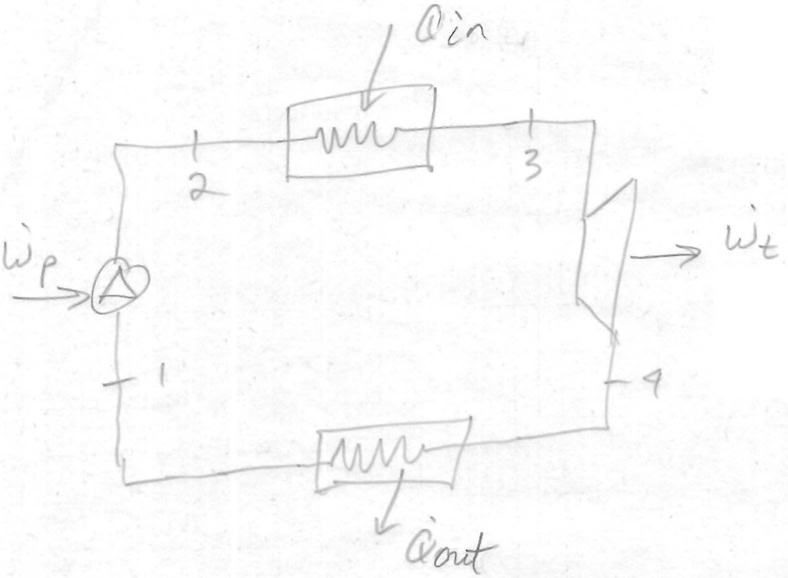
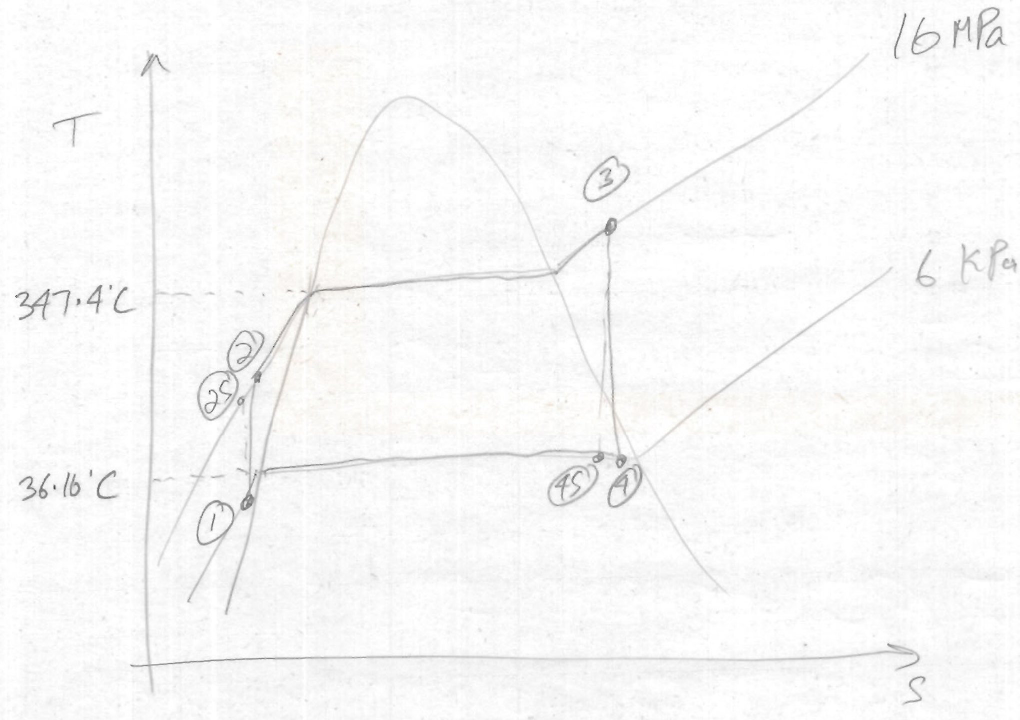
Rankine Cycle Question



a)



b)

T_{sat} (16 MPa) = 347.4°C
T_{sat} (6 KPa) = 36.16°C
} Table A-3

Given: * State (1) 6 KPa and 30°C $\Rightarrow T_1 < T_{sat}(6\,KPa)$ so subcooled liquid.

