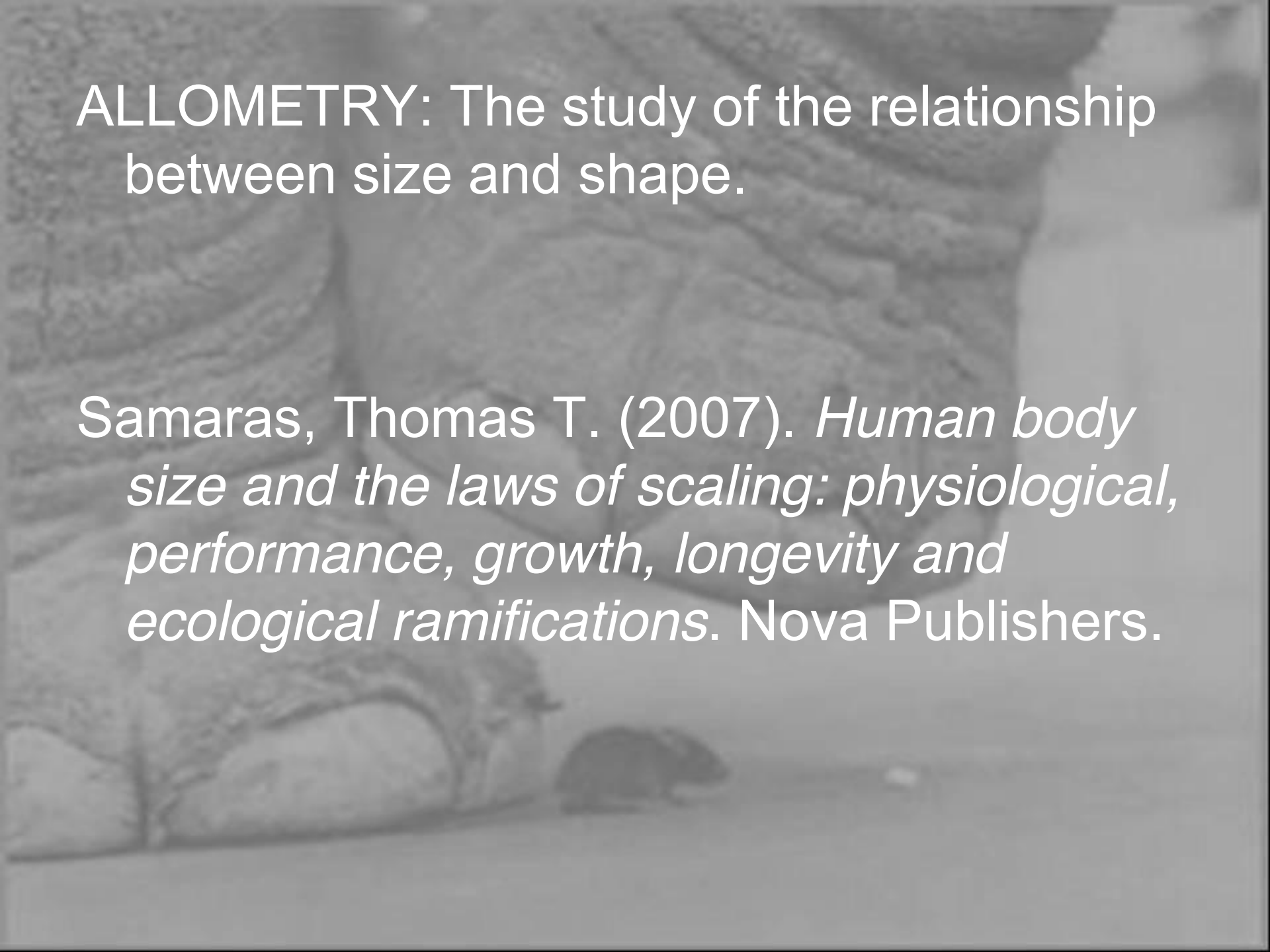


Biology 3UU3: Body Size and Allometry. The Scaling of Organismal Structure and Function

McMaster
University



C. David Rollo



ALLOMETRY: The study of the relationship between size and shape.

Samaras, Thomas T. (2007). *Human body size and the laws of scaling: physiological, performance, growth, longevity and ecological ramifications*. Nova Publishers.



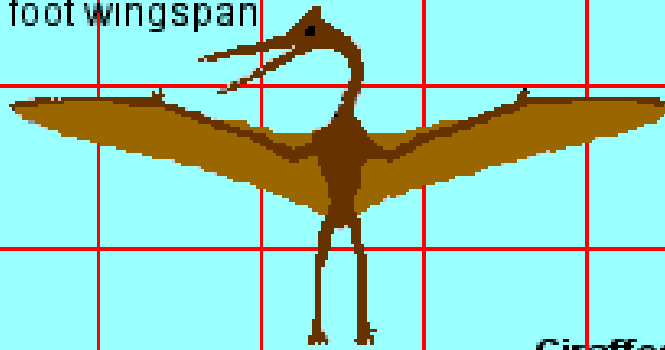
FIGURE 4-13 The adult bumblebee bat weighs only 2 grams. Its size is about the minimum lower limit for an endothermic mammal or bird. (*Photograph courtesy of Bat Conservation International, Merlin D. Tuttle, BCI.*)

