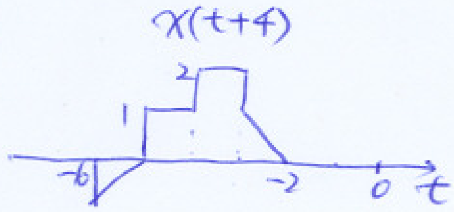
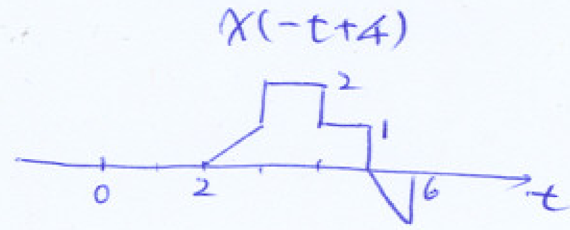
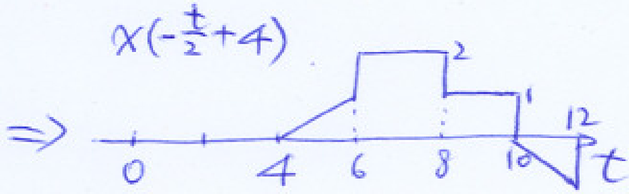
1.21(d)

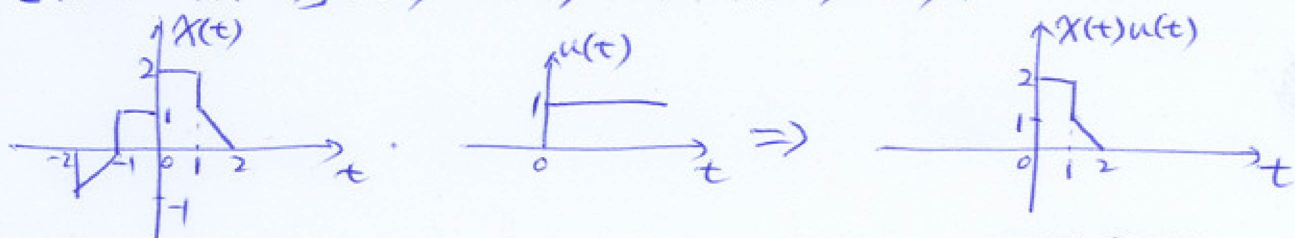


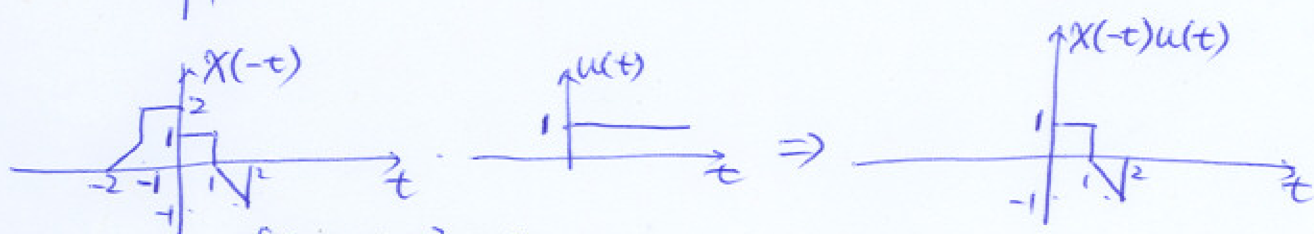
$\Rightarrow$

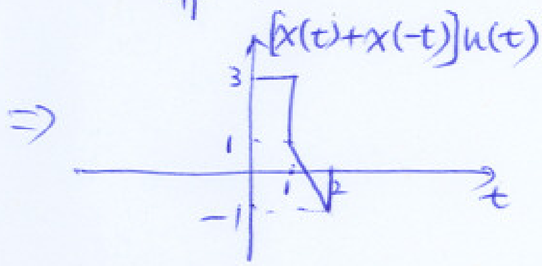




(e) $[x(t)+x(-t)]u(t) = x(t)u(t) + x(-t)u(t)$



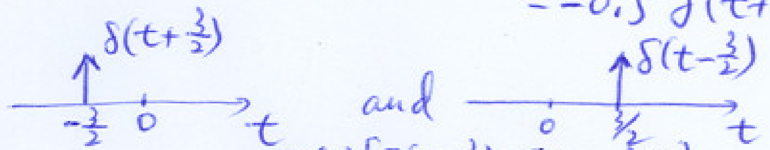


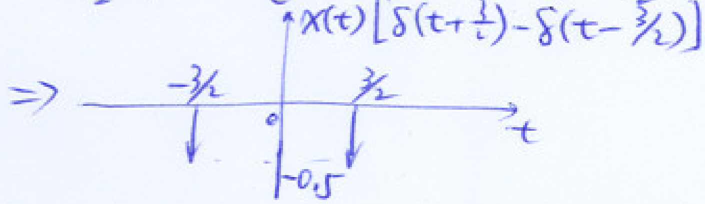


(f) $x(t)\left[\delta(t+\frac{3}{2})-\delta(t-\frac{3}{2})\right] = x(t)\,\delta(t+\frac{3}{2}) - x(t)\,\delta(t-\frac{3}{2})$

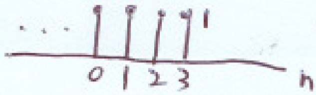
$= x(-\frac{3}{2})\delta(t+\frac{3}{2}) - x(\frac{3}{2})\delta(t-\frac{3}{2})$