* $\dot{Q}_{in} = 1.5\,GW$

* $\dot{m} = 440\ \frac{kg}{s}$

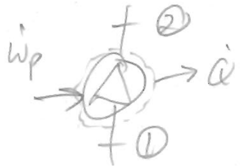
* $P_2 = P_3 = 16\,MPa$

* $P_1 = P_4 = 6\,KPa$

* $\eta_t = 0.91$

* $\eta_p = 0.65$

pump



$$\frac{dE_{cv}}{dt} = \dot{W}_p - \dot{Q} + \dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$

(SS; assume adiabatic; ignore ΔKE & ΔPE)

$$\frac{\dot{W}_p}{\dot{m}} = h_2 - h_1$$

$$\frac{\dot{W}_{ps}}{\dot{m}} = h_{2s} - h_1$$

$$T\,ds = dh - v\,dP$$

$$\int_1^{2s} dh = \int_1^{2s} v\,dP$$

$$h_{2s} - h_1 \approx v_1 (P_2 - P_1)$$ since can treat subcooled liquid as incompressible.

$$v_1 \approx v_f(T_1) = 1.0043 \cdot 10^{-3} \frac{m^3}{kg}$$ (Table A-2).

$$h_1 \approx h_f(T_1) = 125.79 \frac{kJ}{kg}$$ (Table A-2).

$$\frac{\dot{W}_{ps}}{\dot{m}} = h_{2s} - h_1 \approx v_1 (P_2 - P_1)$$

$$= \left(1.0043 \cdot 10^{-3} \frac{m^3}{kg}\right)(16\,000 - 6)\ kPa \left(\frac{1\ kN/m^2}{kPa}\right)\left(\frac{1\ kJ}{kN \cdot m}\right)$$

$$= 16.06 \frac{kJ}{kg}$$

$$\eta_p = \frac{\dot{W}_{ps}/\dot{m}}{\dot{W}_p/\dot{m}} \Rightarrow \frac{\dot{W}_p}{\dot{m}} = \frac{16.06 \frac{kJ}{kg}}{0.65}$$

$$\frac{\dot{W}_p}{\dot{m}} = 24.71 \frac{kJ}{kg}$$

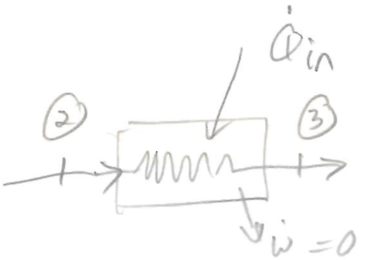
$$\frac{\dot{W}_p}{\dot{m}} = h_2 - h_1 \Rightarrow h_2 = 125.79 + 24.71 \left(\frac{kJ}{kg}\right)$$

d) $\boxed{h_2 = 150.5 \frac{kJ}{kg}}$

$$\dot{W}_p = \left(440 \frac{kg}{s}\right)\left(24.71 \frac{kJ}{kg}\right)\left(\frac{1\ kW}{kJ/s}\right)\left(\frac{1\ MW}{1000\ kW}\right)$$

c) $\boxed{\dot{W}_p = 10.9\ MW}$

boiler



$$\frac{dE_{cv}}{dt} = \dot{Q}_{in} - \dot{W} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_2 - \dot{m}\left(h + \frac{V^2}{2} + gz\right)_3$$

(SS; no work; ignore ΔKE & ΔPE effects)

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_2)$$

$$h_3 = h_2 + \frac{\dot{Q}_{in}}{\dot{m}}$$

$$= 150.5\ \frac{KJ}{Kg} + \frac{(1.5\ GW)\left(\frac{10^6\ KW}{GW}\right)\left(\frac{1\ KJ/s}{KW}\right)}{440\ \frac{Kg}{s}}$$

$$= 3559.6\ \frac{KJ}{Kg}$$ Superheated steam by Table A-4.

