2014 MIDTERM - SOLUTIONS

PART A:

1. Extensive form will specify sets (lists) consisting of players, strategies, information sets, outcomes and describe timing of moves/order of play and specify the payoff functions for each player.
   The normal form does not specify the timing/order of moves or list the information sets.

2. Both players have continuous strategies, a strategy can be any fraction or share taken from the interval from 0 to 1.

$$\pi_J(a,b) = \begin{cases} 0 & \text{if } a+b>1 \text{ and } b>a \\ 5 & \text{if } a+b>1 \text{ and } a=b \\ 10b & \text{if } a+b \leq 1 \text{ or if } a+b>1 \text{ and } a>b \end{cases}$$

PART B

1. Simultaneous and sequential moves:

(a)

	NUMBER of SUBGAMES:	**4**
Player	# of actions in a strategy	# of strategies
1	**4**	**24**
2	**2**	**6**

(b)

		Player 2		
		L	C	R
Player 1	U	$\cdot 7, 2$	$\cdot 8, 5$	$5, 9 \cdot$
	M	$3, 11 \cdot$	$2, 3$	$\cdot 12, 4$
	D	$6, 8 \cdot$	$3, 6$	$4, 2$

By examination of the game matrix - for player 1, D is strictly dominated by U.

After eliminating D from consideration we see that for player 2, C is strictly dominated by R.

No further elimination possible. We are left with:

		Player 2	
		L	R
Player 1	U	$\cdot 7, 2$	$5, 9 \cdot$
	M	$3, 11 \cdot$	$\cdot 12, 4$

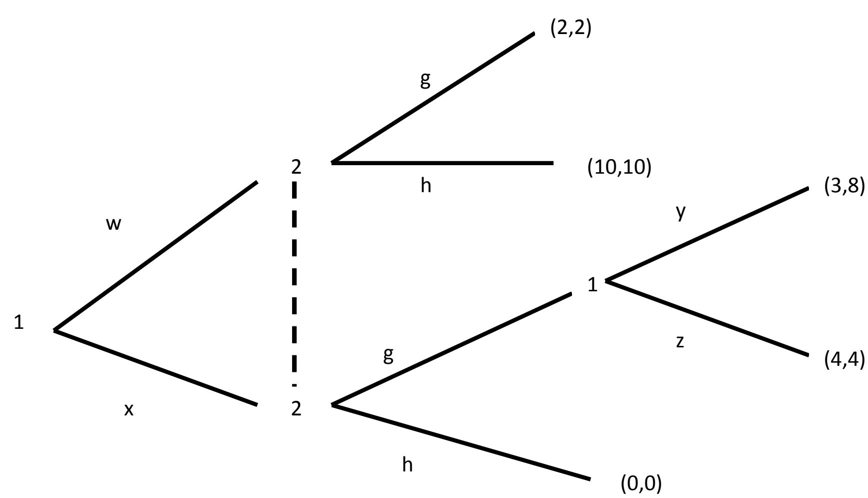
There is no pure strategy Nash equilibrium to this game. There is a mixed strategy Nash equilibrium:

$$\begin{aligned}
\mathbb{E}_1\{U\} &= \mathbb{E}_1\{M\} \\
7\beta + 5(1-\beta) &= 3\beta + 12(1-\beta) \\
4\beta &= 7 - 7\beta \\
\beta &= \frac{7}{11}
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}_2\{L\} &= \mathbb{E}_2\{R\} \\
2\alpha + 11(1-\alpha) &= 9\alpha + 4(1-\alpha) \\
7 - 7\alpha &= 7\alpha \\
\alpha &= \frac{1}{2}
\end{aligned}$$

$$\left\{\left(U \text{ with prob } \frac{1}{2}, M \text{ with prob } \frac{1}{2}\right), \left(L \text{ with prob } \frac{7}{11}, R \text{ with prob } \frac{4}{11}\right)\right\}$$

(c)



In player $1's$ final subgame, player 1 will choose z $(4 > 3)$. There is a simultaneous move subgame that can be shown by the matrix:

