Total mark: 20. Closed book. No calculator is allowed.

Last Name __________ ***First Name*** __________ ***Student Number*** __________

Question 1. [10 Marks] Find the first derivatives of the following functions:

(1) (2 points) $y = (x+1)(x^3 - x^2 + 1)$,

(2) (2 points) $y = \frac{x^3+2x}{x^2-1}$

(3) (2 points) $y = x\sin(3x^2)$

(4) (2 points) $y = e^{\cos(3x+2)}$

(5) (2 points) $y = \left[\sin(2x)\cos(x^2)\right]^3$

Solution:

1)

$$y' = (x^4 - x^2 + x + 1)' = 4x^3 - 2x + 1$$

or

$$\begin{aligned} y' =&(x+1)'(x^3 - x^2 + 1) + (x+1)(x^3 - x^2 + 1)' \\ =&1 * (x^3 - x^2 + 1) + (x+1)(3x^2 - 2x) \\ =&(x^3 - x^2 + 1) + (3x^3 + x^2 - 2x) \\ =&4x^3 - 2x + 1, \end{aligned}$$

2)

$$\begin{aligned} y' =&\frac{(x^3+2x)'(x^2-1) - (x^3+2x)(x^2-1)'}{[x^2-1]^2} \\ =&\frac{(3x^2+2)(x^2-1) - (x^3+2x)(2x)}{[x^2-1]^2} \\ =&\frac{(3x^4 - x^2 - 2) - (2x^4 + 4x^2)}{[x^2+1]^2} \\ =&\frac{x^4 - 5x^2 - 2}{[x^2+1]^2} \end{aligned}$$

3)

$$\begin{aligned} y' =& \left[x\sin(3x^2)\right]' \\ =&(x)'\sin(3x^2) + x[\sin(3x^2)]' \\ =&1 * \sin(3x^2) + x[6x\cos(3x^2)] \\ =&\sin(3x^2) + 6x^2\cos(3x^2) \end{aligned}$$

4) $y' = e^{\cos(3x+2)}[\cos(3x+2)]' = e^{\cos(3x+2)}[-\sin(3x+2)](3x+2)' = -3\sin(3x+2)e^{\cos(3x+2)}$

5)

$$\begin{aligned} y' =&\{[(\sin(2x)\cos(x^2)]^3\}' \\ =&3[(\sin(2x)\cos(x^2)]^2[(\sin(2x)\cos(x^2)]' \\ =&3[(\sin(2x)\cos(x^2)]^2[(\sin(2x))'\cos(x^2) + \sin(2x)(\cos(x^2))'] \\ =&3[(\sin(2x)\cos(x^2)]^2[2\cos(2x)\cos(x^2) + \sin(2x)(2x\sin(x^2)] \\ =&3[(\sin(2x)\cos(x^2)]^2[2\cos(2x)\cos(x^2) + 2x\sin(x^2)\sin(2x)] \end{aligned}$$

Question 2. [4 Marks] Find the second derivatives of the following function $y = \sin(x^2 + x + 1)$

Solution: [**2 marks for each derivative**]

$$y' = (2x+1)\cos(x^2+x+1), \quad y'' = 2\cos(x^2+x+1) - (2x+1)\sin(x^2+x+1)$$

Question 3. [6 Marks] Find an equation for the tangent and normal line to the curve $y = -1 + 2\sin x$ at the point $(\pi/6, 0)$.

Solution: The slop of the tangent line is:[**1 mark1**]

$$(-1+2\sin x)'\big|_{x=\pi/6} = (2\cos x)\big|_{x=\pi/6} = 2\cos(\pi/6) = \sqrt{3}$$

so the slop of the normal is $-\frac{\sqrt{3}}{3}$ since $-\frac{\sqrt{3}}{3} \times \sqrt{3} = -1$.[**1 mark**]
So, the equation of the tangent line is $y - 0 = \sqrt{3}(x - \pi/6)$[**2 marks**]; the equation of the normal line is $y - 0 = -\frac{\sqrt{3}}{3}(x - \pi/6)$.[**2 marks**]