.6 \frac{\text{rev}}{\text{min}}}$$

2 An air puck of mass  $m_1$  is tied to a string and allowed to revolve in a circle of radius  $R$  on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a counterweight of mass  $m_2$  is tied to it (Fig. P6.58). The suspended object remains in equilibrium while the puck on the tabletop revolves. What is (a) the tension in the string? (b) the radial force acting on the puck? (c) the speed of the puck?

SOLUTIONS:

Since the object of mass  $m_2$  is in equilibrium,

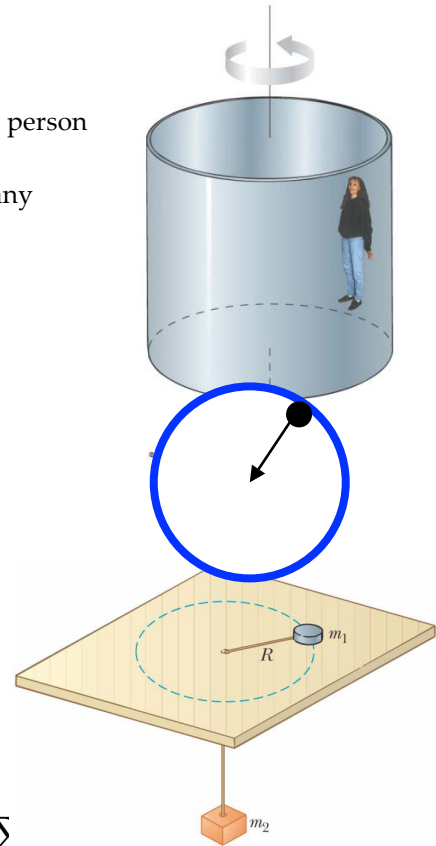
or

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus,

(c) From

we have



$\Sigma$

$$T = \boxed{m_2 g}$$

$$F_c = T = \boxed{m_2 g}$$

$$F_c = \frac{m_1 v^2}{R}$$

$$v = \sqrt{\frac{R F_c}{m_1}} = \sqrt{\left( \frac{m_2}{m_1} \right) g R}$$

3 A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° downward from the horizontal. Find the work done by the shopper on the cart as he moves down an aisle 50.0 m long.

The component of force along the direction of motion is  $F \cos \theta = (35.0 \text{ N}) \cos 25.0^\circ = 31.7 \text{ N}$

The work done by this force is  $W = (F \cos \theta) \Delta r = (31.7 \text{ N})(50.0 \text{ m}) = \boxed{1.59 \times 10^3 \text{ J}}$ .

4. A force  $\mathbf{F} = (4x\hat{i} + 3y\hat{j})\text{N}$  acts on an object as the object moves in the  $x$  direction from the origin to  $x = 5.00$  m. Find the work  $W = \int \mathbf{F} \cdot d\mathbf{r}$  done on the object by the force.

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{r} = \int_0^{5\text{m}} (4x\hat{i} + 3y\hat{j})\text{N} \cdot dx\hat{i} = \int_0^{5\text{m}} (4 \text{ N/m})x dx + 0 = (4 \text{ N/m}) \frac{x^2}{2} \Big|_0^{5\text{m}} = \boxed{50.0 \text{ J}}$$

5. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Assuming the origin is placed where the bullet begins to move, the force (in newtons) exerted by the expanding gas on the bullet is  $15\,000 + 10\,000x - 25\,000x^2$ , where  $x$  is in meters. (a) Determine the work done by the gas on the bullet as the bullet travels the length of the barrel. (b) **What if?** If the barrel is 1.00 m long, how much work is done, and how does this value compare to the work calculated in (a)?

$$(a) \quad W = \int_i^f \mathbf{F} \cdot d\mathbf{r}$$

$$W = \int_0^{0.600 \text{ m}} (15\,000 \text{ N} + 10\,000x \text{ N/m} - 25\,000x^2 \text{ N/m}^2) dx \cos 0^\circ$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3} \Big|_0^{0.600 \text{ m}}$$

$$W = 9.00 \text{ kJ} + 1.80 \text{ kJ} - 1.80 \text{ kJ} = \boxed{9.00 \text{ kJ}}$$

(b) Similarly,

$$W = (15.0 \text{ kN})(1.00 \text{ m}) + \frac{(10.0 \text{ kN/m})(1.00 \text{ m})^2}{2} - \frac{(25.0 \text{ kN/m}^2)(1.00 \text{ m})^3}{3}$$

$$W = \boxed{11.7 \text{ kJ}}, \text{ larger by } 29.6\%$$

6. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between box and floor is 0.300, find (a) the work done by the applied force, (b) the increase in internal energy in the box-floor system due to friction, (c) the work done by the normal force, (d) the work done by the gravitational force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

$$\sum F_y = ma_y: \quad n - 392 \text{ N} = 0$$

$$n = 392 \text{ N}$$

$$f_k = \mu_k n = (0.300)(392 \text{ N}) = 118 \text{ N}$$

$$(a) \quad W_F = F\Delta r \cos \theta = (130)(5.00) \cos 0^\circ = \boxed{650 \text{ J}}$$

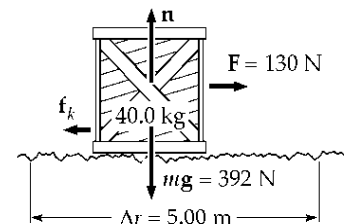
$$(b) \quad \Delta E_{\text{int}} = f_k \Delta x = (118)(5.00) = \boxed{588 \text{ J}}$$

$$(c) \quad W_n = n\Delta r \cos \theta = (392)(5.00) \cos 90^\circ = \boxed{0}$$

$$(d) \quad W_g = mg\Delta r \cos \theta = (392)(5.00) \cos(-90^\circ) = \boxed{0}$$

$$(e) \quad \Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$$

$$\frac{1}{2}mv_f^2 - 0 = 650 \text{ J} - 588 \text{ J} + 0 + 0 = \boxed{62.0 \text{ J}}$$



7. A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. (a) How much work is required to pull him a distance of 60.0 m up a  $30.0^\circ$  slope (assumed frictionless) at a constant speed of 2.00 m/s? (b) A motor of what power is required to perform this task?

(a)  $\sum W = \Delta K$ , but  $\Delta K = 0$  because he moves at constant speed. The skier rises a vertical distance of  $(60.0 \text{ m}) \sin 30.0^\circ = 30.0 \text{ m}$ . Thus,

$$W_{\text{in}} = -W_g = (70.0 \text{ kg})(9.8 \text{ m/s}^2)(30.0 \text{ m}) = \boxed{2.06 \times 10^4 \text{ J}} = \boxed{20.6 \text{ kJ}}.$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$P_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06 \times 10^4 \text{ J}}{30.0 \text{ s}} = \boxed{686 \text{ W}} = 0.919 \text{ hp}$$