		Player 2	
		g	h
Player 1	w, z	$2, 2$	$\cdot 10, 10 \cdot$
	x, z	$\cdot 4, 4 \cdot$	$0, 0$

There are two pure strategy Nash equilibria: $\{(w,z), h\}$ and $\{(x,z), g\}$ and there will be a mixed strategy Nash equilibrium: Let 2 choose g with probability δ and let 1 choose (w,z) with probability λ

$$\begin{aligned}
\mathbb{E}_1\{(w,z)\} &= \mathbb{E}_1\{(x,z)\} \\
2\delta + 10(1-\delta) &= 4\delta + 0(1-\delta) \\
12\delta &= 10 \\
\delta &= \frac{5}{6}
\end{aligned}$$

$$\begin{aligned} \mathbb{E}_2\{g\} &= \mathbb{E}_1\{h\} \\ 2\lambda + 4(1-\lambda) &= 10\lambda + 0(1-\lambda) \\ \lambda &= \frac{1}{3} \end{aligned}$$

$$\left\{\left((w,z) \text{ with prob } \frac{1}{3}\right), \left(g \text{ with prob } \frac{5}{6}\right)\right\}$$

(d) If 1 plays U at the start of the game then 1 will end up with an expected payoff of: $7\beta + 5(1-\beta) = 7\left(\frac{7}{11}\right) + 5\left(\frac{4}{11}\right) = \frac{69}{11} = 6\frac{3}{11}$ from the mixed strategy Nash equilibrium.

If 1 plays D at the start of the game then 1 will end up with

4 if $\{(x,z), g\}$

10 if $\{(w,z), h\}$,

or an expected payoff: $2\delta + 10(1-\delta) = 2\left(\frac{5}{6}\right) + 10\left(\frac{1}{6}\right) = \frac{20}{6} = 3\frac{1}{3}$.if players are playing to the mixed strategy.

Using backwards induction for each of the three Nash equilibria to G_2:

$$\left\{\left(U, \begin{matrix} U \text{ with prob } \frac{1}{2} \\ M \text{ with prob } \frac{1}{2} \end{matrix}, x, z\right), \left(\begin{matrix} L \text{ with prob } \frac{7}{11} \\ R \text{ with prob } \frac{4}{11} \end{matrix}, g\right)\right\}$$

$$\left\{\left(D, \begin{matrix} U \text{ with prob } \frac{1}{2} \\ M \text{ with prob } \frac{1}{2} \end{matrix}, w, z\right), \left(\begin{matrix} L \text{ with prob } \frac{7}{11} \\ R \text{ with prob } \frac{4}{11} \end{matrix}, h\right)\right\}$$

$$\left\{\left(U, \begin{matrix} U \text{ with prob } \frac{1}{2} \\ M \text{ with prob } \frac{1}{2} \end{matrix}, \begin{matrix} w \text{ with prob } \frac{1}{3} \\ x \text{ with prob } \frac{2}{3} \end{matrix}, z\right), \left(\begin{matrix} L \text{ with prob } \frac{7}{11} \\ R \text{ with prob } \frac{4}{11} \end{matrix}, \begin{matrix} g \text{ with prob } \frac{5}{6} \\ h \text{ with prob } \frac{1}{6} \end{matrix}\right)\right\}$$

We can use forward induction to reason that if player 2 observes they are in G_2 (the lower subgame) then player 1 must be expecting to end up at the $\{(w,z), h\}$ Nash equilibrium to this subgame since it is the only outcome that gives a larger payoff than what player 1 knows they will receive by playing Up and going to G_1 (where the payoff to player 1 will be $6\frac{3}{11}$). Expect:

$$\left\{\left(D, \begin{matrix} U \text{ with prob } \frac{1}{2} \\ M \text{ with prob } \frac{1}{2} \end{matrix}, w, z\right), \left(\begin{matrix} L \text{ with prob } \frac{7}{11} \\ R \text{ with prob } \frac{4}{11} \end{matrix}, h\right)\right\}$$

2. Game with Nature

(a) If the state of the economy is normal then:

		Normal	$(p = \frac{2}{3})$
		D 2	
		L	S
D 1	L	$\cdot(10,10)\cdot$	$\cdot(16,8)$
	S	$(8,16)\cdot$	$(6,6)$

D1 will play L and D2 will play L.

