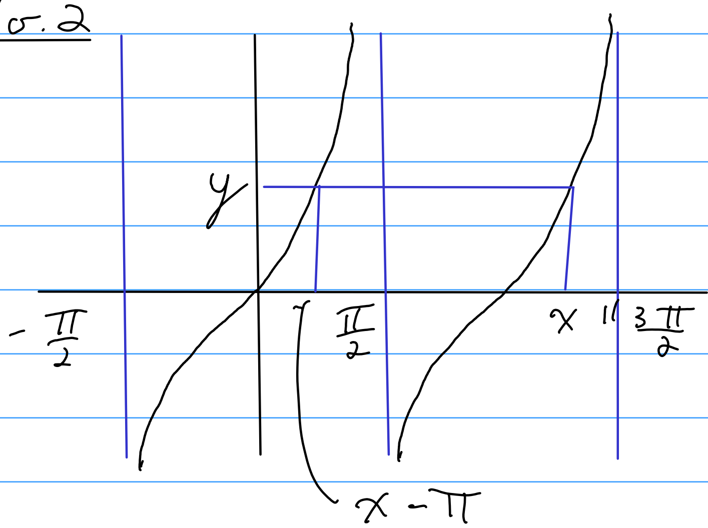
Examen Partiel (a)

No 1

$$16^{\log_4(x)} = 4^{2\log_4(x)} = 4^{\log_4(x^2)} = x^2$$

No. 2



$y = \tan(x)$

$$\arctan(\tan(x)) = \arctan(y) = x - \pi$$

No 3

$$f(x) = e^{3x^2+2}$$

$$f'(x) = \left.\frac{d\, e^z}{dz}\right|_{z=3x^2+2} \cdot \frac{d}{dx}(3x^2+2)$$

$$= \left. e^z\right|_{z=3x^2+2} \cdot (6x) = 6x\, e^{3x^2+2}$$

$$f'(2) = 12\, e^{14}$$

No 4

$2x + y = 1 \implies y = -2x + 1$

On a une droite de pente -2

$y = f(x) = x^2 \implies y' = 2x$

Ainsi $y' = -2 \Rightarrow 2x = -2 \Rightarrow x = -1$

Le point de la courbe $y = x^2$ où la pente est $(-1, 1)$.

L'equation de la droite tangente est

$$\frac{y-1}{x+1} = -2 \Rightarrow y = -2x - 1$$

No 5

$$f(x) = e^{3x}\sin(x^2)$$

$$f'(x) = \frac{d}{dx}(e^{3x})\sin(x^2) + e^{3x}\frac{d}{dx}\sin(x^2)$$

$$= 3e^{3x}\sin(x^2) + 2x\,e^{3x}\cos(x^2)$$

No 6

$$f(x) = (x^2 + 4x + 16)^{1/2}$$

$$f'(x) = \frac{1}{2}(x^2 + 4x + 16)^{-1/2}\frac{d}{dx}(x^2 + 4x + 16)$$

$$= \frac{2x + 4}{2\sqrt{x^2 + 4x + 16}} = \frac{x + 2}{\sqrt{x^2 + 4x + 16}}$$

$$f'(x) = 0 \Rightarrow x = -2$$

No 7

a) $g(x) = (f(x) + 4)^{-1/3}$

$$g'(x) = \left.\frac{d}{dz} z^{-1/3}\right|_{z = f(x)+4} \cdot \frac{d}{dx}(f(x) + 4)$$

$$= \left.-\frac{1}{3} z^{-4/3}\right|_{z = f(x)+4} \cdot f'(x)$$

$$= \frac{-f'(x)}{3\left(f(x)+4\right)^{4/3}}$$

$$g'(2) = \frac{-f'(2)}{3\left(f(2)+4\right)^{4/3}} = \frac{-3}{3(8)^{4/3}} = \frac{-1}{16}$$

b) $g(x) = \sin\left(\frac{\pi}{4} f(x^2-2)\right)$

$$g'(x) = \cos\left(\frac{\pi}{4} f(x^2-2)\right) \cdot \frac{d}{dx}\left(\frac{\pi}{4} f(x^2-2)\right)$$

$$= \frac{\pi}{4} \cos\left(\frac{\pi}{4} f(x^2-2)\right) \cdot f'(x^2-2) \cdot 2x$$

$$= \frac{\pi x}{2} \cos\left(\frac{\pi}{4} f(x^2-2)\right) \cdot f'(x^2-2)$$

$$g'(2) = \pi \cos(\pi) \cdot 3 = -3\pi$$

<u>No 8</u>

$y = f(x) = \ln(6x+7) \Rightarrow e^y = 6x+7$

$$\Rightarrow x = \frac{e^y - 7}{6}$$

Donc $f^{-1}(x) = \dfrac{e^x - 7}{6}$

<u>No 9</u>

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+1) - x(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{x(x+1) + h(x+1) - x(x+1) - xh}{h(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{1}{(x+h+1)(x+1)} = \frac{1}{(x+1)^2}$$

No 10

$$\lim_{x \to 3^+} \frac{x^2-9}{|x-3|} = \lim_{x \to 3^+} \frac{x^2-9}{x-3} = \lim_{x \to 3^+} \frac{(x-3)(x+3)}{x-3}$$

$$= \lim_{x \to 3^+} x+3 = 6$$

et

$$\lim_{x \to 3^-} \frac{x^2-9}{|x-3|} = \lim_{x \to 3^-} \frac{x^2-9}{3-x} = \lim_{x \to 3^-} \frac{(x-3)(x+3)}{3-x}$$

$$= -\lim_{x \to 3^-} x+3 = -6$$

Puisque $\displaystyle\lim_{x \to 3^+} \frac{x^2-9}{|x-3|} \neq \lim_{x \to 3^-} \frac{x^2-9}{|x-3|}$,

on a que $\displaystyle\lim_{x \to 3} \frac{x^2-9}{|x-3|}$ n'existe pas.