University of British Columbia
Department of Economics

Economics 221 (002 & 004)
Introduction to Strategic Thinking
Winter Session Term I, 2011

Midterm Examination - Solutions

PART A:

1. Explain why the Nash equilibrium may be an appropriate solution or prediction of the way a simultaneous-move game will be played, but may not be a very good predictor of how a sequential-move game will be played.
   Nash equilibrium insures that no player has an incentive to unilaterally change behaviour.
   In a simultaneous move game, under a Nash equilibrium no player can be made better off by changing action or strategy.
   In a sequential move game the Nash equilibrium only insures that players are behaving rationally along the equilibrium path. Strategies of a player may include choices that are not rational off the equlibrium path. With common knowledge, the other player will understand this and disregard any non-credible threats or promises.

2. Player 1 and Player 2 play a game where Nature chooses between two different matrices. The choice of Nature is random with the probability of matrix A, $\mathbb{P}(A) = \frac{1}{3}$.

   Nature: A

		Player 2 L	Player 2 R
Player 1	U	$(3,3)$	$(8,6)$
	D	$(6,9)$	$(4,4)$

   Nature: B

		Player 2 L	Player 2 R
Player 1	U	$(9,3)$	$(6,6)$
	D	$(12,6)$	$(4,3)$

   Find a Nash Equilibrium to this game.
   The matrix of expected payoffs is

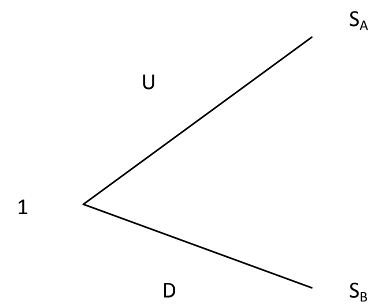
		Player 2 L	Player 2 R
Player 1	U	$(7,3)$	$\cdot(6\frac{2}{3},6)\cdot$
	D	$\cdot(10,7)\cdot$	$(4,3\frac{1}{3})$

   $\{U,R\}$ and $\{D,L\}$ and $\left\{U \text{ with prob}=\frac{11}{20},\ L \text{ with prob}=\frac{8}{17}\right\}$ are the Nash equilibria to this game.

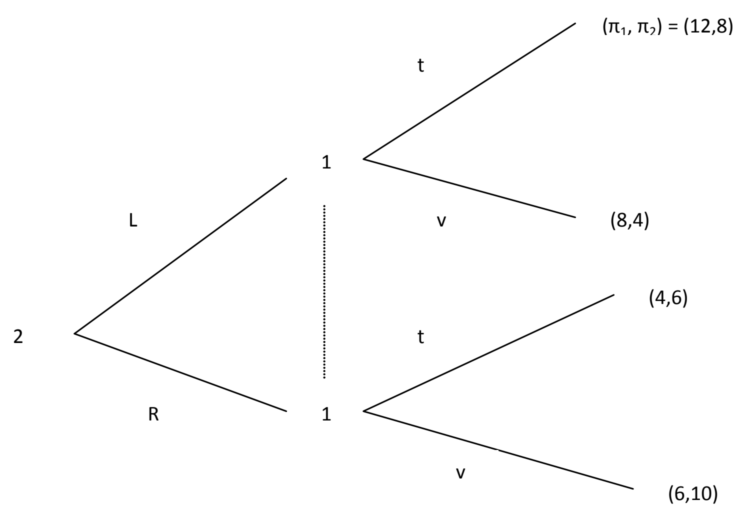
PART B:

1. Player 1 and Player 2 play a game which consists of a mix of sequential and simultaneous moves. Player 1 begins by choosing U or D and the game goes to either subgame

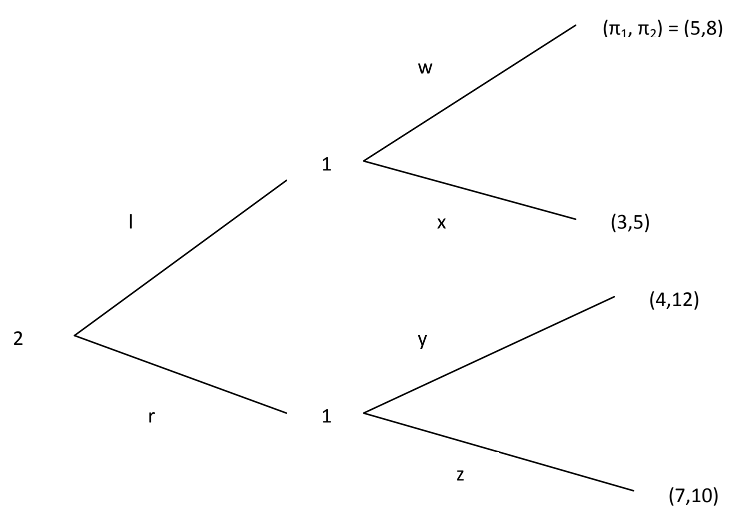
S_A or subgame S_B



Subgame S_A is give by the game tree



And Subgame S_B is give by the game tree



(a) How many subgames are there in this game?