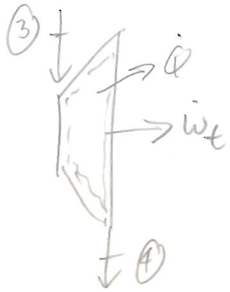
Table A-4 @ 16 MPa: 560°C 3465.4 $\frac{KJ}{Kg}$
600°C 3573.5 $\frac{KJ}{Kg}$ } Table A-4

$$\frac{T_3 - 560}{600 - 560} = \frac{3559.6 - 3465.4}{3573.5 - 3465.4}$$

$$\Rightarrow T_3 = 595\,°C$$

e)

turbine



SS — assume adiabatic — neglect ΔKE & ΔPE effects

$$\frac{dE_{cv}}{dt} = -\dot{W}_t - \dot{Q} + \dot{m}\left(h_3 + \frac{V_3^2}{2} + gz_3\right) - \dot{m}\left(h_4 + \frac{V_4^2}{2} + gz_4\right)$$

$$\frac{\dot{W}_t}{\dot{m}} = (h_3 - h_4)$$

$$\frac{\dot{W}_{ts}}{\dot{m}} = h_3 - h_{4s}$$

(3) 16 MPa, 595°C

$h_3 = 3559.6 \frac{KJ}{Kg}$ (from boiler)

$S_3 = 6.6236 \frac{KJ}{KgK}$ (interpolated from A-4)

(4s) 6 KPa

$S_{4s} = S_3 = 6.6236 \frac{KJ}{KgK}$

$S_f = 0.5210 \frac{KJ}{KgK}$ } Table A-3

$S_g = 8.3304 \frac{KJ}{KgK}$

$$X_{4s} = \frac{S_{4s} - S_f}{S_g - S_f} = 0.781$$

$h_f = 151.53 \frac{KJ}{Kg}$ } Table A-3

$h_{fg} = 2415.9 \frac{KJ}{Kg}$

$$\therefore h_{4s} = 151.53 \frac{kJ}{kg} + (0.781)\left(2415.9 \frac{kJ}{kg}\right)$$

$$= 2039.4 \frac{kJ}{kg}$$

$$\eta_t = \frac{\dot{W}_t/\dot{m}}{\dot{W}_{ts}/\dot{m}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\Rightarrow h_4 = 3559.6 \frac{kJ}{kg} - (0.91)(3559.6 - 2039.4) \frac{kJ}{kg}$$

$$= 2176.2 \frac{kJ}{kg}$$

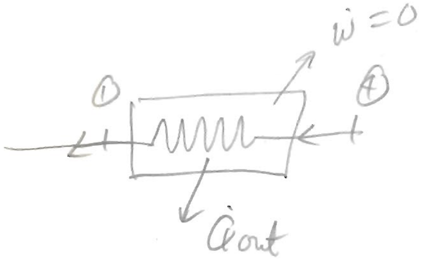
$$\dot{W}_t = \dot{m}(h_3 - h_4) = \left(440 \frac{kg}{s}\right)(3559.6 - 2176.2) \frac{kJ}{kg}$$

g) $\boxed{\dot{W}_t = 608.7 \text{ MW}}$

$$X_4 = \frac{h_4 - h_f}{h_{fg}} = \frac{2176.2 - 151.53}{2415.9}$$

f) $\boxed{X_4 = 0.838}$

condenser



SS; no work; neglect ΔKE & ΔPE effects

$$\frac{dE_{cv}}{dt} = -\dot{Q}_{out} - \dot{W} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_4 - \dot{m}\left(h + \frac{V^2}{2} + gz\right)_1$$

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1$$

$$\dot{Q}_{out} = \left(440\ \frac{kg}{s}\right)\left(2176.2 - 125.79\right)\ \frac{kJ}{kg}$$

h)

$$\boxed{\dot{Q}_{out} = 902.2\ \text{MW}}$$

check EB on entire cycle

$$\dot{W}_p + \dot{Q}_{in} \stackrel{?}{=} \dot{W}_t + \dot{Q}_{out}$$

$$10.9\ \text{MW} + 1500\ \text{MW} \stackrel{?}{=} 608.7\ \text{MW} + 902.2\ \text{MW}\ \checkmark$$

bwr

$$bwr = \frac{\dot{W}_p}{\dot{W}_t} = \frac{10.9\ MW}{608.7\ MW}$$

i) $\boxed{bwr = 0.018}$

cycle efficiency

$$\eta_{cycle} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_t - \dot{W}_p}{\dot{Q}_{in}} = \frac{608.7 - 10.9}{1500}\ \frac{(MW)}{(MW)}$$