Kitti's Hognosed Bat = 1.5 g

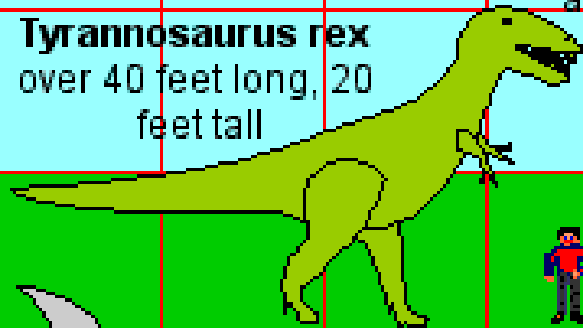


BLUE WHALE
136,000,000 g

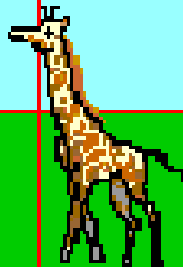
Quetzalcoatlus
40 foot wingspan



Tyrannosaurus rex
over 40 feet long, 20 feet tall

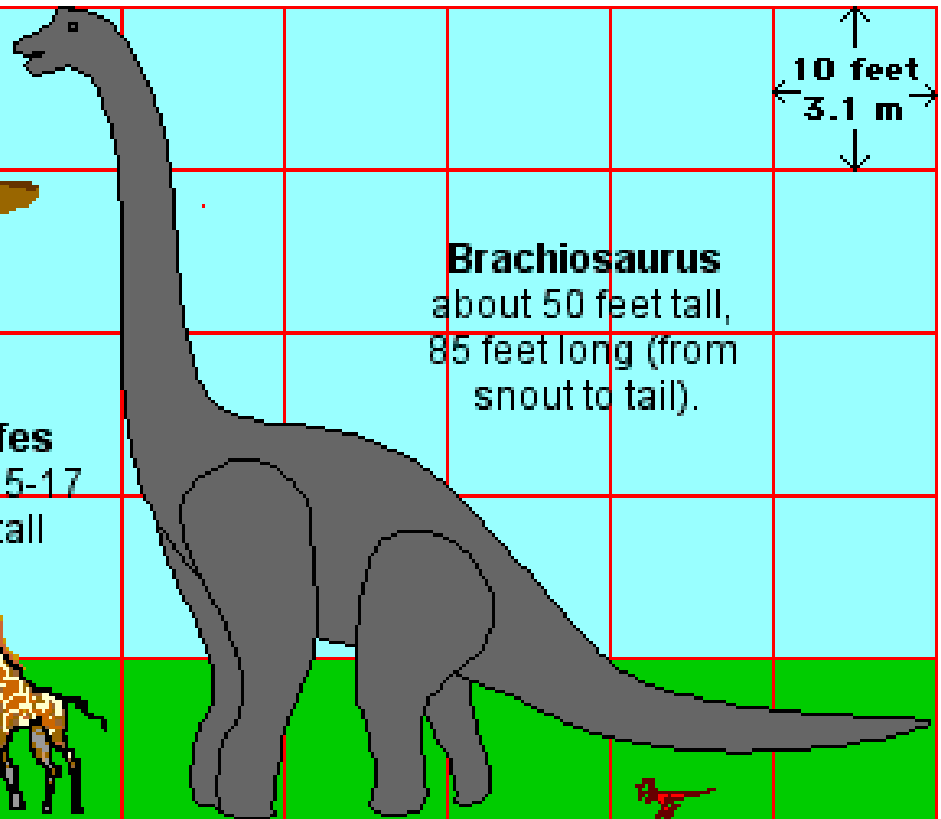


Giraffes
about 15-17 feet tall



Homo sapiens (people)
about 5-6 feet tall.

Brachiosaurus
about 50 feet tall,
85 feet long (from snout to tail).

