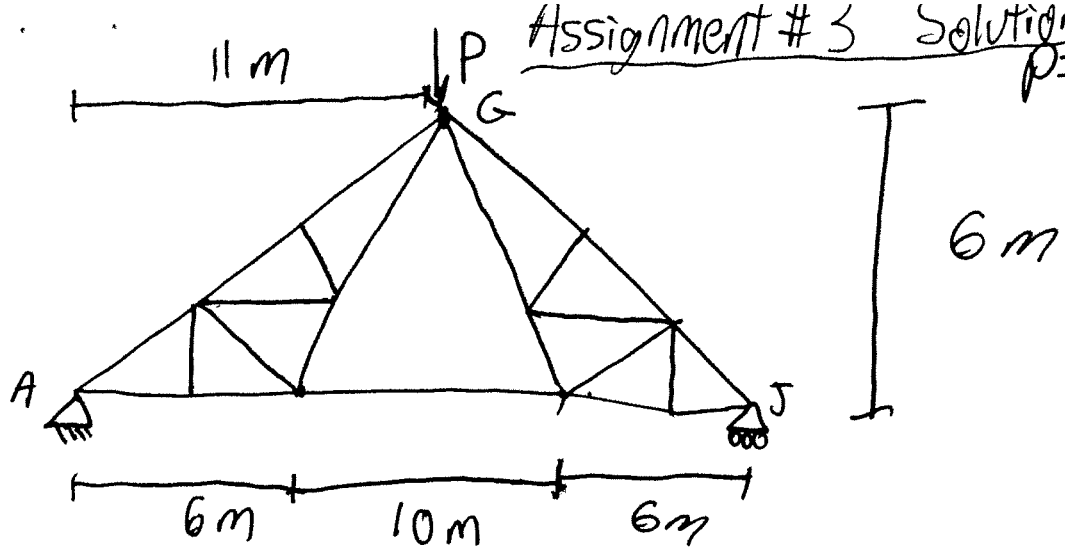
Assignment #3 Solutions

1. 

P = 60KN

a) Find Force couple at A

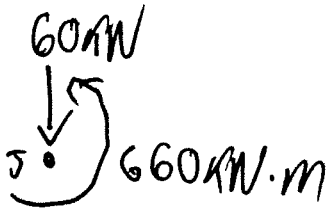
$R_x = \Sigma F_x = 0 \quad R_y = \Sigma F_y = -60KN$,