j) $\boxed{\eta_{cycle} = 0.399}$

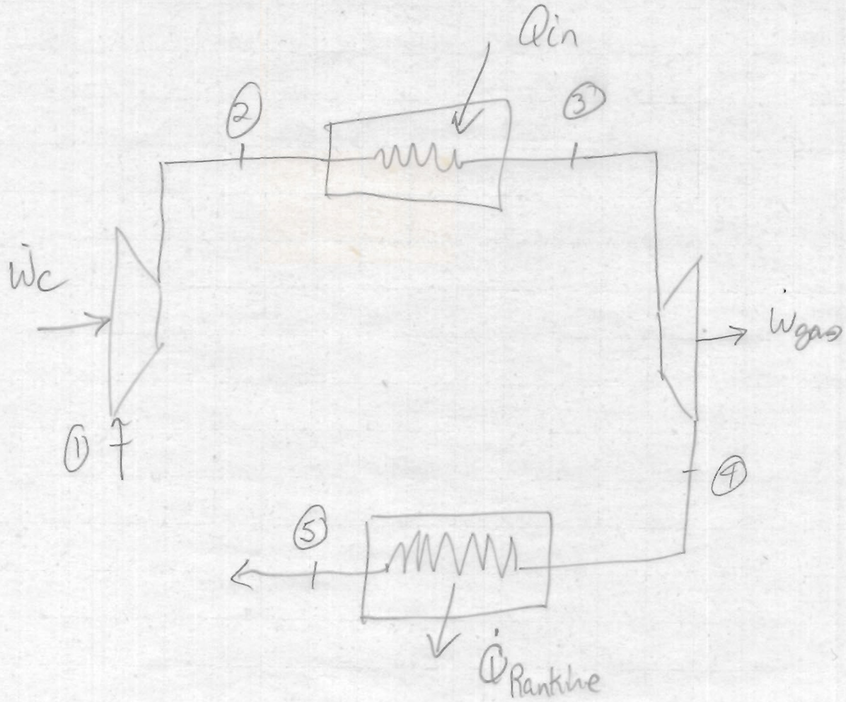
k) $X_4 = 0.838$. This is far too long. This much liquid will cause material and structural degradation to the turbine.

The quality at state 4 could be raised by further superheating state 3 (i.e raise its temperature).

Another method to raise the quality at state 4 would be to have two-stage expansion in the turbine with reheat between the stages.

Combined cycle question

Brayton cycle portion:



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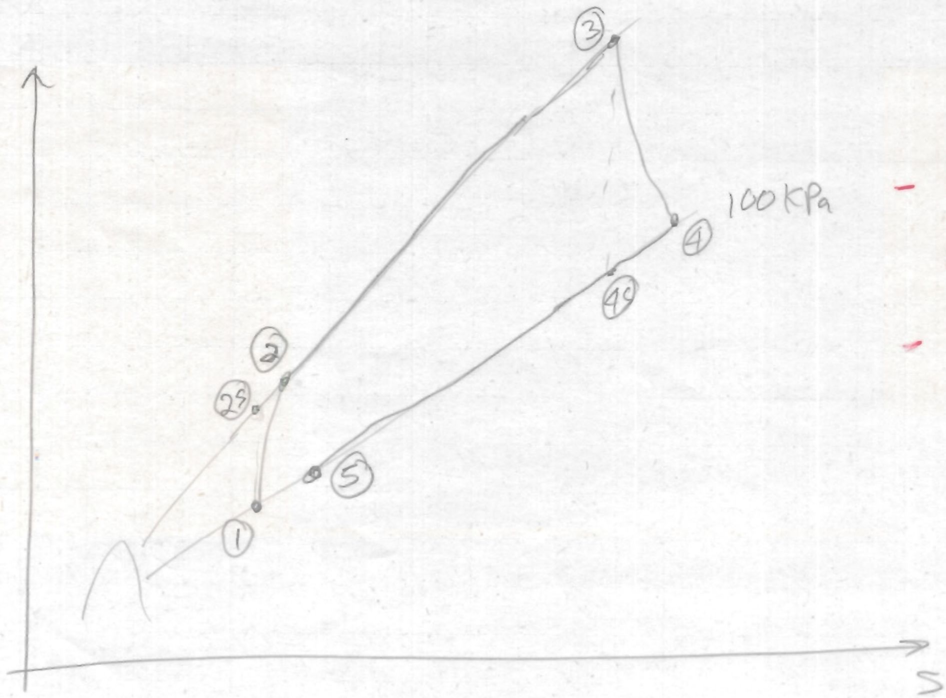
a)