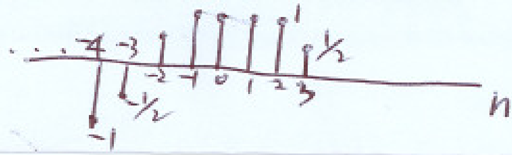
$= -0.5\,\delta(t+\frac{3}{2}) - 0.5\,\delta(t-\frac{3}{2})$



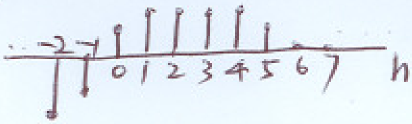


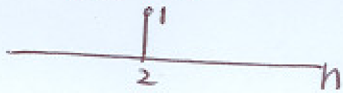
1.22 (e) $u[n+3]$ $\Rightarrow$ $u[-n+3]$ $\Rightarrow$ $x[n]\,u[3-n] = x[n]$

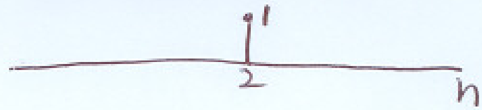




(f) $x[n-2]$ $\quad$ $\delta[n-2]$ $\Rightarrow$ $x[n-2]\,\delta[n-2]$

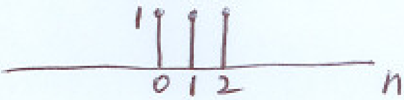






(h) To have $x[(n-1)^2] \neq 0$, $0 \leq (n-1)^2 \leq 3 \Rightarrow n = 0, 1, 2$.

$x[(n-1)^2]$

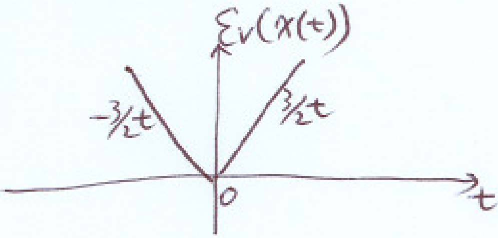


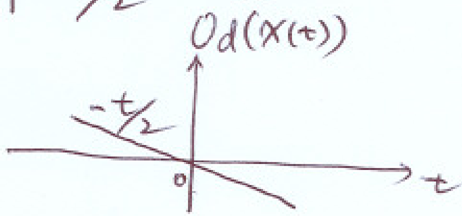
$$\begin{cases} x[(0-1)^2] = 1 \\ x[(1-1)^2] = 1 \\ x[(2-1)^2] = 1 \end{cases}$$