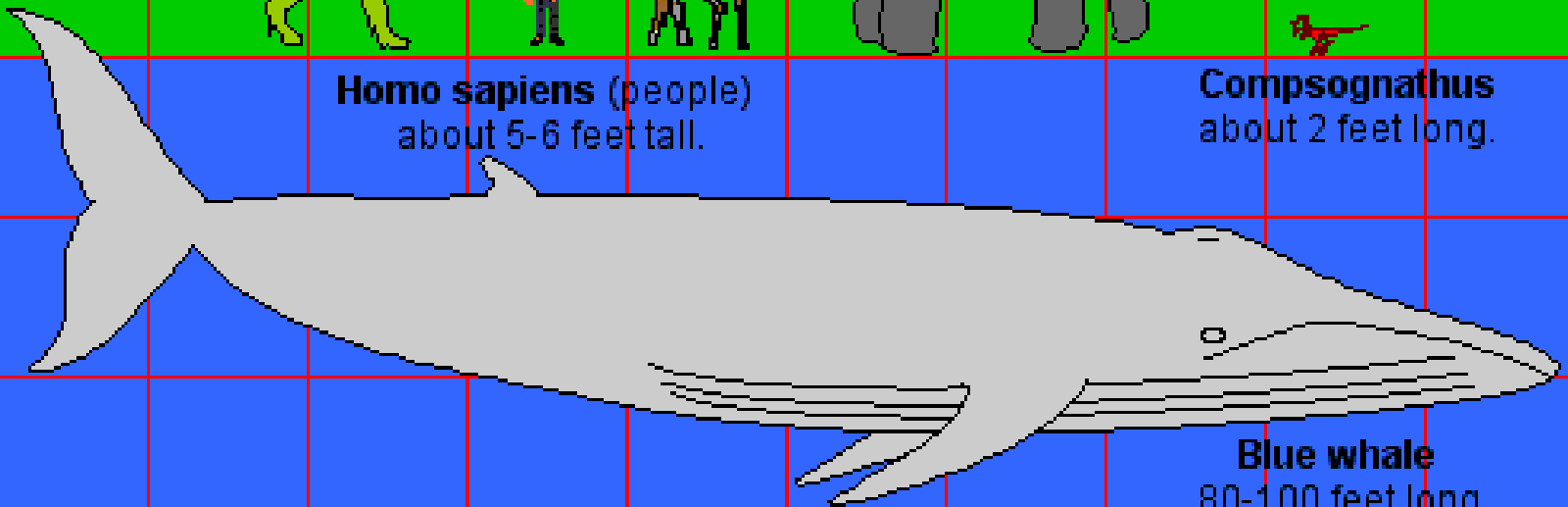


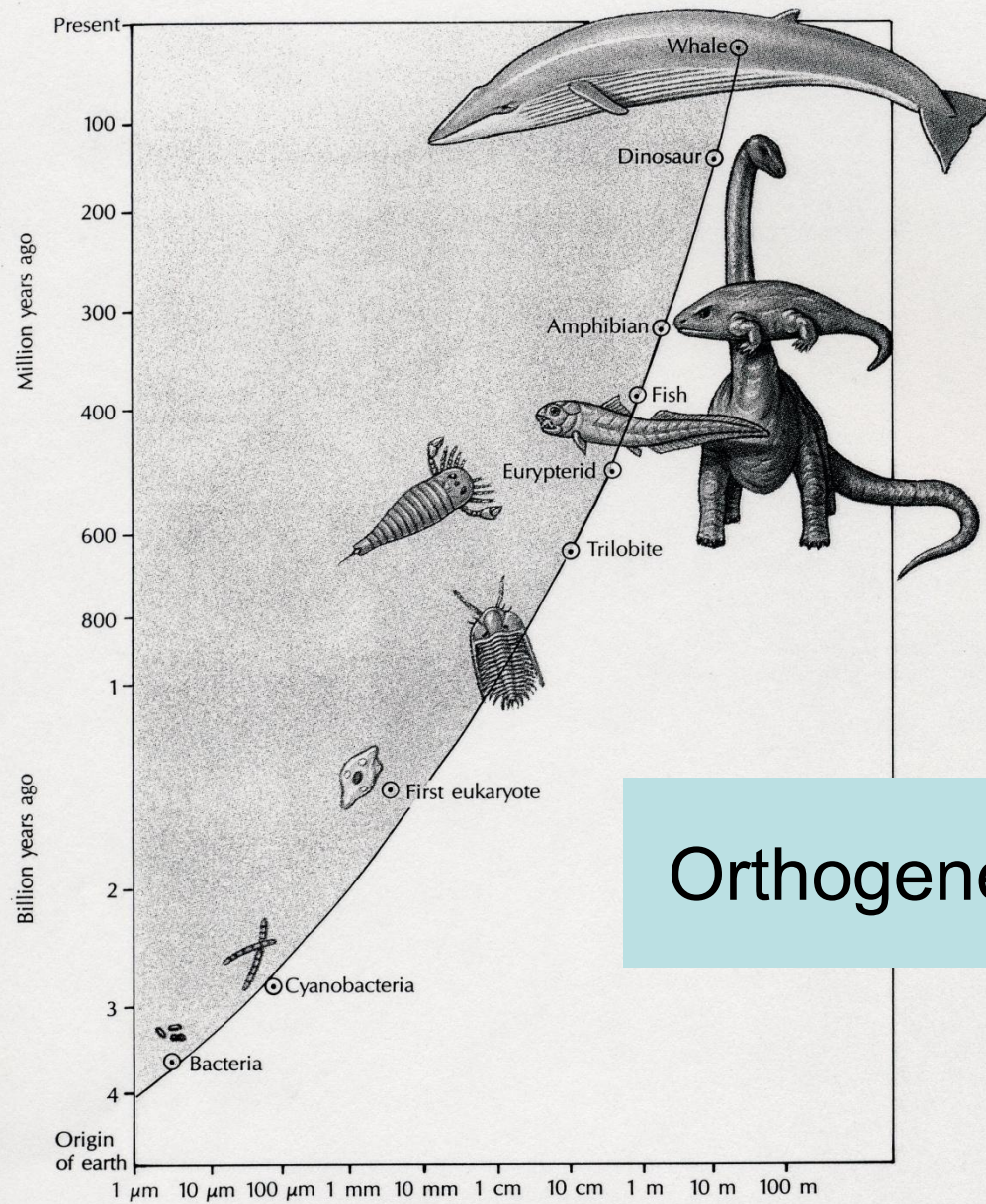
↑
10 feet
3.1 m
↓

Compsognathus
about 2 feet long.



Blue whale
80-100 feet long





Orthogenesis

FIGURE 2.1

The evolution of size (length) increase in organisms at different periods of life on earth. Note that both scales are logarithmic.

Source: Modified from J. T. Bonner, *The Evolution of Complexity*. Princeton University Press, 1988.



Orthogenesis:

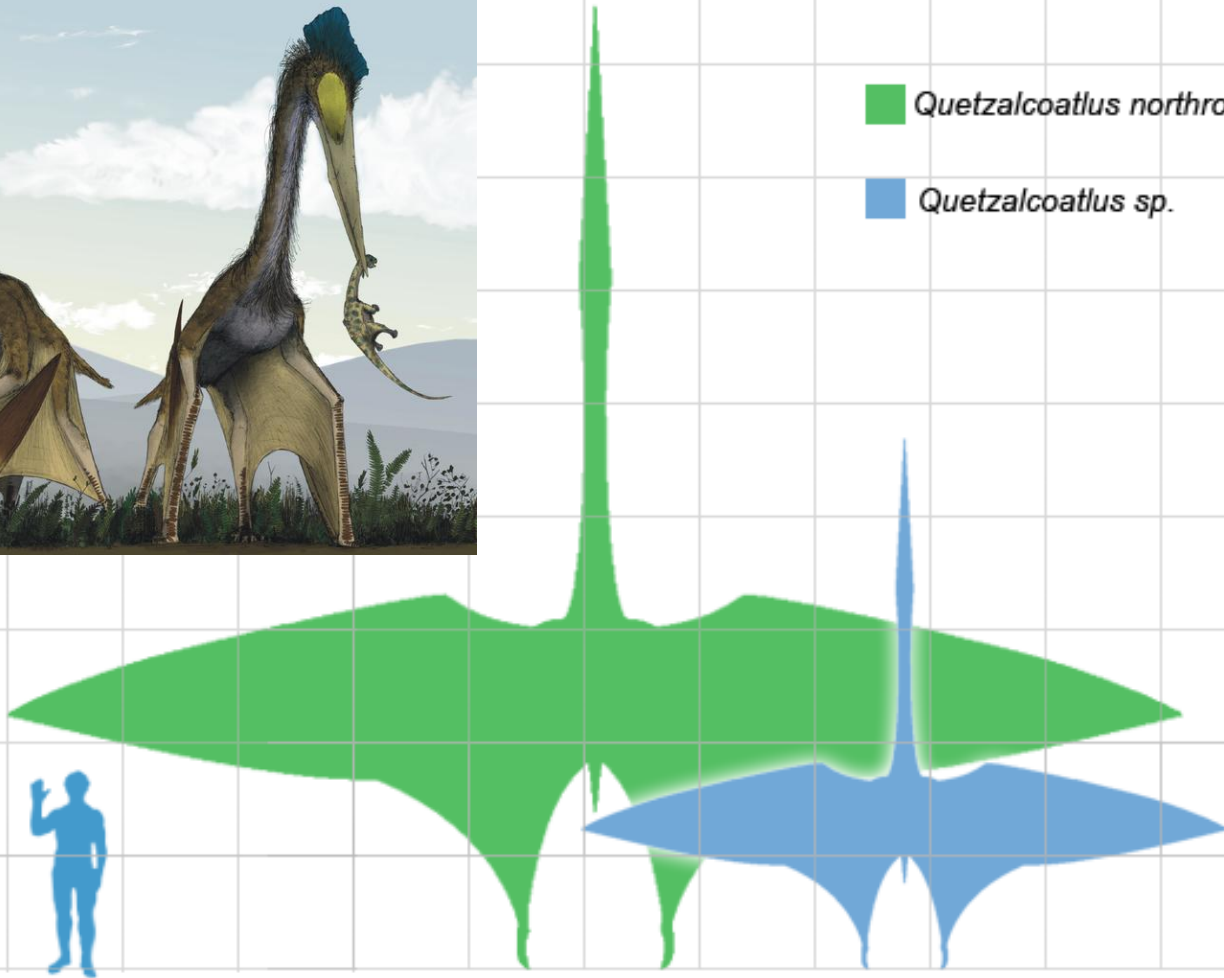
The tendency for lineages to evolve species of increasing size.

10 m Some reached 15 m



■ *Quetzalcoatlus northropi*

■ *Quetzalcoatlus sp.*



One Ton Rat



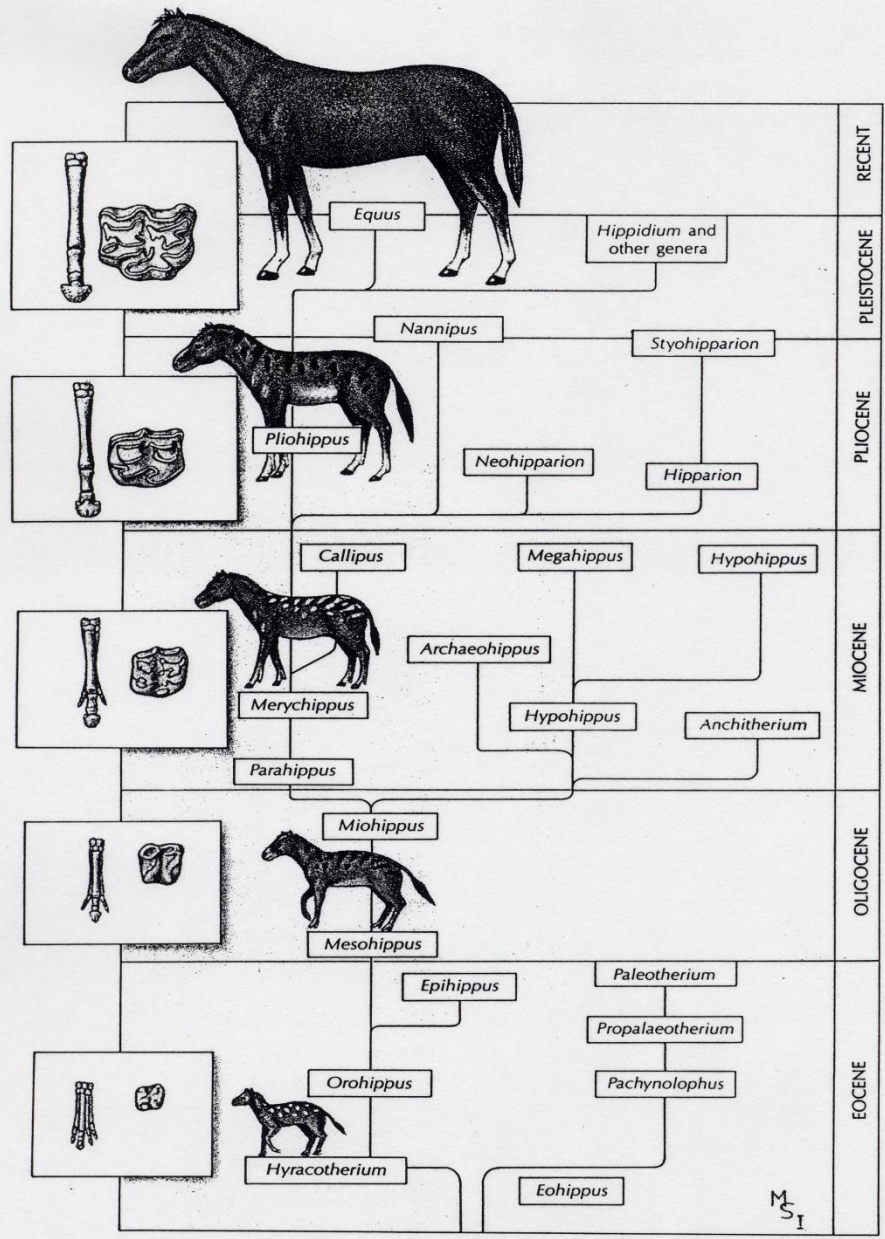


FIGURE 1.11

A reconstruction of the genera of horses from the Eocene to the present. Evolutionary trends toward increased size, elaboration of molars, and loss of toes is shown at left; a hypothetical genealogy of extant and fossil genera is shown on the right.