- 1 mark for each ~~state~~ point
- 1 mark for each isobar

Given:

$T_1 = 300\,K$

$P_1 = 100\,kPa = P_4$

$P_2 = P_3 = 1.1\,MPa$

$\eta_c = 0.9$

$\dot{m} = 100\,kg/s$

$\dot{Q}_{in} = 75\,MW$

$\eta_t = 0.92$

$T_5 = 420\,K$

<u>compressor</u>

2 marks for 1st law + assumptions

(SS; assume adiabatic; ignore ΔKE & ΔPE effects)

$$\frac{dE_{cv}}{dt} = \dot{W}_c - \dot{Q} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_1 - \dot{m}\left(h + \frac{V^2}{2} + gz\right)_2$$

$$\dot{W}_c = \dot{m}\left(h_2 - h_1\right)$$

$$\dot{W}_{cs} = \dot{m}\left(h_{2s} - h_1\right)$$

1 mark isentropic compressor: $\frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1} = \frac{1.1\ \text{MPa}}{100\ \text{kPa}} \cdot \frac{1000\ \text{kPa}}{\text{MPa}}$

$$P_{r2} = (11)(P_{r1})$$

1 mark for lookup

$T_1 = 300\text{K}$: $P_{r1} = 1.3860$, $h_1 = 300.19\ \frac{\text{kJ}}{\text{kg}}$ } Table A-22

$$\therefore P_{r2} = (11)(1.3860) = 15.246$$

$$\frac{T_{2s} - 580}{590 - 580} = \frac{15.246 - 14.38}{15.31 - 14.38}$$

$\Rightarrow T_{2s} = 589.3\text{K}$ (interpolated from Table A-22)

$$\eta_c = \frac{\dot{W}_{cs}/\dot{m}}{\dot{W}_c/\dot{m}} = \frac{h_{2s}-h_1}{h_2-h_1}$$

1 mark for η_c & $\dot{W}_c$ answer

$$\Rightarrow h_2 = h_1 + \frac{1}{\eta_c}(h_{2s}-h_1)$$

$$= 300.19 + \frac{(595.8-300.19)}{0.9}$$

$$= 628.6 \ \frac{kJ}{kg}$$

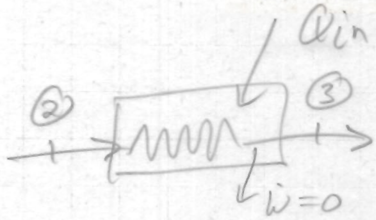
2 marks for h_2 calc & T_2 interpolation

(2) c) $\boxed{T_2 = 620.5^\circ C}$ (interpolated from Table A-22)

$$\dot{W}_c = \dot{m}(h_2-h_1) = \left(100 \ \frac{kg}{s}\right)(628.6-300.19) \ \frac{kJ}{kg} \cdot \frac{1 \ kW}{kJ/s} \cdot \frac{1 \ MW}{1000 \ kW}$$

(6) b) $\boxed{\dot{W}_c = 32.8 \ MW}$

Combustor



1 mark for 1st law + assumptions

SS, no work, neglect ΔKE & ΔPE effect

$$\frac{dE_{cv}}{dt} = \dot{Q}_{in} - \dot{W} + \dot{m}\left(h+\frac{V^2}{2}+gz\right)_2 - \dot{m}\left(h+\frac{V^2}{2}+gz\right)_3$$

$$\dot{Q}_{in} = \dot{m}(h_3-h_2)$$

$$h_3 = \frac{\dot{Q}_{in}}{\dot{m}} + h_2$$

$$= \frac{75 \ MW}{100 \ kg/s} \cdot \frac{1000 \ kW}{MW}\frac{kJ/s}{} + 628.6 \ \frac{kJ}{kg}$$

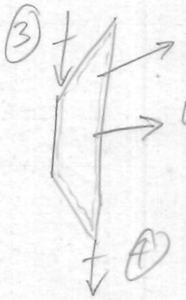
$$h_3 = 1378.6 \ \frac{kJ}{kg}$$ 1 mark

1 mark

(3) d) $\Rightarrow \boxed{T_3 = 1285.4 K}$ (interpolated from Table A-22)

turbine

2 marks for 1st law and assumptions



$$\frac{dE_{cv}}{dt} = -\dot{W}_{gas} - \dot{Q} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_3 - \dot{m}\left(h + \frac{V^2}{2} + gz\right)_4$$

