

PROBLEM 3.116

Four forces act on a 700×375 -mm plate as shown. (a) Find the resultant of these forces. (b) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION

(a)

$$\mathbf{R} = \Sigma \mathbf{F} \quad \text{4 marks for setting up equations}$$

$$= (-400 \text{ N} + 160 \text{ N} - 760 \text{ N})\mathbf{i} \\ + (600 \text{ N} + 300 \text{ N} + 300 \text{ N})\mathbf{j}$$

Can be written as R_x and R_y $= -(1000 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{j}$ 2 marks for correct answers

$$R = \sqrt{(1000 \text{ N})^2 + (1200 \text{ N})^2}$$

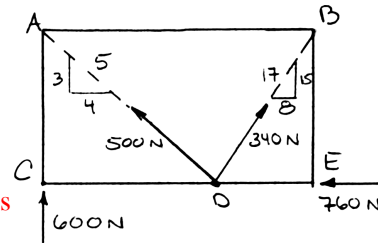
$$= 1562.09 \text{ N} \quad \text{1 mark}$$

$$\tan \theta = \left(-\frac{1200 \text{ N}}{1000 \text{ N}} \right)$$

$$= -1.20000$$

$$\theta = -50.194^\circ \quad \text{1 mark}$$

$$\mathbf{R} = 1562 \text{ N} \nearrow 50.2^\circ \blacktriangleleft$$



(b)

$$\mathbf{M}_C^R = \Sigma \mathbf{r} \times \mathbf{F}$$

$$= (0.5 \text{ m})\mathbf{i} \times (300 \text{ N} + 300 \text{ N})\mathbf{j}$$

$$= (300 \text{ N} \cdot \text{m})\mathbf{k} \quad \text{3 marks}$$

$$(300 \text{ N} \cdot \text{m})\mathbf{k} = x\mathbf{i} \times (1200 \text{ N})\mathbf{j}$$

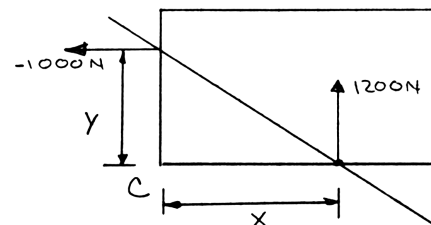
$$x = 0.25000 \text{ m}$$

$$x = 250 \text{ mm} \quad \text{2 marks}$$

$$(300 \text{ N} \cdot \text{m}) = y\mathbf{j} \times (-1000 \text{ N})\mathbf{i}$$

$$y = 0.30000 \text{ m}$$

$$y = 300 \text{ mm} \quad \text{2 marks}$$

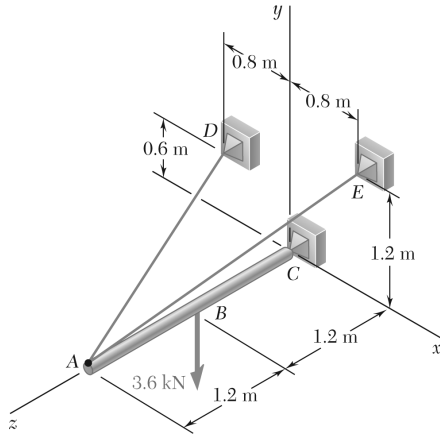


2 marks

Intersection 250 mm to right of C and 300 mm above C \blacktriangleleft

PROBLEM 4.105

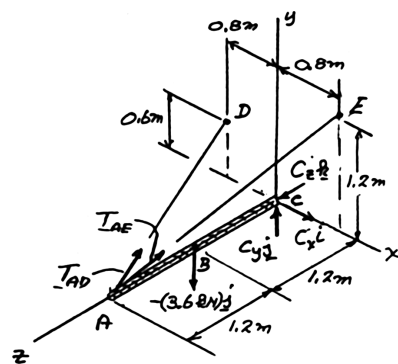
A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.



SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).