5 subgames

(b) How many strategies does player 2 have in this game? How many strategies does player 1 have in this game? How many possible strategy profiles will this game have?

player 2 : 4 strategies

player 1 : 16 strategies

number of strategy profiles: 64

(c) Find all the Nash Equilibria to the simultaneous subgame S_A.

$\{t, L\}$, $\{v, R\}$ and $\left\{t \text{ with prob} = \frac{3}{4}, L \text{ with prob} = \frac{1}{3}\right\}$

(d) Find all the Subgame Perfect Nash equilibria to the game.

$$\begin{array}{l} \{(\text{player } 1), (\text{player } 2)\} \\ \{(D, v, (w, z)), (R, r)\} \\ \left\{\left(D, t \text{ with prob } \frac{3}{4}, (w, z)\right), \left(L \text{ with prob } \frac{1}{3}, r\right)\right\} \\ \{(U, t, (w, z)), (L, r)\} \end{array}$$

2. Two firms compete in a duopoly market. The firms produce homogeneous goods and each firm has constant marginal costs. The marginal cost of firm 1 is $c_1 = \$20$. The marginal cost of firm 2 is $c_2 = \$30$. Market demand is $Q = 360 - 3P$.

(a) If the firms compete in a Cournot duopoly (compete in quantities), derive the best response functions for both firms, $q_1 = R_1(q_2)$ and $q_2 = R_2(q_1)$.

$$\begin{aligned} R_1(q_2) &: \quad q_1 = 150 - \frac{1}{2}q_2 \\ R_2(q_2) &: \quad q_2 = 135 - \frac{1}{2}q_1 \end{aligned}$$

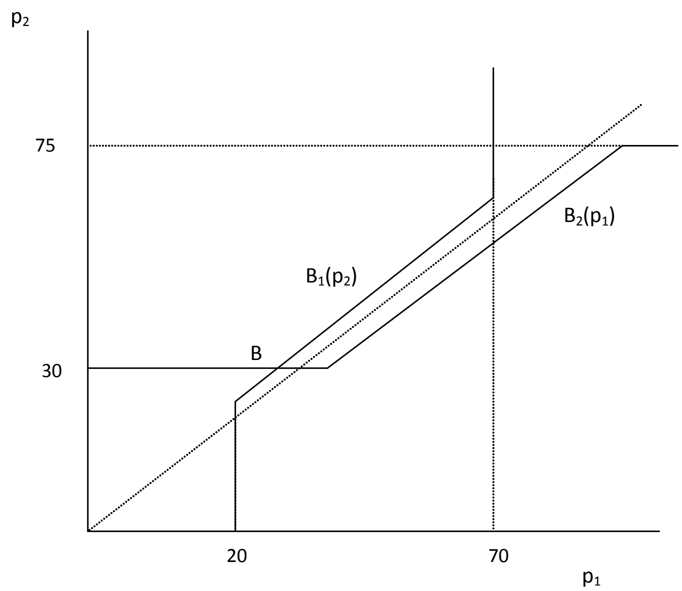
(b) Use the Best Response Functions from part (a) to find the Cournot-Nash equilibrium, $\{q_1^c, q_2^c\}$ and determine the total output and price in the market.

$\{q_1^c = 110, q_2^c = 80\}$

$Q = 190$

$P = \$56.67$

(c) If the firms compete in a Bertrand duopoly (compete in prices), graph the best response functions of both firms and describe the Bertrand-Nash equilibrium. (Show all relevant values on the axes).



(d) Compare Industry profits in the Cournot-Nash equilibrium (part (*b*)) and the Bertrand-Nash equilibrium (part (*c*)). Explain whether industry profits under one competitive structure (Cournot or Bertrand) will always be larger or smaller than the other.

Industry profits will be higher under Cournot because the Cournot price/quantity is closer to monopoly price and quantity.

Industry profits can be larger under either Cournot or Bertrand depending on the difference between costs.