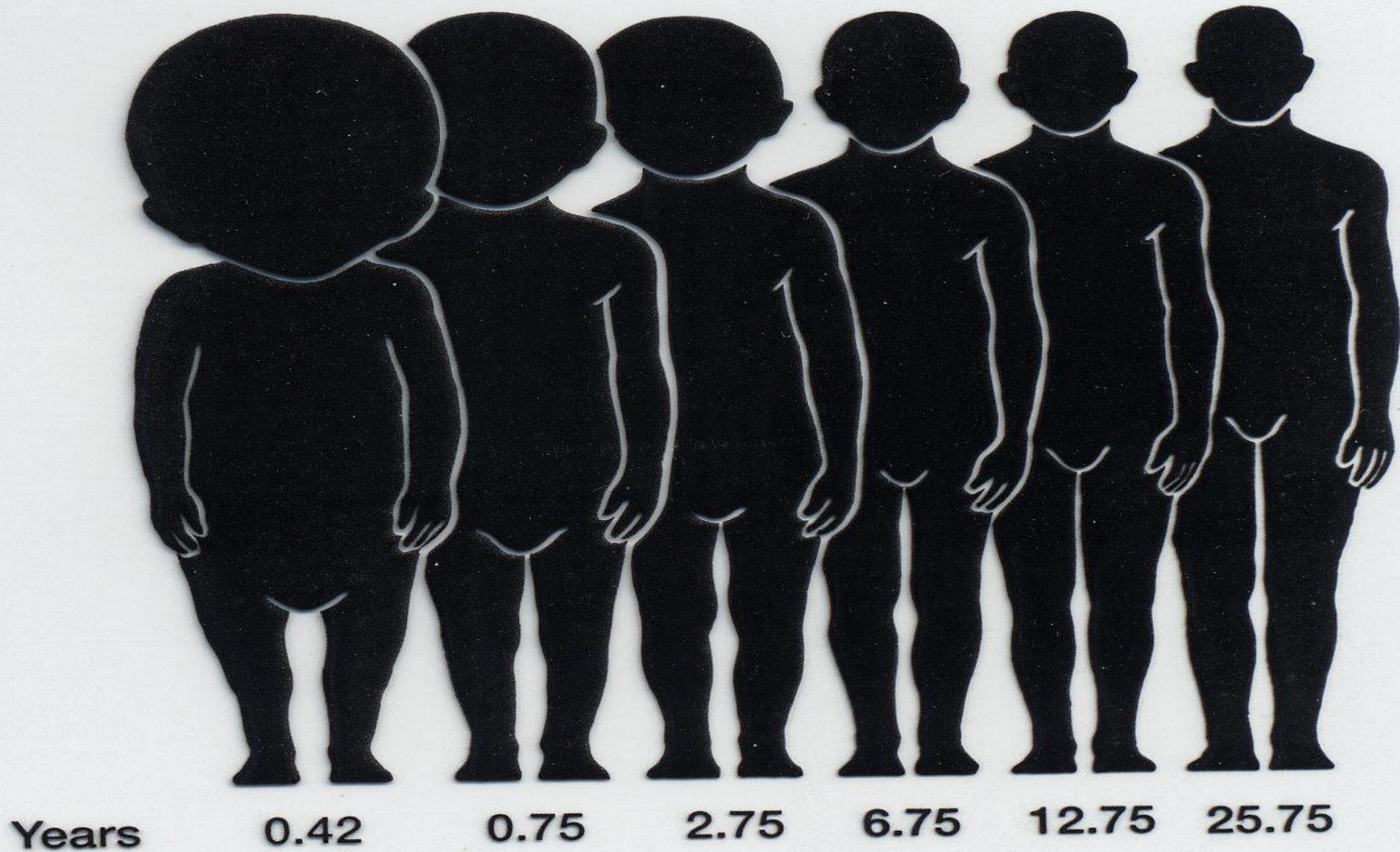


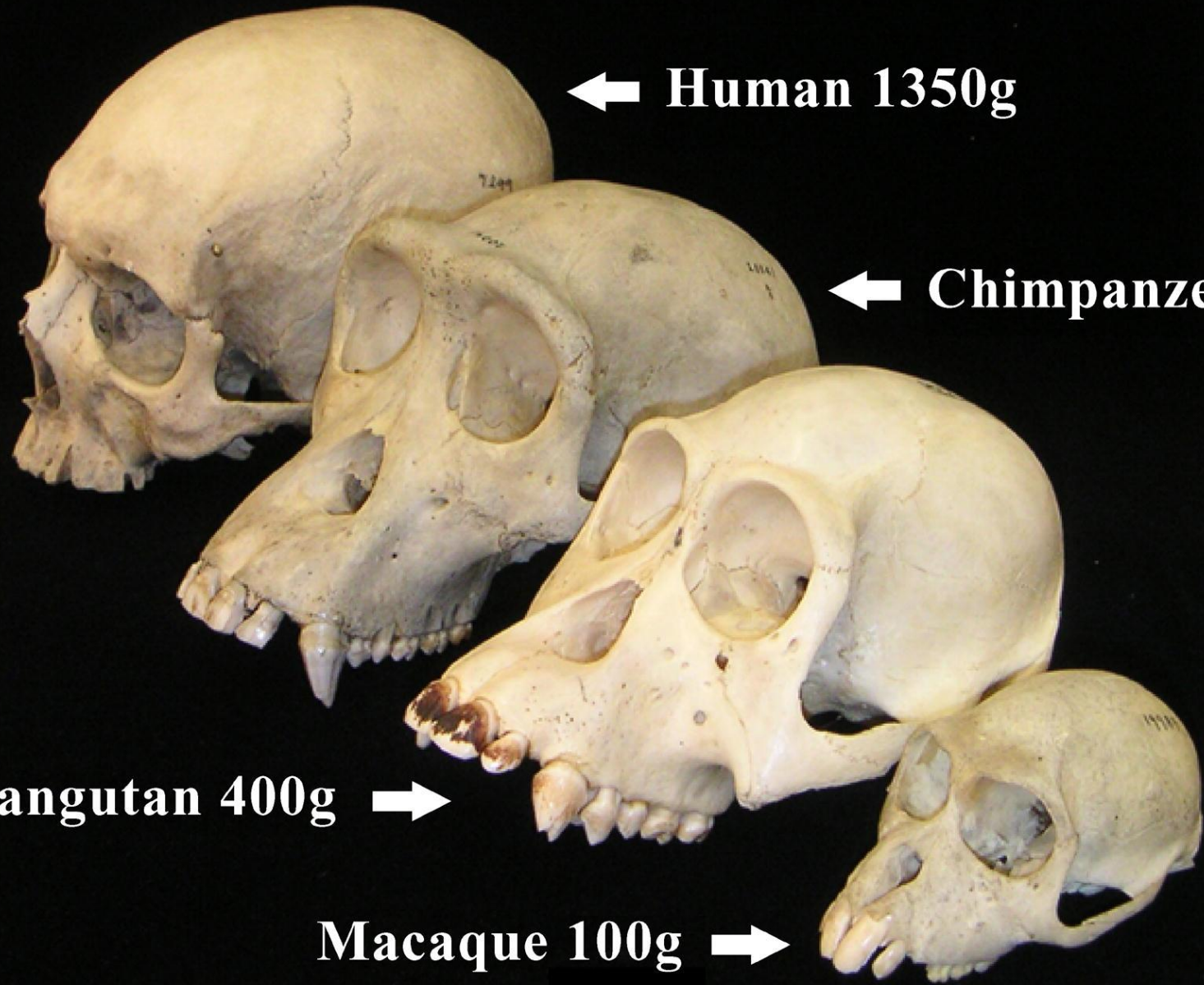
FIGURE 4.9 Allometry in human development. During growth, a person changes shape as well as size. As an infant grows, its head makes up less of its overall height and its trunk and limbs make up more. Ages, in years, are indicated beneath each figure.