$\Sigma M_{AR} = -(60KN)(11m) = -660KN \cdot m$,

b) Find force couple at J

$R_y = -60KN$

$\Sigma M_{JR} = (60)(11m) = 660\ KN \cdot m$,



c) Find reactions A_x, A_y, J_y

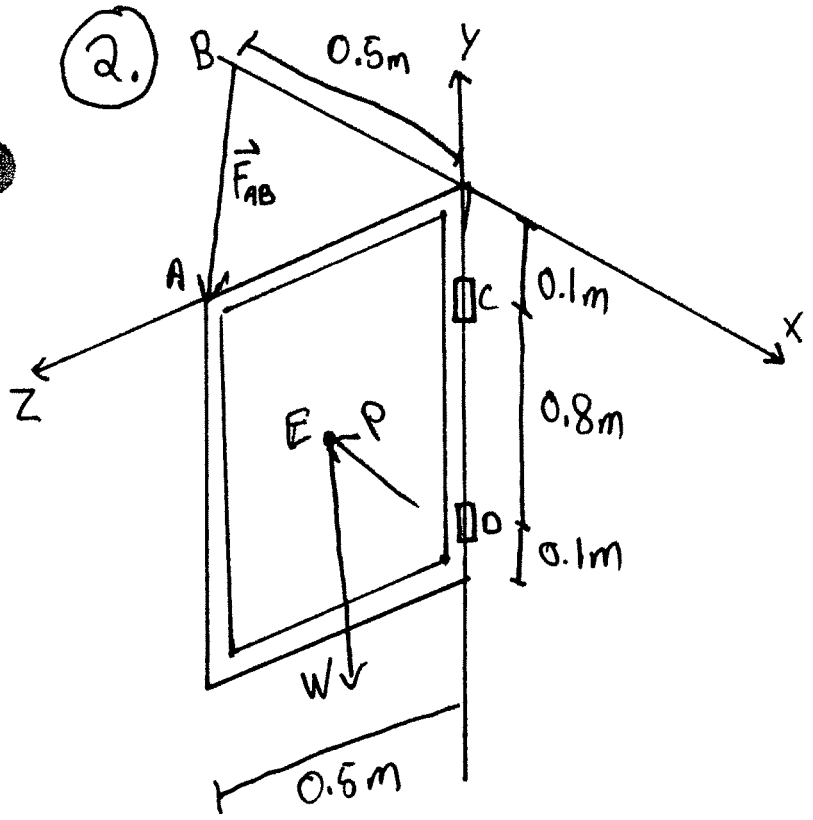
$\Sigma F_x = 0 = \boxed{A_x = 0}$

$\Sigma M_A = -(60KN)(11m) + J_y(22m) \quad \boxed{J_y = 30KN}$,

$\Sigma F_y = 0 = P + J_y + A_y$

$\boxed{A_y = 30KN}$.

(2.)



Given: $W = 200N$

$P = 600N$

- C and D alligned and frictionless (on y-axis, $M_{Cy} = M_{Dy} = 0$)
- Hinge C can support thrust

$C_y \neq 0$

D cannot $D_y = 0$

Want: Reactions at D,

$D_x, \underset{0}{D_y}, D_z, M_{Dx}, \underset{0}{M_{Dy}}, M_{Dz}$

Express $\vec{F}_{BA}$ in vector form:

$$\vec{F}_{BA} = \frac{F_{BA}}{|BA|}\vec{BA} = \frac{F_{BA}}{0.707}(0.5\hat{i} + 0\hat{j} + 0.5\hat{k})$$

$$= F_{BA}\left(\frac{0.5}{.707}\right)\hat{i} + F_{BA}\left(\frac{0.5}{0.707}\right)\hat{k}$$

Forces at E: $\vec{F}_E = -P\hat{i} - W\hat{j} - 0\hat{k} = -600\hat{i} - 200\hat{j} + 0\hat{k}$

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① Start with $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$

$\Sigma F_y = 0 = -W + C_y \Rightarrow C_y = 200N$

$\Sigma F_x = F_{BA_x} - P + C_x + D_x = 0 = F_{BA}\left(\frac{0.5}{.707}\right) - P + C_x + D_x = 0$

$\Sigma F_z = F_{BA_z} + C_z + D_z = 0 = F_{BA}\left(\frac{0.5}{.707}\right) + C_z + D_z = 0$

Combine to get $P - C_x - D_x + C_z + D_z = 0$

-4 unknowns

② Next we can look at $\Sigma M_A = 0$, treating it like a ball and socket $M_{Ax} = M_{Ay} = M_{Az} = 0$

$$\overset{A}{+}\Sigma \vec{M}_A = (M_A)_E + (M_A)_C + (M_A)_D$$

$$= (\vec{AE}) \times (\vec{F}_E) + (\vec{AC}) \times (\vec{F}_C) + (\vec{AD}) \times (\vec{F}_D)$$

$$= (0\hat{i} - 0.5\hat{j} - 0.25\hat{k}) \times (-600\hat{i} - 200\hat{j} + 0\hat{k})$$
$$+ (0\hat{i} - 0.1\hat{j} - 0.5\hat{k}) \times (C_x\hat{i} + C_y\hat{j} + C_z\hat{k})$$
$$+ (0\hat{i} - 0.9\hat{j} - 0.5\hat{k}) \times (D_x\hat{i} + 0\hat{j} + D_z\hat{k})$$

$$= (-50\hat{i} + 150\hat{j} - 300\hat{k}) + (-0.1C_z\hat{i} + 0.5C_y\hat{i} - 0.5C_x\hat{j} + 0.1C_x\hat{i})$$
$$+ (-0.9D_z\hat{i} - 0.5D_x\hat{j} + 0.9D_x\hat{k})$$

