Linear Algebra with Applications to Business and Economics

MATH1119B

Lecture 2-3

Characterization of Invertible Matrices

THE INVERTIBLE MATRIX THEOREM

Theorem 8

Let A be a square matrix nxn matrix. Then the following statements are equivalent. That is, for a given A, the statements are either all true or all false.

a. A is an invertible matrix
b. A is row equivalent to the nxn identity matrix
c. A has n pivot positions
d. The equation $Ax = 0$ has only the trivial solution
e. The columns of A form a linearly independent set
f. The equation $Ax = b$ has at least one solution for each **b** in $\mathbb{R}^n$

THE INVERTIBLE MATRIX THEOREM

g. The columns of A span $\mathcal{R}^n$

h. There is an nxn matrix C such that $CA = I$

i. There is an nxn matrix D such that $AD = I$

j. A^T is an invertible matrix