Linear Algebra with Applications to Business and Economics

MATH1119B

Lecture 2-6

The Leontief Input-output model

THE LEONTIEF INPUT-OUTPUT MODEL

- Suppose that a nation's economy is divided into n sectors that produce goods and services, and let **x** a production vector in $\mathcal{R}^n$ that lists the output of each sector for one year.
- In chapter 1.6, we found the equilibrium prices for each sector of economy, when goods and services produced by each sector of nation's economy is consumed only by the producers themselves.
- However, suppose that there is another part of the economy (called open sector) does not produce goods or services but only consumes them. Let **d** be a final demand vector (or bill of final demands) that lists the values of the goods and services

The Leontief input-output model

demanded from the various sectors by the non-productive part of the economy.

- Now we consider the following: Various sectors produce goods to meet consumer demand, the producers themselves create additional intermediate demand for goods they need as inputs for their own production.
- Leontief asked if there is a production level x such that the amounts produced (or "supplied") will exactly balance the total demand for that production, so that

$$\begin{Bmatrix} amount \\ produced \\ \boldsymbol{x} \end{Bmatrix} = \begin{Bmatrix} Intermediate \\ demand \end{Bmatrix} + \begin{Bmatrix} final \\ demand \\ \boldsymbol{d} \end{Bmatrix}$$

THE LEONTIEF INPUT-OUTPUT MODEL

- The basic assumption of Leontief's input-output model is that for each sector, there is a unit consumption vector in $\mathscr{R}^n$ that lists the inputs needed per unit of output of the sector.

THE LEONTIEF INPUT-OUTPUT MODEL

Ex 2-6-1

An economy divided into three sectors – manufacturing, agriculture and services. The unit consumption vectors are shown in the table.

Purchased From:	Inputs consumed per units of output		
	Manufacturing	Agriculture	Services
Manufacturing	0.10	0.60	0.60
Agriculture	0.30	0.20	0.0
Services	0.30	0.10	0.10
	↑ c1	↑ c2	↑ c3

THE LEONTIEF INPUT-OUTPUT MODEL

a. Construct a consumption matrix for this economy
b. Determine what intermediate demands are created if agriculture plans to produce 100 units

THE LEONTIEF INPUT-OUTPUT MODEL

- The Leontief input-output model or production equation

$$\boldsymbol{x} = \mathbf{C}\,\mathbf{x} + \mathbf{d}$$

Where **x –** amount produced, **Cx** – Intermediate demand and **d** – Final demand

- The above equation may also be written as $I\boldsymbol{x} - C\boldsymbol{x} = \boldsymbol{d}$ or $(I - C)\,\boldsymbol{x} = \boldsymbol{d}$

THE LEONTIEF INPUT-OUTPUT MODEL

Ex 2.6.1 (continued)

c. Determine the production levels needed to satisfy a final demand of 18 units for agriculture, with no final demand for the other sectors.

d. Determine the production levels needed to satisfy a final demand of 18 units for manufacturing, with no final demand for the other sectors.

e. Determine the production levels needed to satisfy a final demand of 18 units of manufacturing , 18 units for agriculture and 0 units for services.

THE LEONTIEF INPUT-OUTPUT MODEL

- In the theorem, the term column sum denotes the sum of the entries in a column of a matrix. Under ordinary circumstances, the column sums of a consumption matrix are less than 1 because a sector should require less than one unit's worth of inputs to produce on unit of output.

THE LEONTIEF INPUT-OUTPUT MODEL

Theorem 11:

Let C be the consumption matrix for an economy, and let **d** be the final demand. If C and **d** have nonnegative entries and if each column sum of C is less than 1, then $(I - C)^{-1}$ exists and the production vector

$$\boldsymbol{x} = (I - C)^{-1}\boldsymbol{d}$$

has nonnegative entries and is the unique solution of

$$\boldsymbol{x} = C\boldsymbol{x} + \boldsymbol{d}$$