(ss; assume adiabatic; ignore ΔKE & ΔPE effects)

$$\dot{W}_{gas} = \dot{m}\,(h_3 - h_4)$$

$$\dot{W}_{gas,s} = \dot{m}\,(h_3 - h_{4s})$$

isentropic turbine: $\frac{P_{r4}}{P_{r3}} = \frac{P_4}{P_3}$ 1 mark

$$P_{r4} = \frac{100\ kPa}{1100\ kPa} \cdot (315.9)$$ 1 mark

(315.9 interpolated from Table A-22)

$$P_{r4} = 28.72$$

$\Rightarrow T_{4s} = 699.5$ (interpolated from Table A-22)

$\Rightarrow h_{4s} = 712.7\ \frac{kJ}{kg}$ 1 mark

$$\eta_t = \frac{\dot{W}_{gas}/\dot{m}}{\dot{W}_{gas,s}/\dot{m}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\Rightarrow h_4 = h_3 - \eta_t\,(h_3 - h_{4s})$$ 1 mark for η_t & $\dot{W}_{gas}$ answer

$$= 1378.6 - (0.92)(1378.6 - 712.7)\ \frac{kJ}{kg}$$

$$= 766.0\ \frac{kJ}{}$$

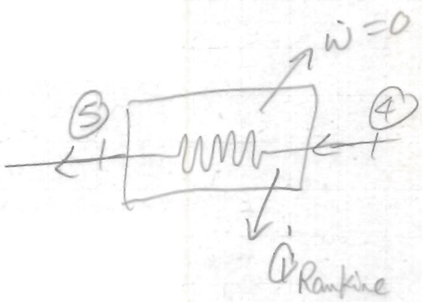
2 marks for h_4 calc and T_4 interpolation

(2) f) $\Rightarrow$ $\boxed{T_4 = 748.8 \text{ K}}$

$$\dot{W}_{gas} = \dot{m}(h_3 - h_4) = \left(100 \tfrac{kg}{s}\right)(1378.6 - 766.0)\tfrac{kJ}{kg} \cdot \tfrac{1 kW}{1 kJ/s}$$

(6) e) $\boxed{\dot{W}_{gas} = 61.3 \text{ MW}}$

heat exchanger



2 marks for 1st law + assumptions

SS, no work, ignore ΔKE & ΔPE

$$\frac{dE_{cv}}{dt} = -\dot{Q}_{Rankine} - \dot{W} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_4 - \dot{m}\left(h + \frac{V^2}{2} + gz\right)_5$$

$$\dot{Q}_{Rankine} = \dot{m}(h_4 - h_5)$$

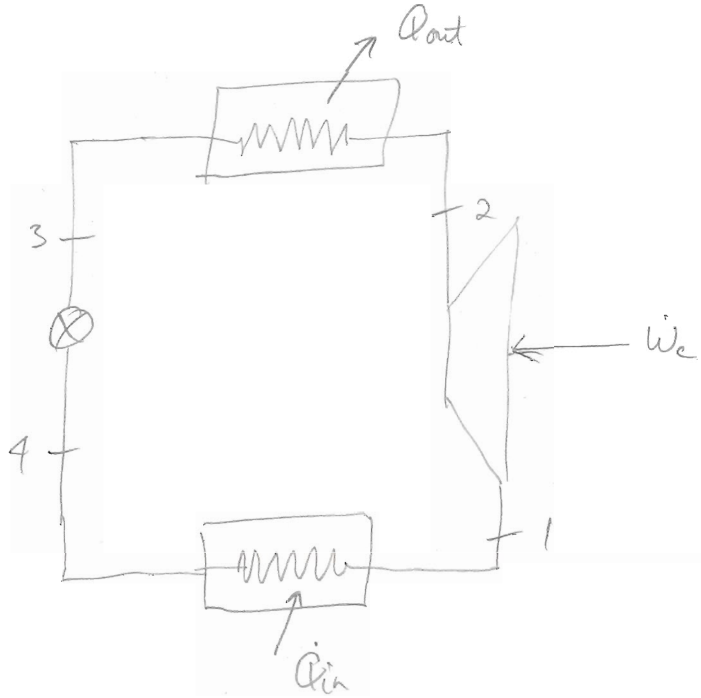
$T_5 = 420 \text{ K}$ (given) $\Rightarrow h_5 = 421.26 \ \frac{kJ}{kg}$ — 1 mark for h_5 calc

h_4 solved when treating turbine

$$\dot{Q}_{Rankine} = \left(100 \tfrac{kg}{s}\right)(766.0 - 421.26)\tfrac{kJ}{kg}$$ — 1 mark for $\dot{Q}_{Rankine}$ calc