From McMahon and Bonner; modified from Medawar.



Fig. 61. Baby and adult chimpanzee from Naef, 1926b. Naef remarks: "Of all animal pictures known to me, this is the most manlike" (p. 448).





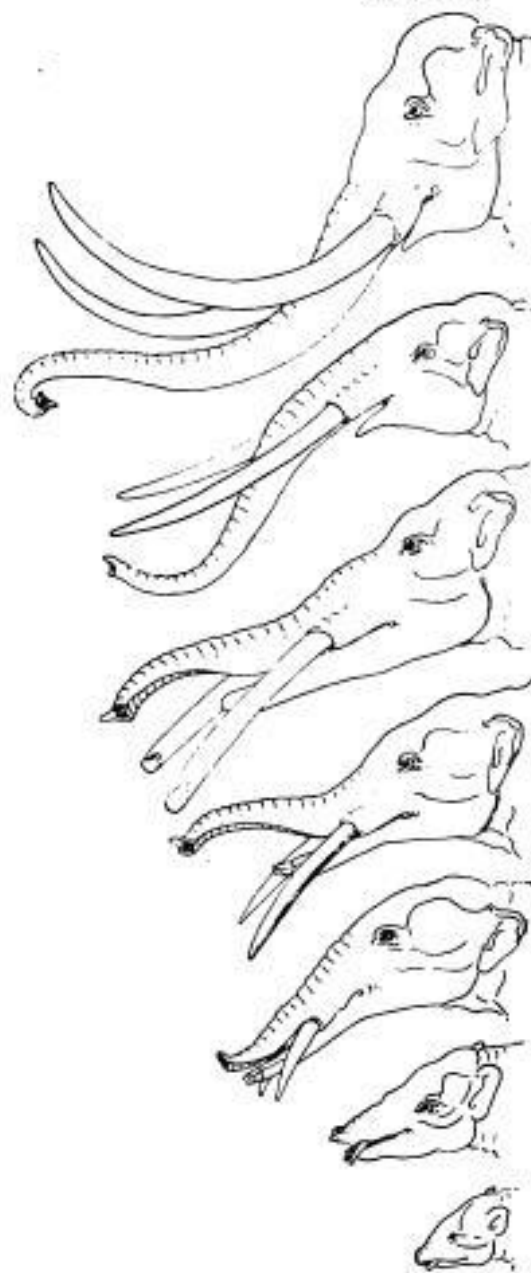
← **Human 1350g**

← **Chimpanzee 400g**

Orangutan 400g →

Macaque 100g →

PLATE IV.



