

MATH 209/4 all sections except EC: - Fundamental Mathematics II
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Midterm - Sunday, March 2, 2014 (1h30min) 14:00-15:30
Only approved calculators are permitted.

MARKS

- [7] 1. (a) Find $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x^2 - x - 6}$.
- [8] (b) Give an example of a function $g(x)$ and a function $h(x)$ with the following properties:
- (i) $\lim_{x \rightarrow 7} g(x) = 0$ (ii) $\lim_{x \rightarrow 7} h(x) = 0$ (iii) $\lim_{x \rightarrow 7} \frac{[g(x)]^2}{h(x)} = 2$
- [7] 2. Let $k(x) = x^3 - 5$. Work out the following in detail: $\lim_{s \rightarrow 0} \frac{k(x+s) - k(x)}{s}$
- [12] 3. (a) If $f(x) = 4x^{\frac{3}{4}} - x^{-6}$, find $f'(1)$. You don't have to simplify the answer.
- (b) If $g(x) = [4x^3 + 7] [3 - \ln(x^2)]$, find $g'(2)$. You don't have to simplify the answer.
- (c) Find $h'(x)$ if $h(x) = \frac{x^2 - \frac{1}{x}}{e^x - x^2}$. You don't have to simplify the answer.
- (d) Find the value of dy if $y = \ln(x + 1)$, $x = 3$, and the change in x is 0.2.
- [6] 4. A stock grew from \$35 to \$120,000 in 43 years. Assuming continuous compounding, what is the associated annual rate of growth?
- [10] 5. The total profit (in dollars) from the sale of x lawn mowers is $P(x) = 30x - 0.03x^2 - 750$, $0 \leq x \leq 1,000$.
- (a) Find the average profit per mower if 50 mowers are produced.
- (b) Find the marginal average profit at a production level of 50 mowers, and interpret the results.
- (c) Use the results from parts (a) and (b) to estimate the average profit per mower if 51 mowers are produced.
- [10] 6. Find x' for the function $x = x(t)$ defined implicitly by $1 + x \ln t = te^x$ and evaluate x' at $(t, x) = (1, 0)$.
- [10] 7. A person who is new on an assembly line performs an operation in T minutes after x performances of the operation, as given by

$$T = 6 \left(1 + \frac{1}{\sqrt{x}} \right)$$

If $dx/dt = 6$ operations per hour, where t is time in hours, find dT/dt after 36 performances of the operation.