MATH 1104B **Tutorial #4** **November 9, 2015**

1a) $\begin{bmatrix} a+b \\ b+c \\ a+c \\ b+c \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + c\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$H = \text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}\right\}$, hence a subspace

Name ____________________ Student #____________________

Name ____________________ Student #____________________

Name ____________________ Student #____________________

Name ____________________ Student #____________________

1 b) Show that all colums have a pivot position!

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

1. Let $H = \begin{bmatrix} a+b \\ b+c \\ a+c \\ b+c \end{bmatrix}, a,b,c \in \Re$

a) Show that H is a subspace of $\Re^4$.

b) Find a basis for H.

c) What is the dimension of H?

1 c) dim(H)=3 the number of vectors

2a) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & -2 \end{array}\right] \Rightarrow [x]_B = \begin{bmatrix} -6 \\ 9 \\ -2 \end{bmatrix}$

2. Let $B = \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\right\}$ be a basis of $\Re^3$.

2 b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix}$

a) If $x = \begin{bmatrix} 6 \\ 5 \\ -2 \end{bmatrix}$, then find $[x]_B$ the co-ordinate vector of x, relative to basis B.

for x solve augmented matrix

b) Find the vector y where $[y]_B = \begin{bmatrix} 7 \\ -5 \\ 3 \end{bmatrix}$.

for y multiply matrix by vector

3. Let A be a 5×7 matrix.

a) What is the minimum value of nullity of A? 3a) min of nul(A) = 2

b) What is the nullity of A, given that dim $Col\ A = 4$? 3b) nul(A) = 3

4. (a) Let $T : \Re^4 \to \Re^2$ be a linear transformation. Suppose $T(u) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T(v) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

4a) $\begin{bmatrix} 3 \\ 6 \end{bmatrix} - \begin{bmatrix} 6 \\ 10 \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$

Calculate $T(3u - 2v)$.

(b) Let the transformation $T : \Re^2 \to \Re^2$ rotate points about origin through an angle of $\pi/3$ radians. What is the standard matrix of the transformation?

4b) $T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\pi/3) \\ \sin(\pi/3) \end{bmatrix}$

$T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin(\pi/3) \\ \cos(\pi/3) \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) \\ \sin(\pi/3) & \cos(\pi/3) \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$

(c) [4] Let $T : \Re^4 \to \Re^3$ be a linear transformation s.t. $T\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$, $T\begin{bmatrix} -1 \\ 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$.

Determine $T(x)$ where $x = \begin{bmatrix} 5 \\ -1 \\ 2 \\ -4 \end{bmatrix}$

4c) $\left[\begin{array}{cc|c} 1 & -1 & 5 \\ 1 & 1 & -1 \\ 1 & 0 & 2 \\ 1 & 2 & -4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow x = \begin{bmatrix} 5 \\ -1 \\ 2 \\ -4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$

$T(x) = 2T\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - 3T\begin{bmatrix} -1 \\ 1 \\ 0 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ -5 \end{bmatrix}$

5. Let $T : \overset{n}{\Re^4} \to \overset{m}{\Re^3}$ be a linear transformation such that

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 + 3x_3 + 4x_4 \\ x_2 + 2x_3 + 3x_4 \\ x_3 + 2x_4 \end{bmatrix}$$

5a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

5b) $\underset{3\times 4}{\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} 1+4+9+16 \\ 2+6+12 \\ 3+8 \end{bmatrix}} = \begin{bmatrix} 30 \\ 20 \\ 11 \end{bmatrix}$

(a) Give a matrix A s.t. $T(x) = Ax$ for any $x \in \Re^4$.

(b) Find the image of the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

5c) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 2 & 1 \end{array}\right]$ system is consistent ⇒ vector in the range of T ✓

5d) Solve for $Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, x_4 is a free variable

$x_3 = -2x_4$, $x_2 = -2x_3 - 3x_4 = +4x_4 - 3x_4 = x_4$

$x_1 = -2x_2 - 3x_3 - 4x_4 = -2x_4 + 6x_4 - 4x_4 = 0$

$x = \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix} x_4$. So $\begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix}$ is a basis for the Ker(T).

(c) Is the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in the range of T? Explain.

(nullspace)

(d) Give a basis for the Kernel of T, $Ker(T)$

(e) Is T one-to-one (injective)? Explain.

(f) Is T onto (surjective)? Explain.

5e) Since each column does not have a pivot, the columns of the standard matrix are not linearly independent ⇒ T is not one-to-one!

5f) Since each row has a pivot, the columns of A span $\mathbb{R}^3$. T is onto! $\dim(\text{col } A) = 3 = m = 3$

One-to-one: Iff $T(x)=0 \Rightarrow x=0$ or $Ax=0 \Rightarrow x=0$; $\text{null}(A)=0$

Onto: Iff $\text{im } T = \mathbb{R}^m$ or $\dim(\text{im } T) = \dim(\text{col } A) = m$