SOLUTION

$$L.S. = \frac{1+\cot x}{1+\tan x} = \frac{1+\frac{\cos x}{\sin x}\ 1}{1+\frac{\sin x}{\cos x}\ 1} = \left.\frac{\frac{\sin x+\cos x}{\sin x}}{\frac{\cos x+\sin x}{\cos x}}\right\} 1$$

$$= \underbrace{\frac{\sin x+\cos x}{\sin x}\cdot\frac{\cos x}{\cos x+\sin x}}_{1} = \frac{\cos x}{\sin x}\ 1 = \overset{1}{\cot x} = R.S.\ 1$$

proper form

SOLUTION

(a) $\lim_{x\to 4} \frac{x-4}{x^2-16} = \lim_{x\to 4} \frac{x-4}{(x-4)(x+4)} = \lim_{x\to 4} \frac{1}{x+4} = \frac{1}{8}$ 2-work 1 1

(b) $\lim_{x\to 2} \frac{x^2-4}{x+2} = \frac{(2)^2-4}{(2)+2} = \frac{0}{4} = 0$ 2-work 2-answer

(c) $\lim_{x\to 0} \frac{\sin 2x}{\sin 4x} = \frac{2}{2}\frac{4}{4}\lim_{x\to 0}\frac{\sin 2x}{\sin 4x}\cdot\frac{x}{x} = \frac{2}{4}\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{4x}{\sin 4x}$

3-work $= \frac{1}{2}\cdot\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\lim_{x\to 0}\frac{4x}{\sin 4x} = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ 1

(d) $\lim_{x\to 0}\frac{1-\cos 3x+\sin 3x}{x} = 3\lim_{x\to 0}\frac{1-\cos 3x+\sin 3x}{3x}$ 1

$= 3\left(\underbrace{\lim_{x\to 0}\frac{1-\cos 3x}{3x}}_{1} + \underbrace{\lim_{x\to 0}\frac{\sin 3x}{3x}}_{1}\right) = 3(0+1) = 3$ 1

(e) $\lim_{x\to\infty}\frac{x^2}{2x^2+\sqrt{3x^3+4x^4}} = \lim_{x\to\infty}\frac{x^2/x^2}{2x^2/x^2+\sqrt{3x^3/x^4+4x^4/x^4}}$ 2-divide by $x^2 = \sqrt{x^4}$

$= \lim_{x\to\infty}\frac{1}{2+\sqrt{\frac{3}{x}+4}} = \frac{1}{2+\sqrt{0+4}} = \frac{1}{4}$ 1-work 1

(f) $\lim_{x\to\infty} x-\sqrt{x^2-4} = \lim_{x\to\infty}\frac{x-\sqrt{x^2-4}}{1}\cdot\frac{x+\sqrt{x^2-4}}{x+\sqrt{x^2-4}} = \lim_{x\to\infty}\frac{x^2-(x^2-4)}{x+\sqrt{x^2-4}}$ 2

$= \lim_{x\to\infty}\frac{4}{x+\sqrt{x^2-4}} = 0$ 1 1

(g) $\lim_{x\to-\infty}\frac{x^3}{x+2} = \lim_{x\to-\infty}\frac{x^3/x}{x/x+2/x} = \lim_{x\to-\infty}\frac{x^2}{1+2/x} = +\infty$ 2-divide by x 1-work 1

SOLUTION

(a) $f_1(x) = \begin{cases} 3x-2, & \text{if } x \ge 0 \\ x-2, & \text{if } x < 0 \end{cases}$

continuous! 1

(b) $f_2(x) = \begin{cases} 3x-2, & \text{if } x \ge 0 \\ 2-x, & \text{if } x < 0 \end{cases}$

$\lim_{x \to 0^+} 3x-2 = -2$ 1 $\neq$ 1 $2 = \lim_{x \to 0^-} 2-x$

Since the limit does not exist, $f_2(x)$ is ~~discont~~ discontinuous at $x=0$ 1

(c) $f_3(x) = \begin{cases} 3x-2, & \text{if } x > 0 \\ x-2, & \text{if } x < 0 \end{cases}$

$f_3(0)$ is undefined, 1
so $f_3(x)$ is disc. at $x=0$ 1

(d) $f_4(x) = \begin{cases} 5x-3, & \text{if } x \ge 2 \\ 3x-2, & \text{if } 0 < x < 2 \\ x-2, & \text{if } x \le 0 \end{cases}$

f is continuous at $x=0$ 1

...they must have checked.

At $x=2$: $\lim_{x \to 2^+} 5x-3 = 5(2)-3 = 7$ 1 NOT EQUAL

$\lim_{x \to 2^-} 3x-2 = 3(2)-2 = 4$ 1

So $f_4(x)$ is discontinuous at $x=2$. 1

SOLUTION

(a) $f(x)$ is discontinuous at $\underline{x=-1}$ (2)

(b) $\lim_{x \to -1^+} \frac{|x+1|}{x+1} + 2 = 3$ (2)

$\lim_{x \to -1^-} \frac{|x+1|}{x+1} + 2 = 1$ (2)

* award part marks for some work/effort