If the state of the economy is a downturn then:

		Downturn		$(p = \frac{1}{3})$
			D 2	
			L	S
D 1	L		$\cdot(7,7)\cdot$	$\cdot(10,4)$
	S		$(4,10)\cdot$	$(8,8)$

D1 will play L and $D2$ will play L.

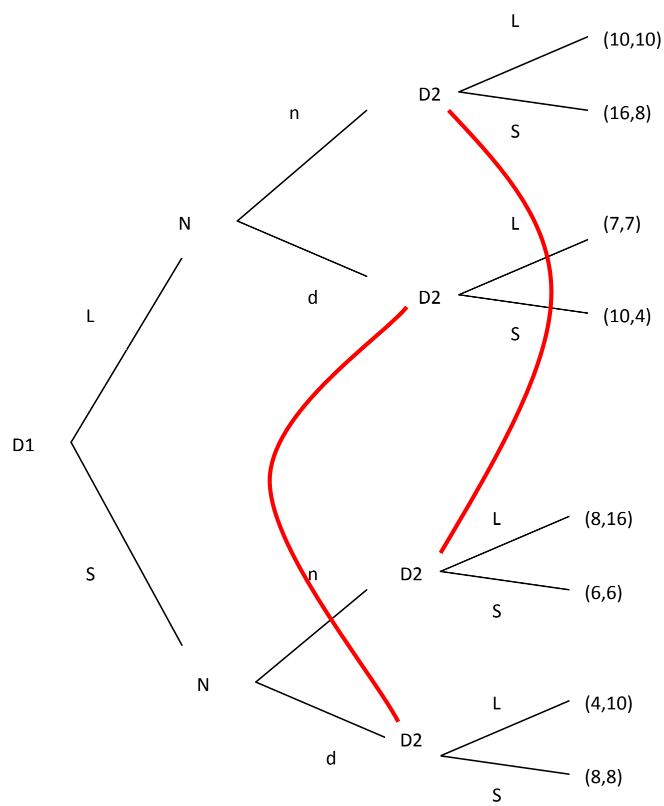
The strategy of each player is a pair of actions that desribes:

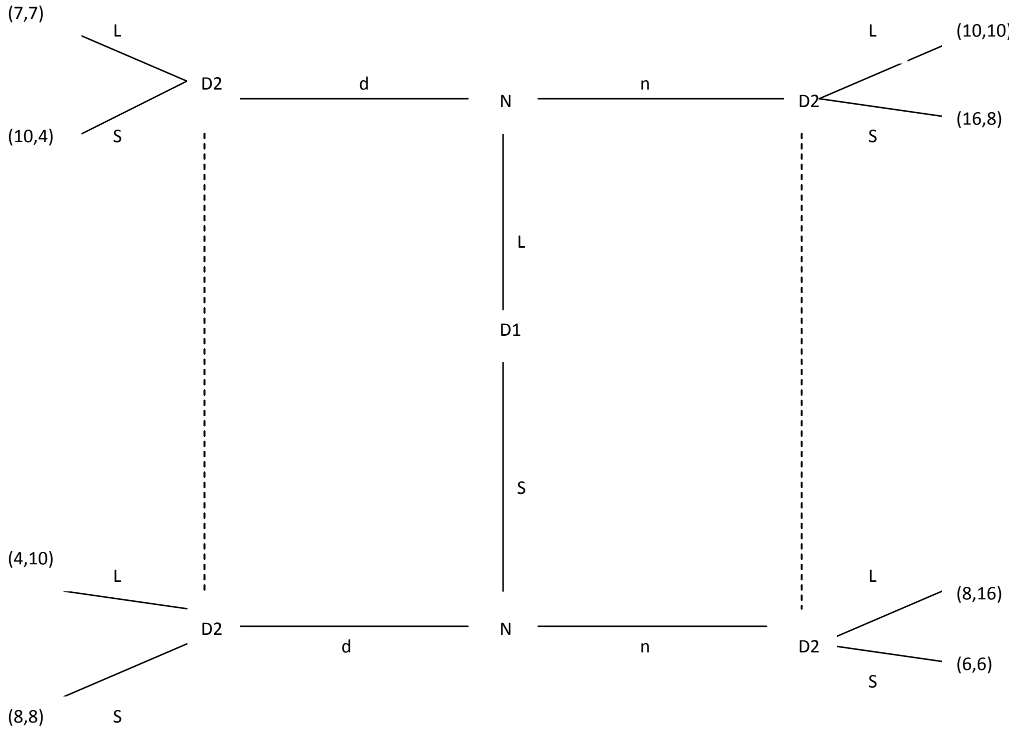
action if Normal, action if downturn

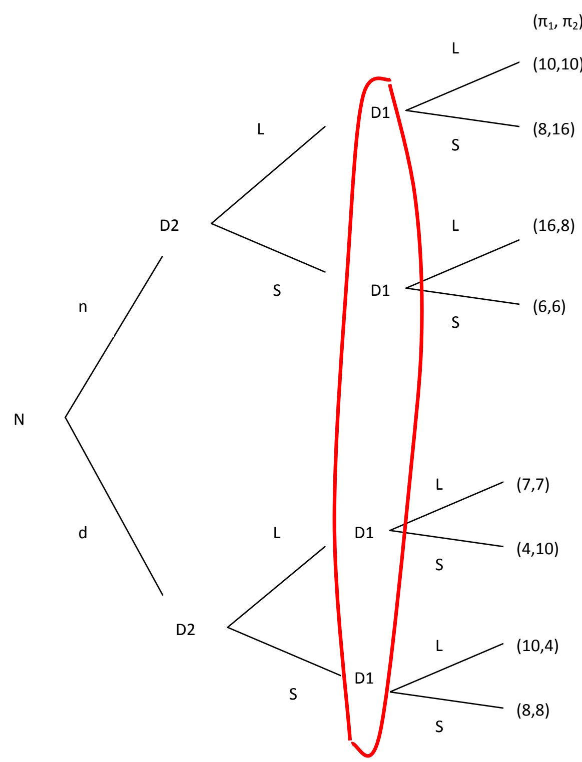
Player 1 : (L, L)

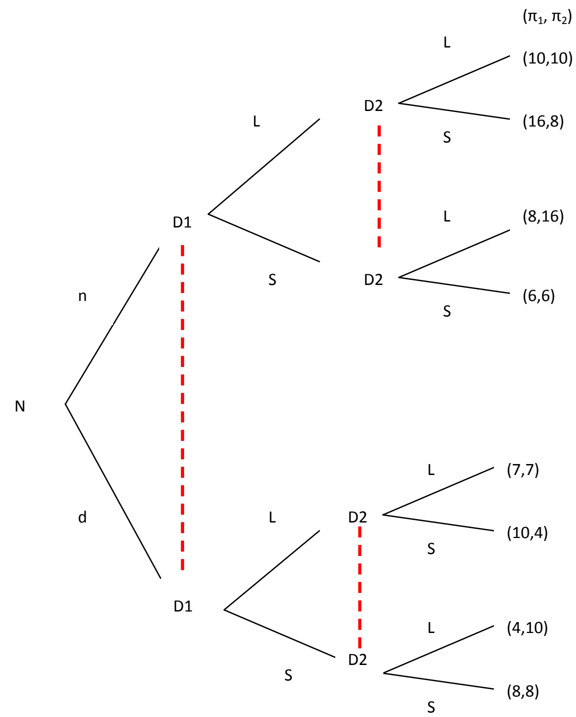
Player 2 : (L, L)

(b) There are a number of ways this can be shown in a game tree: In each case - $D1$ will have only two available strategies, L or S while $D2$ will have four available strategies since the choice of action can be made conditional on the observed state of nature.