(4) g) $\boxed{\dot{Q}_{Rankine} = 34.5 \text{ MW}}$

Vapour Compression Refrigeration Question



a)

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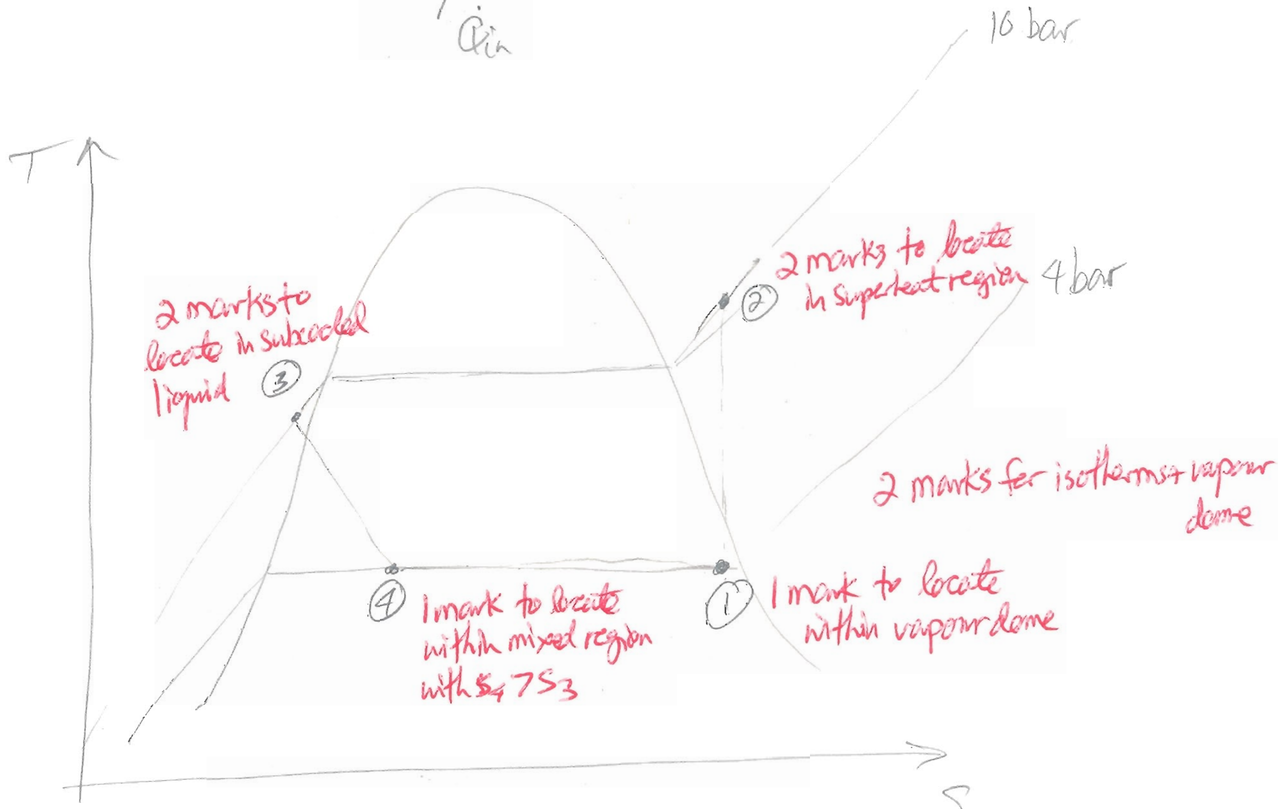


Given: $P_1 = P_4 = 4\ bar$

$P_2 = P_3 = 10\ bar$

$v_1 = 0.0504\ \frac{m^3}{kg}$

$T_3 = 38^\circ C$

R134a

(1) $P_1 = 4\ bar$, $v_1 = 0.0504\ \frac{m^3}{kg}$ } within mixed region by Table A-11.