- Combine like moments

$\hat{i}$ comp. $\quad \Sigma M_{Ax} = 0 = -50 - 0.1C_z + 0.5C_y - 0.9D_z \Rightarrow C_z = 500 - 9D_z$ (*)

$\hat{j}$ comps. $\quad \Sigma M_{Ay} = 0 = 150 - 0.5C_x - 0.5D_x \Rightarrow C_x = 300 - D_x$

$\hat{k}$ $\quad \Sigma M_{Az} = 0 = -300 + 0.1C_x + 0.9D_x \Rightarrow$ Plug in C_x ↑

$$-300 + 0.1(300 - D_x) + 0.9D_x = 0$$
$$270 = 0.8D_x$$

$\boxed{D_x = 337.5\ N}$ $\quad \boxed{C_x = -37.5}$

Plug D_x, C_x and (*) into $P - C_x - D_x + C_z + D_z = 0$

$$600 - (-37.5) - 337.5 + 500 - 9D_z + D_z = 0$$

$\Rightarrow \boxed{D_z = 100\ N}$ $\quad \boxed{C_z = -400\ N}$

③ Finally, to get M_{Dx} and M_{Dz} we must take moment about D.

$$+\circlearrowleft \Sigma \vec{M}_O = \vec{M}_O + (M_O)_E + (M_O)_C + (M_O)_A$$

$$= M_{Dx}\hat{i} + M_{Dz}\hat{k} + \vec{DE} \times \vec{F}_E + \vec{DC} \times \vec{F}_C + \vec{DA} \times \vec{F}_{BA}$$

$$= M_{Dx}\hat{i} + M_{Dz}\hat{k} + (0\hat{i} + 0.4\hat{j} + 0.25\hat{k}) \times (-600\hat{i} - 200\hat{j} + 0\hat{k})$$
$$+ (0\hat{i} + 0.8\hat{j} + 0\hat{k}) \times (C_x\hat{i} + C_y\hat{j} + C_z\hat{k})$$
$$+ (0\hat{i} + 0.9\hat{j} + 0.5\hat{k}) \times (0.707F_{BA}\hat{i} + 0\hat{j} + 0.707F_{BA}\hat{k})$$

$$+\circlearrowleft \Sigma \vec{M}_D = M_{Dx}\hat{i} + M_{Dz}\hat{k} + (50\hat{i} - 150\hat{j} + 240\hat{k}) + (0.8C_z\hat{i} + 0\hat{j} - 0.8C_x\hat{k})$$
$$+ (0.9(.707F_{BA})\hat{i} + 0.5(.707F_{BA})\hat{j} - 0.9(0.707F_{BA})\hat{k})$$

Combine $\hat{j}$ terms $\Sigma M_{Dy} = -150 + 0.5(0.707F_{BA}) \Rightarrow F_{BA} = 424\,N$

" $\hat{i}$ terms $\Sigma M_{Dx} = M_{Dx} + 50 + 0.8C_z + 0.9(0.707F_{BA})$

$$M_{Dx} = -50 - 0.8(-400) + 0.9(.707(424))$$
$$= 0.2 \approx 0\ N\cdot m$$

" $\hat{k}$ terms $\Sigma M_{Dz} = M_{Dz} + 240 - 0.8C_x - 0.9(.707F_{BA})$

$$M_{Dz} = -240 + 0.8(-37.5) + 0.9(.707(424))$$
$$= -0.2 \approx 0\,N\cdot m$$

∴ Reaction at D ⇒ $D_x = 337.5\,N$ $\quad M_{Dx} = 0\,N\cdot m$

$D_z = 100\,N$ $\quad M_{Dz} = 0\,N\cdot m$