1.23 (c) $x(t) = \begin{cases} -2t & t<0 \\ t & t>0 \end{cases} \xRightarrow[\text{over time } t]{\text{flip}} x(-t) = \begin{cases} -t & t<0 \\ 2t & t>0 \end{cases}$

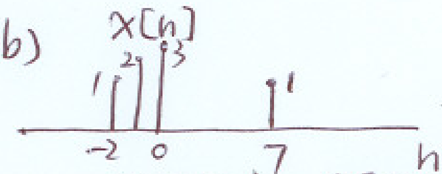
$$\mathcal{E}v(x(t)) = \frac{x(t)+x(-t)}{2} = \begin{cases} -\frac{3}{2}t & t<0 \\ \frac{3}{2}t & t>0 \end{cases}$$

$$\mathcal{O}d(x(t)) = \frac{x(t)-x(-t)}{2} = \begin{cases} -t/2 & t<0 \\ -t/2 & t>0 \end{cases}$$

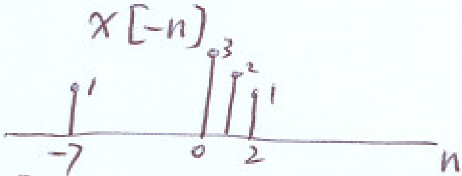




1.24 (b)

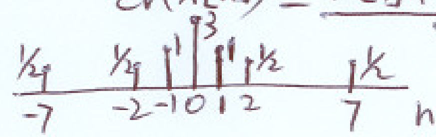


$\Rightarrow$



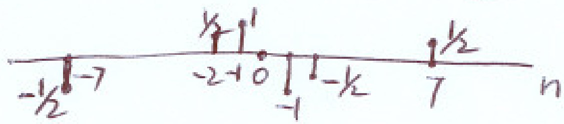
$\mathcal{E}v(x[n]) = \frac{x[n]+x[-n]}{2}$

$\Rightarrow$



,

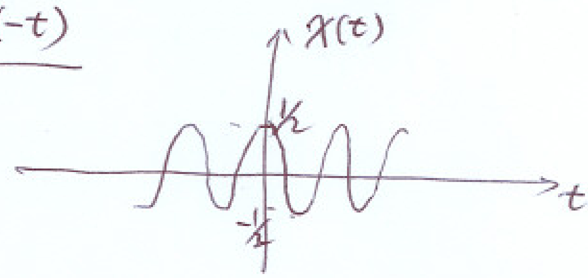
$\mathcal{O}d(x[n]) = \frac{x[n]-x[-n]}{2}$



1.25 (d) $x(t) = \dfrac{\cos(4\pi t)u(t) + \cos(-4\pi t)u(-t)}{2}$

$= \dfrac{\cos(4\pi t)(u(t)+u(-t))}{2}$

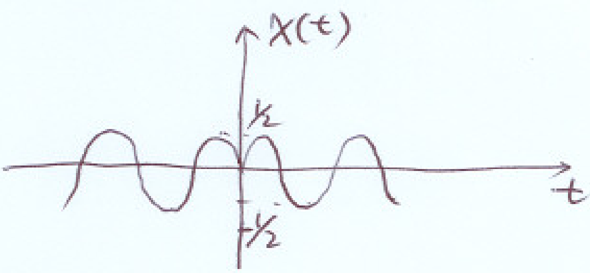
$= \frac{1}{2}\cos(4\pi t)$



$\therefore$ periodic, period $= \frac{2\pi}{4\pi} = 1/2$.

(e) $x(t) = \dfrac{\sin(4\pi t)u(t) + \sin(-4\pi t)u(-t)}{2}$

$= \dfrac{\sin(4\pi t)u(t) - \sin(4\pi t)u(-t)}{2} \Rightarrow$ Not periodic.



