University of British Columbia
Department of Economics

Economics 221 (002 & 003)
Introduction to Strategic Thinking
Winter Session Term I, 2010

Midterm Examination - Solutions

PART A:

1. Find conditions on γ that make the following game solvable by iterated elimination of strictly dominated strategies. Find the equilibrium by iterative elimination of strictly dominated strategies.

		Player 2	
		x	y
	a	$(9,5)$	$(0,0)$
Player 1	b	$(\gamma,2)$	$(2,3)$
	c	$(7,3)$	$(3,2)$

Let SD = strictly dominated

if $\gamma < 7$ then b is SD by c

eliminate $b \Longrightarrow y$ is SD by x

eliminate $y \Longrightarrow c$ is SD by a

eliminate $c \Longrightarrow \{a, x\}$ is equilibrium

2. What is a strategy for a firm in the Cournot model? Does a firm in the Cournot model have a strictly dominant strategy? Does a firm in the Cournot model have any strictly dominated strategies? You may use a figure to illustrate.

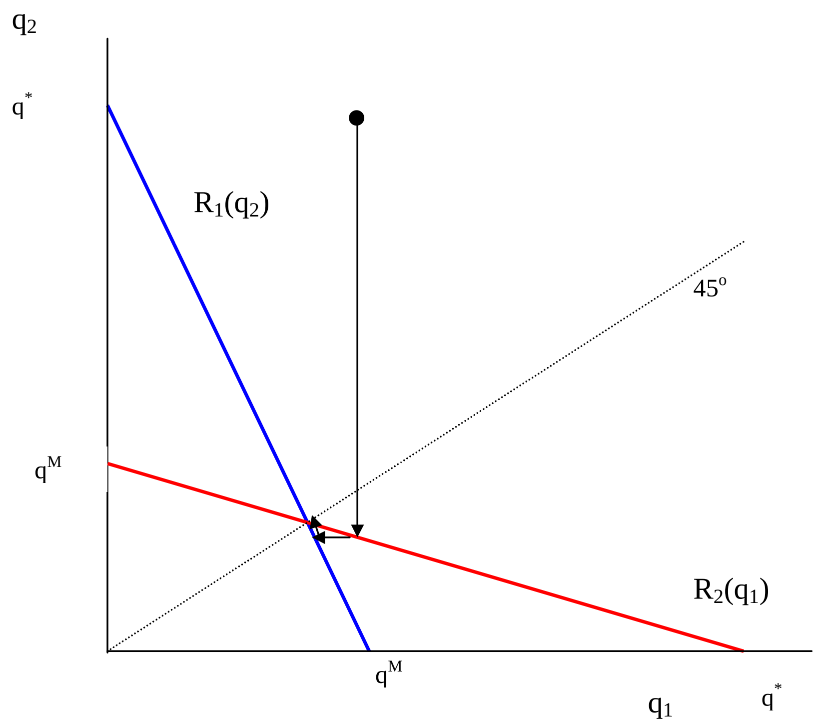
strategy: choice of output, $q \geq 0$

firms do not have a strictly dominant strategy, we know this because $R_i(q_j) \neq k$ where k is a constant - the graph of the BRF of a firm is not a vertical (firm 1) or horizontal (firm 2) line.

firms do have strictly dominated strategies. For firm 1, any $q_1 > q_1^M$ is strictly dominated by q_1^M.

3. Suppose two firms in a Cournot duopoly have identical marginal costs. Starting at $q_1 = q^M$ and $q_2 = q^*$ where q^* is the competitive output level, use a figure that shows the Best response functions for both firms to demonstrate the stability of the Cournot

equilibrium.



4. Explain the importance of the Common knowledge assumption for finding solutions to the following 3 types of games:

   (i) all players have strictly dominant strategies - common knowledge is not required, rational players will choose strictly dominant strategies.

