GNG 1105

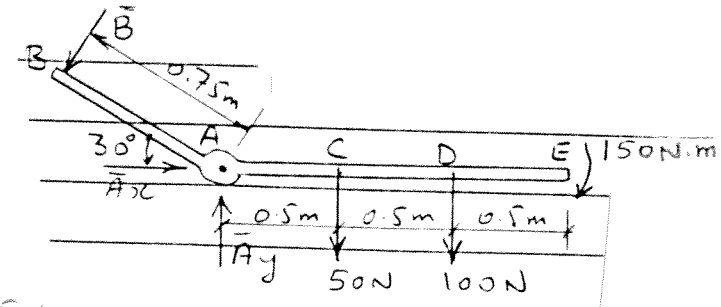
ENGINEERING MECHANICS

MID-TERM EXAM

SOLUTIONS

1. a)

$R = 100 + 50 = 150\,N \downarrow$



$+\circlearrowleft \Sigma M_A = -50\times0.5 - 100\times1.0$

$-150\,N.m$

$= -25 - 100 - 150 = -275\,N.m.$

∴ Equivalent force-couple system at A is:

$R = 150\,N \downarrow$

$M_A = 275\,N.m \circlearrowright$ — ANS.

b)

$+\circlearrowleft \Sigma M_A = 0$

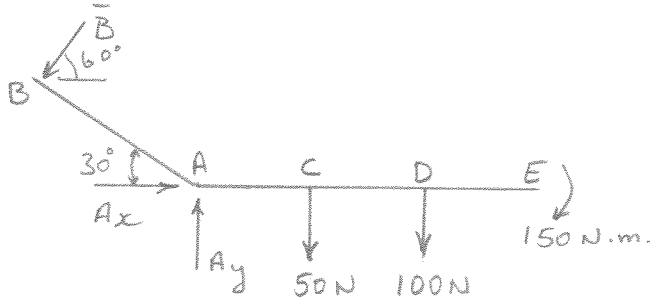
$B\times0.75 - 50\times0.5 - 100\times1.0$

$-150\,N.m = 0$

$0.75B - 25 - 100 - 150 = 0$

$0.75B = 275\,N$

$\therefore B = \frac{275}{0.75} = 367\,N$



i.e. $B = 367\,N$ — ANS.

$\rightarrow \Sigma F_x = 0$; $A_x - 367\cos60° = 0$

$\therefore A_x = 183.5\,N$

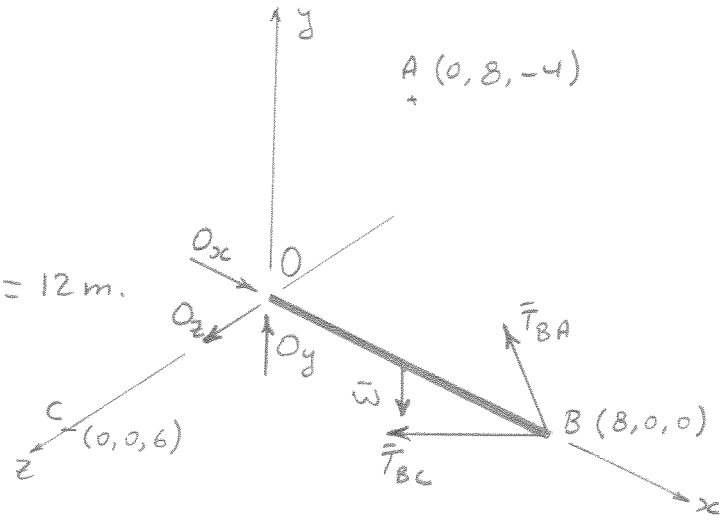
$+\uparrow \Sigma F_y = 0$; $A_y - 367\sin60° - 50 - 100 = 0$. $\therefore A_y = 467.8\,N$

$A = \sqrt{(183.5)^2 + (467.8)^2} = 502.5\,N$

$\angle\theta = \tan^{-1}\frac{467.8}{183.5} = 68.6°$ ∴ $A = 502.5\,N$ at $68.6°$ — ANS.

2.

a) See diagram



b) $\bar{BA} = -8\bar{i} + 8\bar{j} - 4\bar{k}$

$BA = \sqrt{(-8)^2 + (8)^2 + (-4)^2} = 12\,m.$

$\bar{BC} = -8\bar{i} + 0\bar{j} + 6\bar{k}$

$BC = \sqrt{(-8)^2 + (6)^2} = 10\,m.$

$\bar{T}_{BA} = T_{BA}\,\bar{\lambda}_{BA} = T_{BA}\,\dfrac{\bar{BA}}{BA}$

$\bar{T}_{BA} = \dfrac{T_{BA}}{12}(-8\bar{i} + 8\bar{j} - 4\bar{k}) = T_{BA}\left(-\dfrac{8}{12}\bar{i} + \dfrac{8}{12}\bar{j} - \dfrac{4}{12}\bar{k}\right)$

$= T_{BA}\left(-\dfrac{2}{3}\bar{i} + \dfrac{2}{3}\bar{j} - \dfrac{1}{3}\bar{k}\right)$ } — ANS.

