

Total mark: 20. Closed book. No calculator is allowed.

Last Name _____ First Name _____ Student Number _____

Question 1. [4 Marks] Simplify the following expression:

- (1) (2 points) $(16)^{-\frac{3}{2}}$,
- (2) (2 points) $(\sqrt{5})^{\log_5 9}$

Solution:

- 1) $(16)^{-\frac{3}{2}} = \frac{1}{64}$,
- 2) $(\sqrt{5})^{\log_5 9} = 3$

Question 2. [2 Marks] Solve for x : $e^{2x} = e^{10}$

Solution: $x = 5$,

Question 3. [3 Marks] Let $f(x) = \sqrt{x-9}$, $g(x) = x^2$. Find $f \circ g$ and the domain of $f \circ g$.

Solution: [2 marks for the function, 1 mark for the domain]

$$f \circ g = \sqrt{x^2 - 9}, \text{ Domain: } |x| \geq 3,$$

Question 4. [3 Marks] Find f^{-1} for $f(x) = 10x - x^2$, $x \geq 5$.

Solution: [2 marks for the function, 1 mark for the domain]

$$f^{-1}(x) = 5 + \sqrt{25 - x}, \quad x \leq 25$$

Question 5. [5 Marks] Find the exact value of the following expression:

- (1) (2 points) $\arcsin\left(\frac{1}{2}\right)$
- (2) (3 points) $\cos^2\left(\frac{\pi}{12}\right)$

Solution:

- 1) $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$
- 2)

$$\cos^2\left(\frac{\pi}{12}\right) = \frac{1}{2} \left(1 + \cos\left(\frac{2\pi}{12}\right)\right) = \frac{1}{2} \left(1 + \cos\left(\frac{\pi}{6}\right)\right) = \frac{1}{2} \left(1 + \frac{\sqrt{3}}{2}\right) = \frac{2 + \sqrt{3}}{4}$$

Question 6. [3 Marks] Given $\cos \theta = \frac{3}{5}$, $\theta \in [0, \frac{\pi}{2}]$, find $\sin \theta$.

Solution:

$$\sin \theta = \frac{4}{5}.$$