ADM2303 formula sheet - Fall 2015

Probability theory

Rule of sum of probabilities:

$$P(S) = 1$$

Subtraction rule (Let A^c be compliment of A, i.e., Not A):

$$P(A) = 1 - P(A^c)$$

Addition rule for two mutually exclusive events (where $\cup$ connotes "or" aka union):

$$P(A \cup B) = P(A) + P(B)$$

Addition rule for two not mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication rule for two independent events (where $\cap$ connotes "and" aka intersection):

$$P(A \cap B) = P(A) \times P(B)$$

Multiplication rule for n independent events:

$$P(A_1 \cap A_2 \cap ...A_n) = P(A_1) \times P(A_2) \times ... \times P(A_n)$$

Multiplication rule for dependent events:

$$P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$$

Partition rule: for a partition $B_1, B_2, ..., B_k$:

$$P(A) = \sum_{i=1}^{k} P(A \cap B_i) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Checking for independence of event A and B :

$$\text{If } P(A|B) = P(A) \text{ and } P(B|A) = P(B)$$
$$\Rightarrow \text{A and B are independent}$$

Bayes' formula:

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)} = \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

Random variables (RV)

Expected value of discrete RV X :

$$E(X) = \mu = \sum_{i=1}^{n} x_i P(X = x_i)$$

Variance of discrete RV X:

$$Var(X) = \sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 P(X = x_i) = \sum_{i=1}^{n} x_i^2 P(X = x_i) - \mu^2$$

Standard deviation of discrete RV X:

$$SD(X) = \sigma = \sqrt{Var(X)}$$

Coefficient of variation of discrete RV X:

$$CV(X) = \frac{SD(X)}{E(X)}$$

Combining random variables

Adding a constant c to random variable X:

$$E(X \pm c) = E(X) \pm c$$
$$Var(X \pm c) = Var(X)$$

Multiplying random variable X by a constant a :

$$E(aX) = aE(X)$$
$$Var(aX) = a^2 Var(X)$$

Expected value of linear combination of RVs:

$$E(aX \pm bY \pm c) = aE(X) \pm bE(Y) \pm c$$

Variance of linear combination of **independent** RVs:

$$Var(aX \pm bY \pm c) = a^2 Var(X) + b^2 Var(Y)$$

Discrete distributions

The Binomial probability distribution:

$$P(X = x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \quad \text{for } x = 0, 1, ..., n$$
$$E(X) = np, Var(X) = np(1-p)$$

The Poisson probability distribution (if approx'n of binomial, $\lambda = np$):

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x = 0, 1, 2, ...$$
$$E(X) = \lambda, Var(X) = \lambda, e = 2.718$$

The Geometric probability distribution:

$$P(X = x) = (1-p)^{x-1} p \quad \text{for } x = 1, 2, ...$$
$$E(X) = \frac{1}{p}, Var(X) = \frac{1-p}{p^2}$$

Continuous distributions

The Normal distribution:

$$X \sim N(\mu, \sigma) \Rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2\right)$$

$$Z = \frac{X - \mu}{\sigma} \sim N(0,1) \Rightarrow P(Z \leq z) = \text{using normal table}$$

The Exponential distribution :

$$X \sim Expo(\lambda) \Rightarrow f(x) = \lambda e^{-\lambda x}$$
$$P(X \leq a) = 1 - e^{-a\lambda}$$
$$E(X) = \frac{1}{\lambda}, Var(X) = (\frac{1}{\lambda})^2$$

The Uniform distribution:

$$X \sim \text{Uniform}(a, b) \Rightarrow f(x) = \frac{1}{b-a} \quad \text{for } a \leq x \leq b$$
$$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b-a}$$
$$E(X) = \frac{a+b}{2}, Var(X) = \frac{b-a}{\sqrt{12}}$$

Normal approximation to Binomial :

If $X \sim \text{Binomial}(n, p)$

If n is large i.e. $np \geq 10$ and $n(1-p) \geq 10$

$$\Rightarrow X \sim N\left(\mu_x = np, \sigma_x = \sqrt{np(1-p)}\right)$$