Students can use the 6 equations of equilibrium ($F_x, F_y, F_z, M_x, M_y, M_z$) instead of the vector approach shown below



$$\mathbf{r}_B = 1.2\mathbf{k}$$

$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overline{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \quad AD = 2.6 \text{ m} \quad 1 \text{ mark}$$

$$\overline{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \quad AE = 2.8 \text{ m} \quad 1 \text{ mark}$$

$$T_{AD} = \frac{\overline{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k}) \quad 3 \text{ marks for finding the component forces of } T_{AD} \text{ and } T_{AE}$$

$$T_{AE} = \frac{\overline{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

FBD: 5 marks
(coordinates of D & E not necessary)

$$\Sigma M_C = 0: \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3.6 \text{ kN})\mathbf{j} = 0$$

3 marks for setting up the moment equations about point C (x, y, z)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: -0.55385T_{AD} - 1.02857T_{AE} + 4.32 = 0 \quad (1)$$

$$\mathbf{j}: -0.73846T_{AD} + 0.68671T_{AE} = 0$$

$$T_{AD} = 0.92857T_{AE} \quad (2)$$

From Eq. (1): $-0.55385(0.92857)T_{AE} - 1.02857T_{AE} + 4.32 = 0$

$$1.54286T_{AE} = 4.32$$

1 mark

$$T_{AE} = 2.800 \text{ kN}$$

$$T_{AE} = 2.80 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 4.105 (Continued)

1 mark

From Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600 \text{ kN}$

$T_{AD} = 2.60 \text{ kN} \blacktriangleleft$

3 marks for
setting up
equilibrium
equations

$$\Sigma F_x = 0: C_x - \frac{0.8}{2.6}(2.6 \text{ kN}) + \frac{0.8}{2.8}(2.8 \text{ kN}) = 0$$

$C_x = 0$ 1 mark

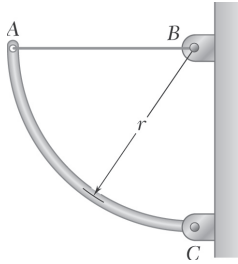
$$\Sigma F_y = 0: C_y + \frac{0.6}{2.6}(2.6 \text{ kN}) + \frac{1.2}{2.8}(2.8 \text{ kN}) - (3.6 \text{ kN}) = 0$$

$C_y = 1.800 \text{ kN}$ 1 mark

$$\Sigma F_z = 0: C_z - \frac{2.4}{2.6}(2.6 \text{ kN}) - \frac{2.4}{2.8}(2.8 \text{ kN}) = 0$$

$C_z = 4.80 \text{ kN}$ 1 mark

$\mathbf{C} = (1.800 \text{ kN})\mathbf{j} + (4.80 \text{ kN})\mathbf{k} \blacktriangleleft$



PROBLEM 5.31

A uniform circular rod of weight 40 N and radius 10 cm is attached to a pin at C and to the cable AB. Determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION

For quarter circle

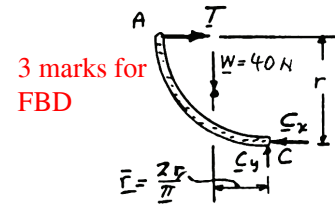
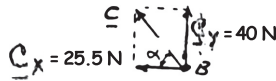
$$\bar{r} = \frac{2r}{\pi} \quad \text{2 marks for proper location of c.g.}$$

$$(a) \quad +\curvearrowright \Sigma M_C = 0: \quad W \left(\frac{2r}{\pi} \right) - Tr = 0 \quad \text{2 marks for moment equation}$$

$$T = W \left(\frac{2}{\pi} \right) = (40 \text{ N}) \left(\frac{2}{\pi} \right)$$

$$(b) \quad \text{1 mark} \quad +\rightarrow \Sigma F_x = 0: \quad T - C_x = 0 \quad 25.5 \text{ N} - C_x = 0$$

$$\text{1 mark} \quad +\uparrow \Sigma F_y = 0: \quad C_y - W = 0 \quad C_y - 40 \text{ N} = 0$$



3 marks for FBD

$$\text{1 mark} \quad T = 25.5 \text{ N} \quad \blacktriangleleft$$

$$\text{1 mark} \quad C_x = 25.5 \text{ N} \quad \leftarrow$$

$$\text{1 mark} \quad C_y = 40 \text{ N} \quad \uparrow$$

$$C = 47.4 \text{ N} \quad \searrow 57.5^\circ \quad \blacktriangleleft$$