29 September, 2015
MATH 1004, Section C
Total marks: 51

**TEST 1**

1. Prove that [7 marks],
$$\frac{1+\cot x}{1+\tan x} = \cot x$$

2. Solve the following limits. If you know *L'Hospital's Rule*, you MAY NOT use it! [4 marks each]

   (a)
   $$\lim_{x\to 4} \frac{x-4}{x^2-16}$$

   (b)
   $$\lim_{x\to 2} \frac{x^2-4}{x+2}$$

   (c)
   $$\lim_{x\to 0} \frac{\sin 2x}{\sin 4x}$$

   (d)
   $$\lim_{x\to 0} \frac{1-\cos 3x+\sin 3x}{x}$$

   (e)
   $$\lim_{x\to \infty} \frac{x^2}{2x^2+\sqrt{3x^3+4x^4}}$$

   (f)
   $$\lim_{x\to \infty} x-\sqrt{x^2-4}$$

   (g)
   $$\lim_{x\to -\infty} \frac{x^3}{x+2}$$

3. For each of the piecewise functions below, determine whether or not they are continuous. If the function is continuous, simply state that it is. If the function is NOT, indicate the point(s) of discontinuity and justify your answer [10 marks].

   (a)

$$f_1(x) = \begin{cases} 3x - 2, \text{ if } x \geq 0 \\ x - 2, \text{ if } x < 0 \end{cases}$$

   (b)

$$f_2(x) = \begin{cases} 3x - 2, \text{ if } x \geq 0 \\ 2 - x, \text{ if } x < 0 \end{cases}$$

   (c)

$$f_3(x) = \begin{cases} 3x - 2, \text{ if } x > 0 \\ x - 2, \text{ if } x < 0 \end{cases}$$

   (d)

$$f_4(x) = \begin{cases} 5x - 3, \text{ if } x \geq 2 \\ 3x - 2, \text{ if } 0 < x < 2 \\ x - 2, \text{ if } x \leq 0 \end{cases}$$

4. Consider the function,

$$f(x) = \frac{|x+1|}{x+1} + 2$$

   (a) This function is discontinuous at some value $x = a$. What is $a$? [2 marks]

   (b) Solve the limits as $x \to a^+$ and $x \to a^-$ [2 marks each]:

$$\lim_{x \to a^+} \frac{|x+1|}{x+1} + 2$$

$$\lim_{x \to a^-} \frac{|x+1|}{x+1} + 2$$