7. Mammoth.
Pleistocene.
Three species, the Columbian, Imperial, and Primitive are abundant in Nebraska.

6. Mastodon.
Mastodon americanus.
Common in Nebraska.
Pleistocene.

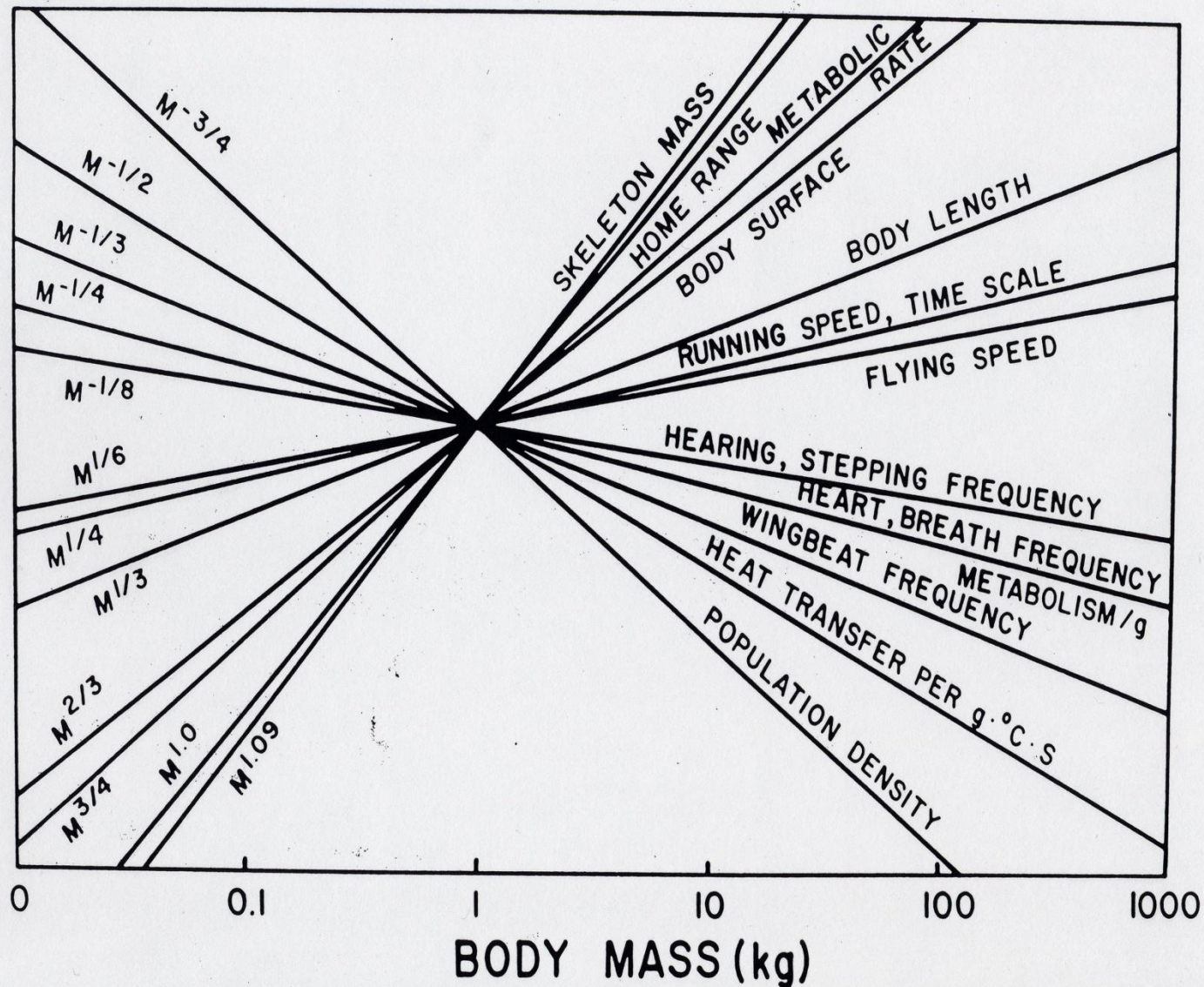
5. *Eubelodon morrilli*.
A trilophodont mastodon
Pliocene.
Devil's Gulch, Nebraska.

4. *Tetrabelodon willistoni*.
A late four-tusker.
A trilophodont mastodon.
Pliocene of Nebraska.

