# MATH1004F — Solution-Test 3 — 7:35–8:25, Oct. 22 2014

## Multiple Choice (No Partial Mark), circle the best possible answer

1. [2 points ] Solve the inequality $x^2 - 3x + 2 < 0$ for $x$

(a)$\{x : -2 < x < -1\}$ (b) $\{x : 2 < x < \infty\}$
(c) $\{x : -\infty < x < 1\}$ (d) $\{x : 1 < x < 2\}(**)$

2. [2 points ] Let $f(x) = (\sin x)^{2x}$. Evaluate $f'(x)$. In other word, find the derivative of $f$ at $x$.

(a) $(\sin x)^{2x}\{2\ln(\sin x) + 2x\cot x\}$ (**) (b) $(\sin x)^{2x}\{2x\ln(\sin x)\}$
(c) $(2\ln(\sin x) + 2x\tan x)$ (d) $(\cos x)^{2x}\{2\ln(\sin x) + 2x\tan x\}$

3. [2 points=1+1] Let $h(x) = \frac{2x^2}{x^2-4}$, then

the vertical asymptote is: -2, 2 the horizontal asymptote is: 2

4. [2 points] Which of the following is a critical number of

$$f(x) = \frac{\ln x}{x}$$

(a) $e$ (**) (b) 0 (c) 1 (d) $\sqrt{2}$

5. [2 points] The slope of the tangent line to the curve $f(x) = xe^x$ at the point $(1, e)$ is:

(a) 1 (b) $e$ (c) $2e$ (**) (d) $3e$

## Long Answer Questions, you have to show your steps.

6. [6 points = 0.5 + 1 + 2.5 + 2] Consider the function

$$f(x) = 4x^3 - 2x^2.$$

a) State the domain of $f(x)$.
Sol: domain $= \{x | x \in \mathbb{R}\}$.

b) Find all the critical numbers of $f(x)$.
Sol: From $f'(x) = 12x^2 - 4x = 4x(3x - 1) = 0$ we have $x = 0, 1/3$. Critical numbers are: $x = 0, 1/3$.

c) Find all the intervals of increasing and decreasing, and local extreme (Minimum and Maximum).

Sol: Look at the following table

| $x$ | $-\infty < x < 0$ | $0 < x < 1/3$ | $1/3 < x < \infty$ |
|---|---|---|---|
| $f'(x)$ | + | - | + |
| $f(x)$ | increasing | decreasing | increasing |

By the First Derivative Test, $f(0) = 0$ is a local maximum, $f(1/3) = -2/27$ is a local minimum.

d) Study the concavity and find all the point(s) of inflection.

Sol: Note that $f''(x) = 24x - 4 = 0$ at $x = 1/6$. Look at the following table

| $x$ | $-\infty < x < 1/6$ | $1/6 < x < \infty$ |
|---|---|---|
| $f''(x)$ | - | + |
| $f(x)$ | concave down | concave up |

$x = 1/6$ or $(1/6, f(1/6))$ is a point of inflection.

7. [4 points =2 + 2] Find the derivative of each of the following functions.

$$3^x \log_3(x^2+1), \qquad e^{x^2} \ln(\sin x)$$

sol: $\frac{d}{dx} 3^x \log_3(x^2+1) = 3^x \ln 3 \log_3(x^2+1) + 3^x \frac{2x}{x^2+1} \frac{1}{\ln 3}$

$\frac{d}{dx} e^{x^2} \ln(\sin x) = e^{x^2} \frac{\cos x}{\sin x} + e^{x^2}(2x) \ln(\sin x) = e^{x^2}(\cot(x) + (2x) \ln(\sin x))$