$x_1 = 0.990$

$s_1 = 0.9078\ \frac{kJ}{kgK}$ by Table A-11

$h_1 = 250.42\ \frac{kJ}{kg}$

(2) $P_2 = 10\ bar$

$s_2 \geq s_1 = 0.9078\ \frac{kJ}{kgK}$

$s_g = 0.9043\ \frac{kJ}{kgK}$ (Table A-11)

$\therefore s_2 \geq s_g \Rightarrow$ superheated

(3) $P_3 = 10\ bar$

$T_3 = 38^\circ C$

$T_{sat} = 39.39^\circ C$

$\therefore T_3 < T_{sat} \Rightarrow$ subcooled liquid

$h_3 \approx h_f(T_3) = 103.21\ \frac{kJ}{kg}$ (Table A-10)

(4) $P_4 = 4$ bar

$$h_f = 62.0 \ \frac{kJ}{kg}$$
$$h_g = 252.32 \ \frac{kJ}{kg}$$
} Table A-11

1st law: $\frac{dE_{cv}}{dt} = -\dot{Q} - \dot{W} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_3 - \dot{m}\left(h + \frac{V^2}{2} + gz\right)_4$

(ss; assume adiabatic; no work; neglect ΔKE & ΔPE effects)

$\therefore h_4 = h_3 = 103.21 \ \frac{kJ}{kg}$

$$X_4 = \frac{103.21 - 62.0}{252.32 - 62.0} = 0.216$$

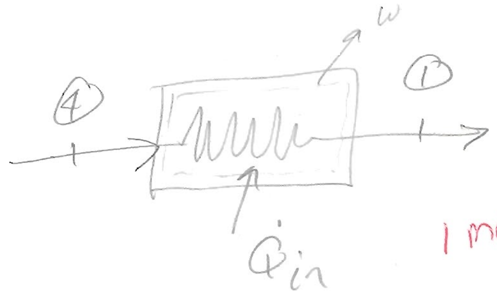
(3) b) $T_1 = T_4 = T_{sat}(4 \text{ bar}) = 8.93°C$ 2 marks

To effect a heat transfer from the wine cabinet (the cold reservoir) to the working fluid at the evaporator, there must be a finite temperature difference. Therefore, the wine cabinet must be warmer that 8.93°C.

1 mark to state that wine cabinet must be warmer than Tsat

(4)

c)



1 mark for 1st law + assumptions

1st law: $\frac{dE_{CV}}{dt} = \dot{Q}_{in} - \dot{W} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_4 - \dot{m}\left(h + \frac{V^2}{2} + gz\right)_1$

(SS; no work; neglect ΔKE & ΔPE effects)

$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$

1 mark for h_1 1 mark for h_4

$= 250.42 - 103.21 \left(\frac{kJ}{kg}\right)$

$$\boxed{\frac{\dot{Q}_{in}}{\dot{m}} = 147.21 \frac{kJ}{kg}}$$

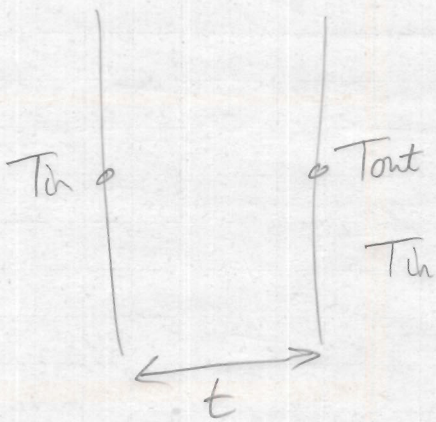
1 mark for $\frac{\dot{Q}_{in}}{\dot{m}}$

Heat Transfer Question

a) Radiation

b) Conduction

c) Fourier's law: $\frac{q_x}{A} = -k\frac{\partial T}{\partial x} = -k\frac{dT}{dx}$ since 1D



$T_{in} - T_{out} = 200°C$

$\frac{q_x}{A} = -k\frac{\Delta T}{\Delta x}$ since $\frac{dT}{dx} = \text{constant}$

$\Delta x = \frac{-k\Delta T}{q_x/A} = \frac{(0.04\frac{W}{mK})(-200°C)}{400\,W/m^2}$

$\boxed{\Delta x = 0.02\,m}$