MATH 1104B **Tutorial #1** **September 21, 2015**

Please work in teams of 4. At the end of the tutorial every team hands in one set of solutions with everybody's name and student number PRINTED and everybody's signature.

The main goal of the tutorial is to learn by working together. Do NOT divide up problems between you and work on them separately. Groups doing this will be marked in a tougher fashion. You and your group should work together on all problems sharing insights and difficulties as you progress. Don't worry if you can't finish all the questions; what you haven't finished in class, finish at home. Your TA is here to help you- don't be shy to ask questions!

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Name _____ Student #_____
Name _____ Student #_____
Name _____ Student #_____
Name _____ Student #_____

1. Consider the following system of equations:

$$\begin{aligned}
x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 - 6x_6 &= 12 \\
7x_1 - 8x_2 + 9x_3 - 10x_4 + 11x_5 - 12x_6 &= 6 \\
13x_1 + 14x_2 + 15x_3 + 16x_4 + 17x_5 + 18x_6 &= 18 \\
19x_1 - 20x_2 - 21x_3 - 22x_4 - 23x_5 - 24x_6 &= 15 \\
25x_1 + 26x_2 + 27x_3 + 28x_4 + 29x_5 + 30x_6 &= 24
\end{aligned}$$

a) This is system of ____5_____ equations with ____6_____ unknowns.
Thus $m =$ ____5____ and $n =$ ____6____ .

b) Give an augmented matrix to represent this linear system of equations.

ANS:
$$\left[\begin{array}{cccccc|c}
1 & -2 & 3 & -4 & 5 & -6 & 12 \\
7 & -8 & 9 & -10 & 11 & -12 & 6 \\
13 & 14 & 15 & 16 & 17 & 18 & 18 \\
19 & -20 & -21 & -22 & -23 & -24 & 15 \\
25 & 26 & 27 & 28 & 29 & 30 & 24
\end{array}\right]$$

2. For each matrix state whether it is in Row Echelon Form (REF), Reduced Row Echelon Form (RREF) or neither.
If the matrix is not REF or RREF, explain why?
If the matrix is in REF, determine the pivot columns.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Neither; zero row is not in the bottom.

(b) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ RREF; pivot columns 1 and 2.

(c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ REF; pivot columns 1 and 2.

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ RREF; pivot columns 1 and 3.

(e) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Neither; leading entry of row 2 is to the left of the leading entry in row

.(f) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ REF; pivot columns 1, 3 and 4.

(g) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ neither; Leading entry of row 3 is not to the left of the leading entry in row 4.

3. a) Elementary row operations on an augmented matrix never change the solution set of the associated linear system. Answer (circle one): **TRUE** FALSE
b) An inconsistent system has more than one solution.
Answer (circle one): TRUE **FALSE**
c) The echelon form of a matrix is unique.
Answer (circle one): TRUE **FALSE**
d) The general solution of a system is an explicit description of all solutions of the system. Answer (circle one): **TRUE** FALSE

4. a) Find the general solution of the linear system in parametric vector form whose augmented matrix is given by

$$\left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{array}\right]$$

ANS:

$$\left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{array}\right] \sim (R_3' = R_3 + R_1) \left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{array}\right]$$

$$\sim (R_3' = R_3 + 4R_2) \left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Column 1 and 3 are pivot columns.
Hence, x *and* z are basic variables. y *and* w are free variables.
So $z - 2w = -3 \Rightarrow z = 2w - 3$
And $x - 7y + 6w = 5 \Rightarrow x = 7y - 6w + 5$
So, the general solution of the above system is,

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7y - 6w + 5 \\ y \\ 2w - 3 \\ w \end{bmatrix} = y \begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} -6 \\ 0 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ -3 \\ 0 \end{bmatrix}$$

b) Give the solution to the corresponding homogeneous liner system.
ANS:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = y \begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} -6 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$