$\bar{T}_{BC} = T_{BC}\,\bar{\lambda}_{BC} = T_{BC}\,\dfrac{\bar{BC}}{BC}$

$\bar{T}_{BC} = \dfrac{T_{BC}}{10}(-8\bar{i} + 0\bar{j} + 6\bar{k}) = T_{BC}\left(-\dfrac{8}{10}\bar{i} + \dfrac{6}{10}\bar{k}\right) = T_{BC}\left(-\dfrac{4}{5}\bar{i} + \dfrac{3}{5}\bar{k}\right)$ — ANS.

$\bar{W} = -1.5\,KN\,\bar{j}$

c) $\Sigma \bar{M}_O = 0$

$= \bar{OB} \times \bar{T}_{BA} + \bar{OB} \times \bar{T}_{BC} + \bar{OG} \times \bar{W} = 0 \qquad (\bar{OB} = 8\bar{i};\ OG = 4\bar{i})$

$= 8\bar{i} \times T_{BA}\left(-\dfrac{2}{3}\bar{i} + \dfrac{2}{3}\bar{j} - \dfrac{1}{3}\bar{k}\right) + 8\bar{i} \times T_{BC}\left(-\dfrac{4}{5}\bar{i} + \dfrac{3}{5}\bar{k}\right)$

$+ 4\bar{i} \times (-1.5\,KN)\bar{j} = 0$

$\therefore \Sigma \bar{M}_O = \dfrac{16}{3}T_{BA}\bar{k} + \dfrac{8}{3}T_{BA}\bar{j} - \dfrac{24}{5}T_{BC}\bar{j} - 6\bar{k} = 0$

Equate Coeff. of $\bar{j}$ and $\bar{k}$ to zero.

$\bar{j}$: $\dfrac{8}{3}T_{BA} - \dfrac{24}{5}T_{BC} = 0$ ——— (1)

$\bar{k}$: $\dfrac{16}{3}T_{BA} - 6 = 0$ ——— (2)

From eq. (2) : $\dfrac{16}{3}T_{BA} = 6$, $\therefore T_{BA} = \dfrac{3 \times 6}{16} = \underline{1.125\,KN}$ ANS.

Insert in (1) : $\dfrac{8}{3} \times 1.125 - \dfrac{24}{5}T_{BC} = 0$; $\dfrac{24}{5}T_{BC} = 3$

$\therefore T_{BC} = \dfrac{3 \times 5}{24} = \underline{0.625\,KN}$ ANS.

2. (Cont'd)

c) Another Method:

$$\Sigma \bar{M}_O = 0 = \bar{OB} \times \bar{T}_{BA} + \bar{OB} \times T_{BC} + OG \times \bar{W}$$

$$= T_{BA} \begin{vmatrix} (+)\ \bar{i} & (-)\ \bar{j} & (+)\ \bar{k} \\ 8 & 0 & 0 \\ -\frac{2}{3} & +\frac{2}{3} & -\frac{1}{3} \end{vmatrix} + T_{BC} \begin{vmatrix} (+)\ \bar{i} & (-)\ \bar{j} & (+)\ \bar{k} \\ 8 & 0 & 0 \\ -\frac{4}{5} & 0 & \frac{3}{5} \end{vmatrix} + 4\bar{i} \times (-1.5)\bar{j} = 0$$

$$= -8T_{BA} \times \left(-\frac{1}{3}\right)\bar{j} + 8T_{BA} \times \frac{2}{3}\bar{k} + \left(-8T_{BC} \times \frac{3}{5}\right)\bar{j} - 6\bar{k} = 0$$

Equate Coeff. of $\bar{j}$ and $\bar{k}$ to zero:

$\bar{j}$: $\frac{8}{3}T_{BA} - \frac{24}{5}T_{BC} = 0$ ——— (1)

$\bar{k}$: $+\frac{16}{3}T_{BA} - 6 = 0$ ——— (2)

Eq. (2): $\frac{16}{3}T_{BA} = 6$; $\therefore T_{BA} = \frac{6 \times 3}{16} = \underline{1.125\ KN}$ ANS.

Insert in eq. (1):

$$\frac{8}{3} \times 1.125 - \frac{24}{5}T_{BC} = 0, \quad \therefore T_{BC} = \frac{3 \times 5}{24} = \underline{0.625\ KN}$$ ANS.

(3)