Total mark: 20. Closed book. No calculator is allowed.

Last Name _____ ***First Name*** _____ ***Student Number*** _____

Question 1. [8 Marks] Find the following limits:

(1) (2 points) $\lim_{x\to\infty} \frac{5x^4+7x^3+5x}{3x^4+4x^3-3x}$,

(2) (2 points) $\lim_{x\to 0} \cot\left(\pi \sin(\frac{\pi}{6(x+1)})\right)$

(3) (2 points) $\lim_{\theta\to 0} \frac{\sin(3\theta)}{\sin(4\theta)}$

(4) (2 points) $\lim_{x\to\infty} \sqrt{x^2+3x} - \sqrt{x^2+x}$

Solution:

1) $\lim_{x\to\infty} \frac{5x^4+7x^3+5x}{3x^4+4x^3-3x} = \frac{5}{3}$,
2) $\lim_{x\to 0} \cot\left(\pi \sin(\frac{\pi}{6(x+1)})\right) = 0$
3) $\lim_{\theta\to 0} \frac{\sin(3\theta)}{\sin(4\theta)} = \frac{3}{4}$
4) $\lim_{x\to\infty} \sqrt{x^2+3x} - \sqrt{x^2+x} = 1$

Question 2. [4 Marks] Define $f(5)$ in a way the extends $f(x) = \frac{x^2-2x-15}{x^2-25}$ to be continuous at $x = 5$?

Solution: $f(5) = \frac{4}{5}$

Question 3. [5 Marks] Let

$$f(x) = \begin{cases} 3-2x & x < 1 \\ x^2+x-1 & 1 \le x \le 2 \\ 3x^2 & x > 2 \end{cases}$$

Find $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x), \lim_{x\to 2} f(x)$, $\lim_{x\to 1} f(x)$, $\lim_{x\to 3} f(x)$.

Solution:

$$\lim_{x\to 2^-} f(x) = 5, \ \lim_{x\to 2^+} f(x) = 12, \ \lim_{x\to 2} f(x) \text{ does not exist },$$

$$\lim_{x\to 1} f(x) = 1, \ \lim_{x\to 3} f(x) = 27$$

Question 4. [3 Marks] For what value of a is $f(x) = \begin{cases} 3x-3 & x < 1 \\ x^2+x-a & x \ge 1 \end{cases}$ continuous at every x?

Solution: $a = 2$