1. Let S be a set of size 25, and let x be an element of S. The number of subsets of S that contain x is

   (a) 2^{24}

   (b) $2^{25} - 25$

   (c) $2^{25} - 1$

   (d) none of the above.

2. How many bit strings of length 8 start with 00 or end with 1?

   (a) 192

   (b) 160

   (c) 186

   (d) 174

3. The number of different strings that can be made by reordering the letters of the word HELLOHELLO is

   (a) 4!2!2!2!

   (b) $\binom{10}{4}\binom{6}{2}\binom{4}{2}\binom{3}{2}$

   (c) 10!

   (d) none of the above.

4. A club with 14 men and 18 women needs to select a team consisting of 8 players. How many different teams can be made?

   (a) $\binom{14}{8}\binom{18}{8}$

   (b) $\sum_{n=0}^{8}(\binom{14}{n} + \binom{18}{8-n})$

   (c) $\binom{32}{8}$

   (d) $\binom{32}{8}8!$

5. Using the 26-letter alphabet $\{a, b, c, \ldots, z\}$, how many different 20-letter strings are there that start with $abxy$, end with pq, and contain exactly 4 k's?

   (a) $\binom{14}{4}4!10^{25}$

   (b) $\binom{14}{4}10^{25}$

   (c) $\binom{14}{10}25^{10}$

   (d) none of the above.

6. How many solutions are there to the equation $x_1 + x_2 + x_3 = 16$, where $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ are integers?

   (a) $\binom{18}{2}$
   (b) $\binom{19}{2}$
   (c) $\binom{19}{3}$
   (d) $\binom{18}{3}$

7. What is the coefficient of $x^{12}y^{15}$ in the expansion of $(-3x + 7y)^{27}$?

   (a) $\binom{27}{12}$
   (b) $-\binom{27}{15}3^{12}7^{15}$
   (c) $\binom{27}{15}7^{12}3^{15}$
   (d) $\binom{27}{15}3^{12}7^{15}$

8. Let A be a set of size 6 and let B be a set of size 5. How many different functions are there from A to B that are **not** one-to-one?

   (a) 6^5
   (b) $5^6 - 5!$
   (c) 5^6
   (d) $6^5 - \binom{6}{5}$

9. The Fibonacci numbers are defined as follows: $f_0 = 0$, $f_1 = 1$, and $f_{n+1} = f_{n-1} + f_n$ for $n \geq 1$. Which of the following is true?

   (a) For all $n \geq 1$: $f_1^2 + f_2^2 + f_3^2 + \ldots + f_n^2 = (f_n)^2$.
   (b) For all $n \geq 1$: $f_1^2 + f_2^2 + f_3^2 + \ldots + f_n^2 = f_{n-1}f_n$.
   (c) For all $n \geq 1$: $f_1^2 + f_2^2 + f_3^2 + \ldots + f_n^2 = f_n f_{n+1}$.
   (d) None of the above.

10. Consider the following recursive function:

$$\begin{aligned} f(0) &= 7, \\ f(n+1) &= f(n) + 6n + 1 \text{ for all integers } n \geq 0. \end{aligned}$$

Which of the following is true?

(a) For all $n \geq 0$: $f(n) = 2n^2 + 2n + 7$.

(b) For all $n \geq 0$: $f(n) = 3n^2 + 2n + 7$.

(c) For all $n \geq 0$: $f(n) = 2n^2 - 2n + 7$.

(d) For all $n \geq 0$: $f(n) = 3n^2 - 2n + 7$.

11. Let V be a set of size n. How many undirected graphs with vertex set V are there?

(a) $2^{\binom{n}{2}}$ (i.e., 2 to the power $\binom{n}{2}$)

(b) 2^n

(c) $n!$

(d) 2^{n^2} (i.e., 2 to the power n^2)