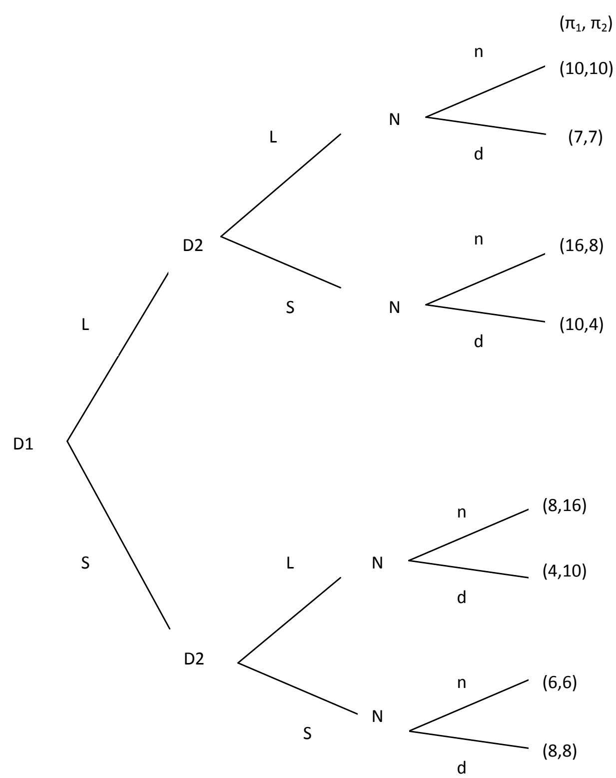



(7,7)
L
D2
d
N
n
D2
L
(10,10)
(10,4)
S
S
(16,8)
L
D1
S
(4,10)
L
L
(8,16)
D2
d
N
n
D2
(6,6)
S
(8,8)
S


(π_1, π_2)
L
(10,10)
L
D1
(8,16)
S
D2
L
(16,8)
S
D1
(6,6)
n
S
N
L
(7,7)
d
L
D1
(4,10)
S
D2
L
(10,4)
D1
S
(8,8)
S

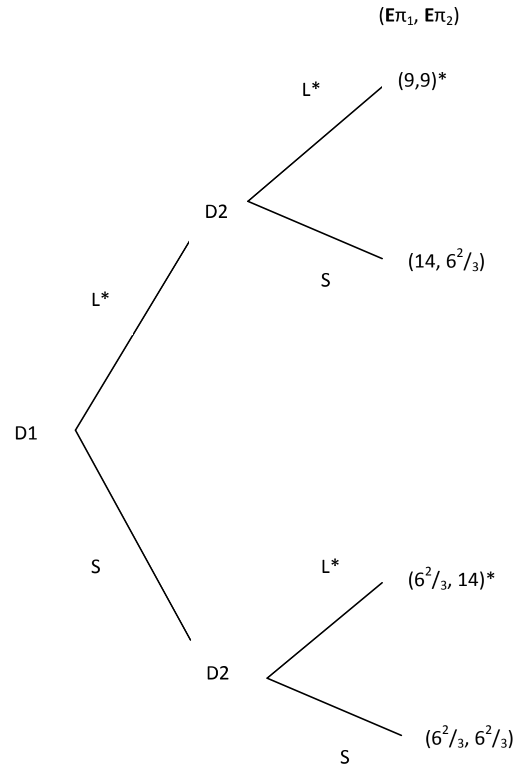


(c)



(d) The game tree in (c) can be re-drawn with expected payoffs in place of the specific state of the economy:

for example: $\mathbb{E}_1\left\{\{L,L\}\right\} = \frac{2}{3}(10) + \frac{1}{3}(7) = \frac{27}{3}$



Using backwards induction:

$$\{L,(L,L)\}$$