1.26 (b) Try to find a period: $\frac{N}{8} = m(2\pi) \Rightarrow N = 16\pi \cdot \underline{\underline{m}}$ ← m is integer.

No integer m will give an integer N. ⇒ Not periodic.

(d) $\frac{\pi}{2} N_1 = m(2\pi) \Rightarrow N_1 = 4m$ Let $m=1$. $N_1 = 4$

$\frac{\pi}{4} N_2 = m(2\pi) \Rightarrow N_2 = 8m$. $N_2 = 8$.

Least Common Multiple (LCM) of N_1 and N_2 is 8.

⇒ periodic with period = 8.

1.27 (b) Clearly, <u>memoryless</u> and <u>causal</u> (since $y(t)$ depends on the input at time t only)

Suppose for input $x_1(t)$ and $x_2(t)$, we have

$y_1(t) = \cos(3t)x_1(t)$, $y_2(t) = \cos(3t)x_2(t)$.

Let $x_3(t) = a x_1(t) + b x_2(t)$.

then $y_3(t) = \cos(3t)x_3(t) = a\cos(3t)x_1(t) + b\cos(3t)x_2(t) = a y_1(t) + b y_2(t)$.

⇒ <u>Linear</u>.

If $x_4(t) = x_1(t - t_0)$

then $y_4(t) = \cos(3t)x_4(t) = \cos(3t)x_1(t-t_0)$

But, for system to be time-invariant, $y_1(t-t_0) = \cos(3(t-t_0))x_1(t-t_0)$

⇒ <u>Not time-invariant</u>.

If $|x(t)| < B$ and $|\cos(3t)| \le 1$. then $|y(t)| < B$ ⇒ <u>Stable</u>.

(c) Since $y(t)$ determines on $x(t)$, $t \in (-\infty, 2t]$,

⇒ <u>Not memoryless</u> and <u>Not causal</u>.

Suppose for input $x_1(t)$ and $x_2(t)$, we have

$y_1(t) = \int_{-\infty}^{2t} x_1(\tau)\,d\tau$, $y_2(t) = \int_{-\infty}^{2t} x_2(\tau)\,d\tau$.

Let $x_3(t) = a x_1(t) + b x_2(t)$, then $y_3(t) = \int_{-\infty}^{2t} x_3(\tau)\,d\tau = a\int_{-\infty}^{2t} x_1(\tau)\,d\tau + b\int_{-\infty}^{2t} x_2(\tau)\,d\tau$

$= a y_1(t) + b y_2(t)$ ⇒ <u>Linear</u>

If $x_1(t)=\delta(t)$, $y_1(t)=\int_{-\infty}^{2t}\delta(\tau)d\tau=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$

Now if $x_2(t)=\delta(t-1)$, $y_2(t)=\int_{-\infty}^{2t}\delta(\tau-1)d\tau=\begin{cases}0, & t<\frac{1}{2}\\ 1, & t>\frac{1}{2}\end{cases}$

While $\delta(t-1)$ is a shifted version of $\delta(t)$, $y_2(t)$ is not. $\Rightarrow$ Not TI.

If $|x(t)|<B$, $y(t)$ may be unbounded. $\Rightarrow$ unstable because the upper bound of integral $2t$ is unbounded.

1.28 (e) Clearly, not memoryless since $y[n]$ depends on the input at time $n+1$ sometimes