   (ii) a solution can be found by iterative elimination of strictly dominated strategies - CK is necessary so that eg. player 1 knows that player 2 is rational, player 1 knows that player 2 knows that player 1 is rational, etc.

   (iii) coordination games such as:

		Player 2	
		L	R
Player 1	U	(x, x)	$(0, 0)$
	D	$(0, 0)$	(y, y)

where $x > y > 0$

   CK is necessary for the Nash equilibria to a game such as this. CK does not permit selection of one equilibrium over the others - $\{U, L\}$ and $\{D, R\}$ are both valid pure strategy Nash equilibria.

PART B:

1. Suppose that the interaction between a traveller driving across the border and a border guard can be modelled as a game with the usual assumptions of rationality and common knowledge. The traveller must choose whether to try to hide (H) a bottle of alcohol

when crossing the border or leave it behind (L). The border guard must choose whether to search (S) the vehicle for any contraband or let the vehicle pass (P) through with no check. If a search is carried out and the traveller has alcohol, it will be found by the border guard. The game can be represented by the following matrix:

		Guard	
		S	P
Traveller	H	$(0,10)\cdot$	$\cdot(12,2)$
	L	$\cdot(5,4)$	$(4,6)\cdot$

(a) Find a Nash equilibrium to the game.

No pure strategy Nash equilibria. There is a mixed strategy Nash equilibrium where T plays H with $\alpha = \frac{1}{5}$ and G plays S with $\beta = \frac{8}{13}$.

(b) What is the probability that the Guard will choose to search and find nothing?

$$\mathbb{P}(L)\,\mathbb{P}(S) = \left(\frac{4}{15}\right)\left(\frac{8}{13}\right) = \frac{32}{65}$$

(c) Suppose that there is a third player to this game. The Manager at the border crossing chooses which of two incentive systems (I_1 or I_2) will be put in place for the guards. If I_2 is chosen, the guard is very motivated to catch people smuggling and doesn't care about wait times at the border. Modifying the game to include the Supervisor we have: (traveller, guard, manager)

M: I_1

		G:	
		S	P
T:	H	$(0,10\cdot,15)\cdot$	$\cdot(12,2,0)\cdot$
	L	$\cdot(5,4,2)\cdot$	$(4,6\cdot,10)\cdot$

M: I_2

		G:	
		S	P
T:	H	$(0,10\cdot,15)\cdot$	$\cdot(12,2,0)\cdot$
	L	$\cdot(5,8\cdot,2)\cdot$	$(4,4,10)\cdot$

Explain what it would mean for player G to have a strictly dominant strategy. Does any player in this game have a strictly dominant strategy?

If G has a strictly dominant strategy then the strictly dominant strategy must give G a larger payoff than playing any other strategy no matter what both M and T are playing.

No players in the game have strictly dominant strategies.

(d) Find a Nash equilibrium to the game in (c).

$$\{L, S, I_2\}$$

2. Two firms compete in a Cournot duopoly market. The firms produce homogeneous goods and each firm has constant marginal costs. The marginal cost of firm 1 is $c_1 = \$80$. The marginal cost of firm 2 is $c_2 = \$80$. Market demand is $q = 8000 - 40p$.

(a) Find the best response function of firm 1.

$$\begin{aligned} P &= 200 - \frac{1}{40}q \\ &= 200 - \frac{1}{40}q_2 - \frac{1}{40}q_1 \\ MR_1 &= 200 - \frac{1}{40}q_2 - \frac{1}{20}q_1 \end{aligned}$$

setting $MR_1 = MC_1$

$$\begin{aligned} 200 - \frac{1}{40}q_2 - \frac{1}{20}q_1 &= 80 = c_1 \\ R_1(q_2) &: \quad q_1 = 2400 - \frac{1}{2}q_2 \end{aligned}$$

(b) What is the monopoly output (when $c = \$80$)?

