ACTU 256 / Fall 2013

Assignment 4

Due: Wednesday, November 6, 2013 (at the beginning of the class)

Please note:

- *Each student should submit their assignments before the beginning of class on the announced due date. Late assignments will not be accepted.*
- *Solutions must be written up carefully, showing all work, for full credit.*
- *No points will be awarded for a numerical answer that is not justified by a demonstration of the steps used.*

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1. Bond A is an n-year bond with quarterly coupons and face value of 1000. Bond B is an $\frac{n}{4}$-year zero-coupon bond. Both bonds have the same redemption amount and nominal yield rate convertible quarterly.
The difference between the price of bond B and the present value of the redemption amount for bond A is equal to 320.40, while the difference of their squared values is 579837.5. If, for bond A, the ratio of the 3-month bond rate to the 3-month yield rate, $\frac{r}{i} = 1.04167$, find the price of bond A.

Solution. Let $C_A = C_B = C$ be the common redemption amount. We are given

$$Cv_i^n - Cv_i^{4n} = 320.40$$

$$(Cv_i^n)^2 - (Cv_i^{4n})^2 = 579837.5,$$

which is equivalent to

$$Cv_i^n - Cv_i^{4n} = 320.40$$

$$(Cv_i^n - Cv_i^{4n})(Cv_i^n + Cv_i^{4n}) = 579837.5,$$

where $a^2 - b^2 = (a-b)(a+b)$ was used. Then,

$$Cv_i^n - Cv_i^{4n} = 320.40$$

$$Cv_i^n + Cv_i^{4n} = 1809.73,$$

leading to $Cv_i^n = 1065.065$. Consequently,

$$Cv_i^n - Cv_i^{4n} = 320.40 \;\Rightarrow\; Cv_i^n(1-v_i^{3n}) = 320.40 \;\Rightarrow\; 1065.065(1-v_i^{3n}) = 320.40$$

$$\Rightarrow \quad v_i^{3n} = 1 - \frac{320.40}{1065.065} = 0.7.$$

So, $v_i^n = 0.8879$ and therefore, $C = \frac{1065.065}{v_i^n} = 1199.53 \approx 1200$. The price of bond A is

$$P_A = Fra_{\overline{4n}|i} + Cv_i^{4n} = 1000 \cdot \frac{r}{i}(1 - v_i^{4n}) + Cv_i^{4n}$$

$$= 1000(1.04167)(1 - 0.8879^4) + 1200 \cdot 0.8879^4 = 1140.$$

2. Mike borrows an amount at an annual interest rate of $8^o/_o$. He repays all interest and principal in a lump sum at the end of ten years from now.
Mike uses the amount borrowed to purchase a 5-year bond with a par value of 1000 with coupons at a nominal rate of $10^o/_o$ payable semiannually, with the first coupon paid at the end of 6-month period from now. The bond is redeemed at par and Mike's yield rate for the bond is $9^o/_o$ convertible semiannually.
As Mike receives each coupon payment, he immediately puts the money into an account earning nominal rate of $5.8^o/_o$ convertible semiannually.
At the end of five years (from now), immediately after Mike receives the final coupon payment, Mike deposits the accumulated value of the coupons and the redemption amount of the bond into a savings account earning an annual interest rate of $6.5^o/_o$. At the end of each year from year 6 through 10, Mike deposits an additional amount of 50 into this savings account.
Find Mike's accumulated value at the end of ten years after the loan is repaid.

Solution. The loan amount, say L, is the purchase price of the bond, that is,

$$L = Fra_{\overline{10}|j} + 1000v_j^{10} = 1000(0.05)a_{\overline{10}|0.045} + 1000v_{0.045}^{10} = 1039.56.$$

At the end of five years (from now), the accumulated value of the coupons (using the nominal rate of $5.8^o/_o$ convertible semiannually) plus the redemption amount is

$$50s_{\overline{10}|\frac{0.058}{2}} + 1000 = 1570.56.$$

After the first five years, using now the savings account which earns an annual interest rate of $6.5^o/_o$, the above amount and the additional deposits of 50 accumulate to

$$1570.56(1 + 0.065)^5 + 50s_{\overline{5}|0.065} = 2436.48$$

at the end of ten years.
Mike's accumulated value at the end of ten years after the loan is repaid is

the accumulated value at the end of ten years in the savings account less the original loan amount L with all interest due at rate of $8^o/_o$:

$$2436.48 - L(1+0.08)^{10} = 2436.48 - 1039.56(1+0.08)^{10} = 192.15$$

3. A 1000 bond is paying coupons at nominal interest rate of $7^o/_o$ payable semiannually. The bond is redeemed at par and matures on November 10, 2029. The nominal yield rate convertible semiannually is quoted as $10.384^o/_o$.
(a) Find the purchase price and the market price of the bond on July 24, 2013 to the nearest 0.001.
(b) What is the purchase price of the bond on July 24, 2013 if it is assumed simple interest between coupon dates?
Note that: May 10 $\rightarrow$ 130 (day of the year); July 24 $\rightarrow$ 205 (day of the year); November 10 $\rightarrow$ 314 (day of the year).

Solution. (a) The price of the bond on May 10, 2013 (after the last coupon is paid before July 24, 2013) is

$$P_0 = 1000 \cdot \left(\frac{0.07}{2}\right) a_{\overline{33}|\frac{0.10384}{2}} + 1000 \cdot \left(1 + \frac{0.10384}{2}\right)^{-33} = 735.439.$$

The value ("price-plus-accrued") of the bond on July 24, 2013 is

$$P_0 \cdot \left(1 + \frac{0.10384}{2}\right)^{\frac{205-130}{314-130}} = P_0 \cdot \left(1 + \frac{0.10384}{2}\right)^{\frac{75}{184}} = 750.769.$$

The market price on July 24, 2013 is the price-plus-accrued less the fractional coupon:

$$750.77 - 1000\left(\frac{0.07}{2}\right) \cdot \frac{75}{184} = 736.504.$$

(b) The purchase price of the bond on July 24, 2013, assuming simple interest rate between coupon dates, is:

$$P_0 \cdot \left(1 + \frac{0.10384}{2} \cdot \frac{75}{184}\right) = 751.003.$$

4. Among a company's assets and accounting records, an actuary finds a 15-year par value bond that was purchased at a premium. From the records, the actuary has determined the following:
(i) The bond pays semi-annual interest.
(ii) The amount for amortization of the premium in the 3rd coupon payment was 981.2.
(iii) The amount for amortization of the premium in the 7th coupon payment was 1085.06.

What is the value of the premium?

Solution. The premium amount is $P - F$, where

$$P - F = F(r-j)a_{\overline{30}|j} = F(r-j) \cdot \frac{s_{\overline{30}|j}}{(1+j)^{30}} = F(r-j)v_j^{30}s_{\overline{30}|j} = PR_1 \cdot s_{\overline{30}|j}, \quad (1)$$

since $PR_1 = F(r-j)v_j^{30}$. Now,

$$PR_7 = PR_3(1+j)^4 \quad \Rightarrow \quad (1+j)^4 = \frac{1083.06}{981.2} = 1.1038 \quad \Rightarrow \quad j = 0.025,$$

and

$$PR_3 = PR_1(1+j)^2 \quad \Rightarrow \quad PR_1 = \frac{981.2}{(1.025)^2} = 933.92.$$

So, using (1), $P - F = 933.92 \cdot s_{\overline{30}|0.025} = 41002.$

5. John purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of $8^o/_o$ convertible semiannually at a price of 1811.49. The bond can be called at par value X on any coupon date starting at the end of 16.5 years after the coupon is paid. Find X such that the price yields a nominal annual rate of interest convertible semiannually of
(a) at least $6^o/_o$.
(b) at least $8^o/_o$.
(c) at least $10^o/_o$.

Solution.
(a) Since $r = 0.04 > 0.03 = j$, the bond is sold at a premium and hence, the minimum yield rate occurs at the earliest redemption date, that is, at $n = 33$:

$$P = 1811.49 = X(0.04)a_{\overline{33}|0.03} + X \cdot \frac{1}{(1.03)^{33}} \quad \Rightarrow \quad X = 1500.$$

(b) Since $r = 0.04 = j$, we have that $P = F$, and hence, $X = 1811.49$.
(c) Since $r = 0.04 < 0.05 = j$, the bond is sold at a discount and hence, the minimum yield rate occurs at the latest redemption date, that is, at $n = 40$:

$$P = 1811.49 = X(0.04)a_{\overline{40}|0.05} + X \cdot \frac{1}{(1.05)^{40}} \quad \Rightarrow X = 2186.71.$$