$\because$ $y[-1]=x[0]$. $\Rightarrow$ not causal

If $|x[n]|<B$, then $|y[n]|<B$ $\Rightarrow$ stable

Suppose for $x_1[n]$ and $x_2[n]$, $y_1[n]=\begin{cases}x_1[n], & n\geq 1\\ 0, & n=0\\ x_1[n+1], & n\leq -1\end{cases}$ $y_2[n]=\begin{cases}x_2[n], & n\geq 1\\ 0, & n=0\\ x_2[n+1], & n\leq -1\end{cases}$

then for $x_3[n]=ax_1[n]+bx_2[n]$,

$$y_3[n]=\begin{cases}x_3[n]\\ 0\\ x_3[n+1]\end{cases}=\begin{cases}ax_1[n]+bx_2[n]\\ 0\\ ax_1[n+1]+bx_2[n+1]\end{cases}=\begin{cases}ay_1[n]+by_2[n], & n\geq 1\\ 0, & n=0\\ ay_1[n]+by_2[n], & n\leq -1\end{cases}\Rightarrow \text{Linear}$$

$\because$ $x[n]=\delta[n]\Rightarrow y[n]=0$. and $x[n]=\delta[n-1]\Rightarrow y[n]=\delta[n-1]$

$\therefore$ Not time-invariant.

(f) Memoryless and causal since $y[n]$ depends on input at present only.

If $|x[n]|<B$, then $|y[n]|<B$ $\Rightarrow$ stable

Suppose for $x_1[n]$ and $x_2[n]$, $y_1[n]=\begin{cases}x_1[n], & n\geq 1\\ 0, & n=0\\ x_1[n], & n\leq -1\end{cases}$ $y_2[n]=\begin{cases}x_2[n], & n\geq 1\\ 0, & n=0\\ x_2[n], & n\leq -1\end{cases}$

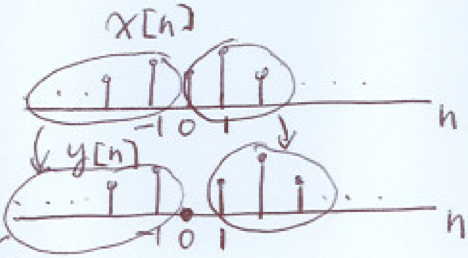
then for $x_3[n]=ax_1[n]+bx_2[n]$,

$$y_3[n]=\begin{cases}x_3[n]\\ 0\\ x_3[n]\end{cases}=\begin{cases}ax_1[n]+bx_2[n]\\ 0\\ ax_1[n]+bx_2[n]\end{cases}=\begin{cases}ay_1[n]+by_2[n]\\ 0\\ ay_1[n]+by_2[n]\end{cases}\Rightarrow \text{Linear}$$

$\because$ $x[n]=\delta[n]\Rightarrow y[n]=0$, and $x[n]=\delta[n-1]\Rightarrow y[n]=\delta[n-1]$.

$\therefore$ Not time-invariant.

1.30 (e)



one-to one mapping

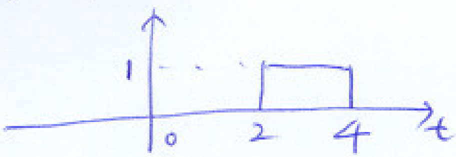
$\Rightarrow$ Invertible

$$w[n] = \begin{cases} y[n+1], & n \geq 0 \\ y[n], & n < 0. \end{cases}$$

(f) Not invertible.

For $x[n] = \delta[n]$. $y[n] = 0$.
$x[n] = 2\delta[n]$. $y[n] = 0$.
so by knowing $y[n]$, you can not recover $x[n]$.

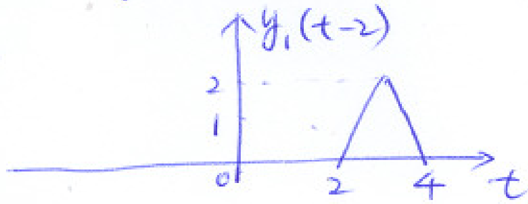
1.31 (a) $x_1(t-2)$



$\therefore\ x_2(t) = x_1(t) - x_1(t-2)$

Now, based on T.I. Property.

$x_1(t-2) \Rightarrow y_1(t-2)$



Next, based on Linear Property.

$x_2(t) = x_1(t) - x_1(t-2)$

$\Rightarrow y_2(t) = y_1(t) - y_1(t-2)$.