if $q_2 = 0$ then $q_1 = q_1^M = R_1(0) = 2400$

(c) Find the Cournot-Nash equilibrium and determine the total output and price in the market. Show your equilibrium in a well-labelled diagram that shows the best response functions of each firm. Be sure to show the values of any intercepts or intersections.

with $c_1 = c_2$ firm 2 will have similar BRF

$$R_2(q_1) : q_2 = 2400 - \frac{1}{2}q_1$$

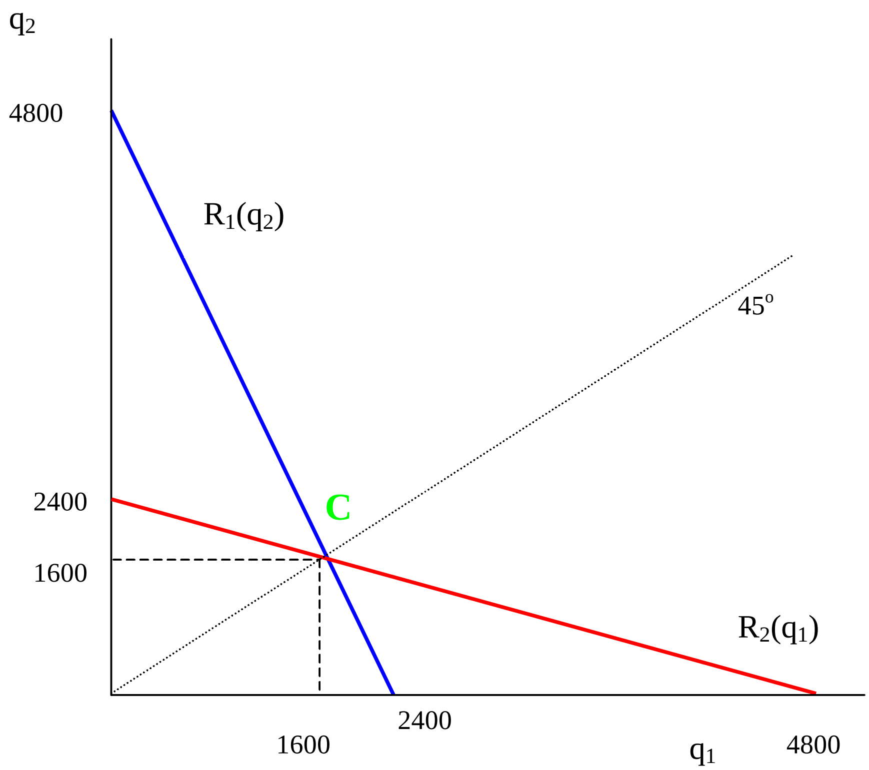
given the symmetry in the problem, we expect $q_1^c = q_2^c = q^c$

$$\begin{aligned} q^c &= 2400 - \frac{1}{2}q^c \\ q^c &= 1600 \end{aligned}$$

Cournot-Nash equilibrium

$$q_1^c = 1600, \;\; q_2^c = 1600$$

Total industry output $Q^c = 3200$, price $P^c = \$120$



(d) Suppose that, recognizing that profits are maximized in a monopoly industry, both firms attempt to replicate this by agreeing to set the total output $q_1 + q_2$ equal to the monopoly level of output, with each firm producing $\frac{1}{2}q^M$. If firm 1 really thinks firm 2 will produce $q_2 = \frac{1}{2}q^M$, will firm 1 really want to stick to $q_1 = \frac{1}{2}q^M$? What output level should firm 1 set? What profits can firm 1 earn if firm 2 produces $q_2 = \frac{1}{2}q^M$?

If $q_2 = \frac{1}{2}q^M = 1200$ then firm 1 will want to set $q_1 = R_1(1200) = 2400 - \frac{1}{2}(1200) = 1800$ rather than $q_1 = 1200$ as agreed upon.

Total industry output will be $Q = 3000$ and price will be $P = \$125$. Profits to firm 1 will be

$$\pi_1 = (125 - 80)(1800) = \$81000$$