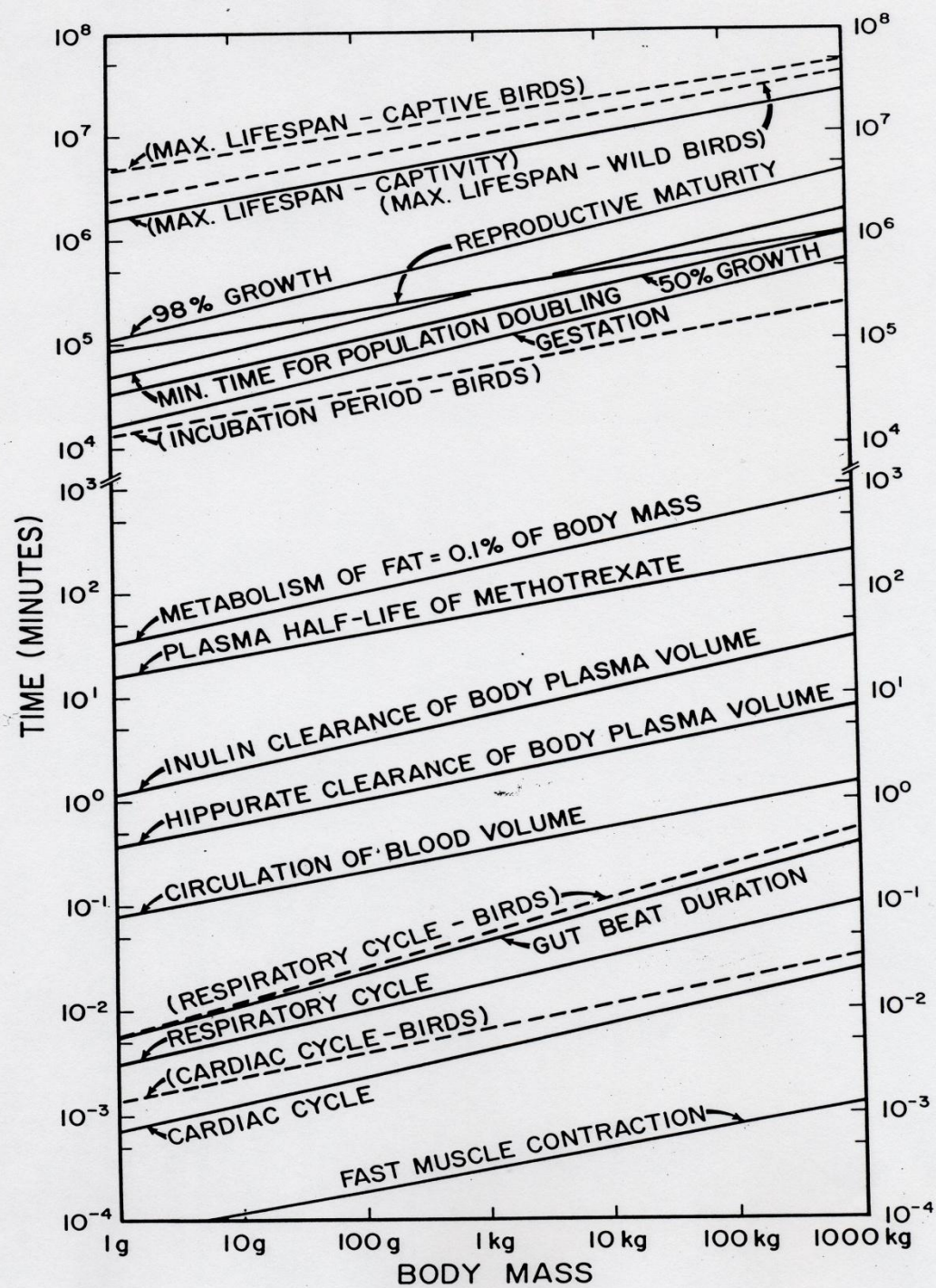
3. *Tetrabelodon*.
An early four-tusked
mastodon.
Miocene

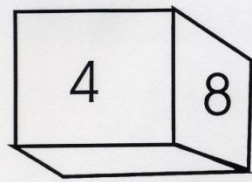
2. *Palaeomastodon*.
Lower Oligocene, Egypt.

1. *Moeritherium*.
Ancestral proboscidean.
Upper Eocene, Egypt.



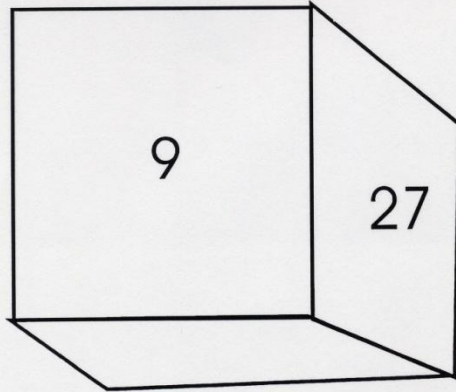
1-2 Some allometric generalizations, mostly for eutherian mammals. When plotted on log-log paper, the fractional exponential powers of body mass appear as the slopes of the lines. For example, body surfaces of animals, like surface areas of geometric figures, are proportional to volume (or mass, assuming constant density) raised to the $\frac{2}{3}$ power. On the log-log plot the $\frac{2}{3}$ becomes the slope, that is, an increase of 10^2 in surface when mass increases by a factor of 10^3 . (Adapted from Zar, 1968a; Calder, 1974; and Bartholomew, 1982.)



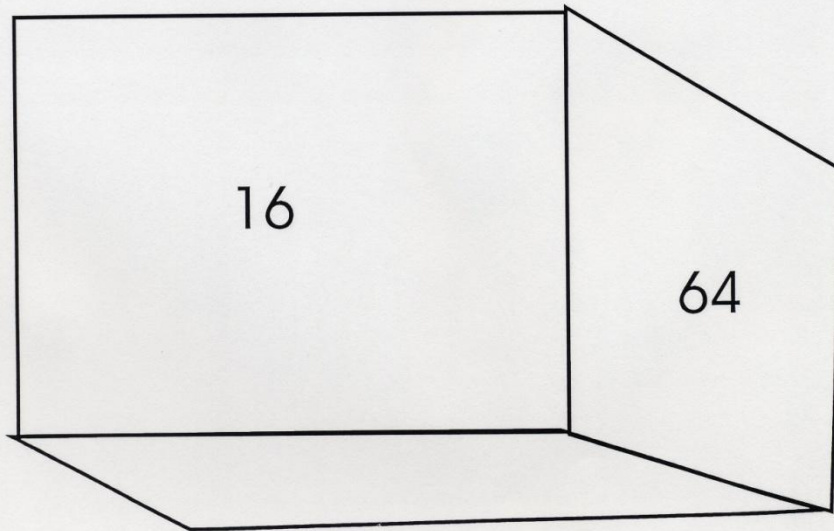


2

ALLOMETRY OF CUBES



3



4

Length $L = 3$

$$Y = aX^b$$

Surface Area $L^2 = 9$

$$= aX^{\text{Surface Area/Volume}}$$

Volume $L^3 = 27$

$$= aX^{\text{SA}(L^2)/\text{VOL}(L^3)}$$

$$= aX^{2/3} = aX^{0.67}$$

$$X^{0.67} = 27^{0.67} = 9.0$$

ALLOMETRIC EQUATIONS

Most factors that change with body size across and within species can be described by equations of the form:

$$Y = a \times X^b$$

or

$$\text{logarithm (Y)} = \text{logarithm (a)} + b(\text{logarithm [X]})$$

Where Y = dependent variable, a = intercept of the line, b = exponent or slope, X = body size

For most factors, inter-specific scaling has an exponent of 0.75

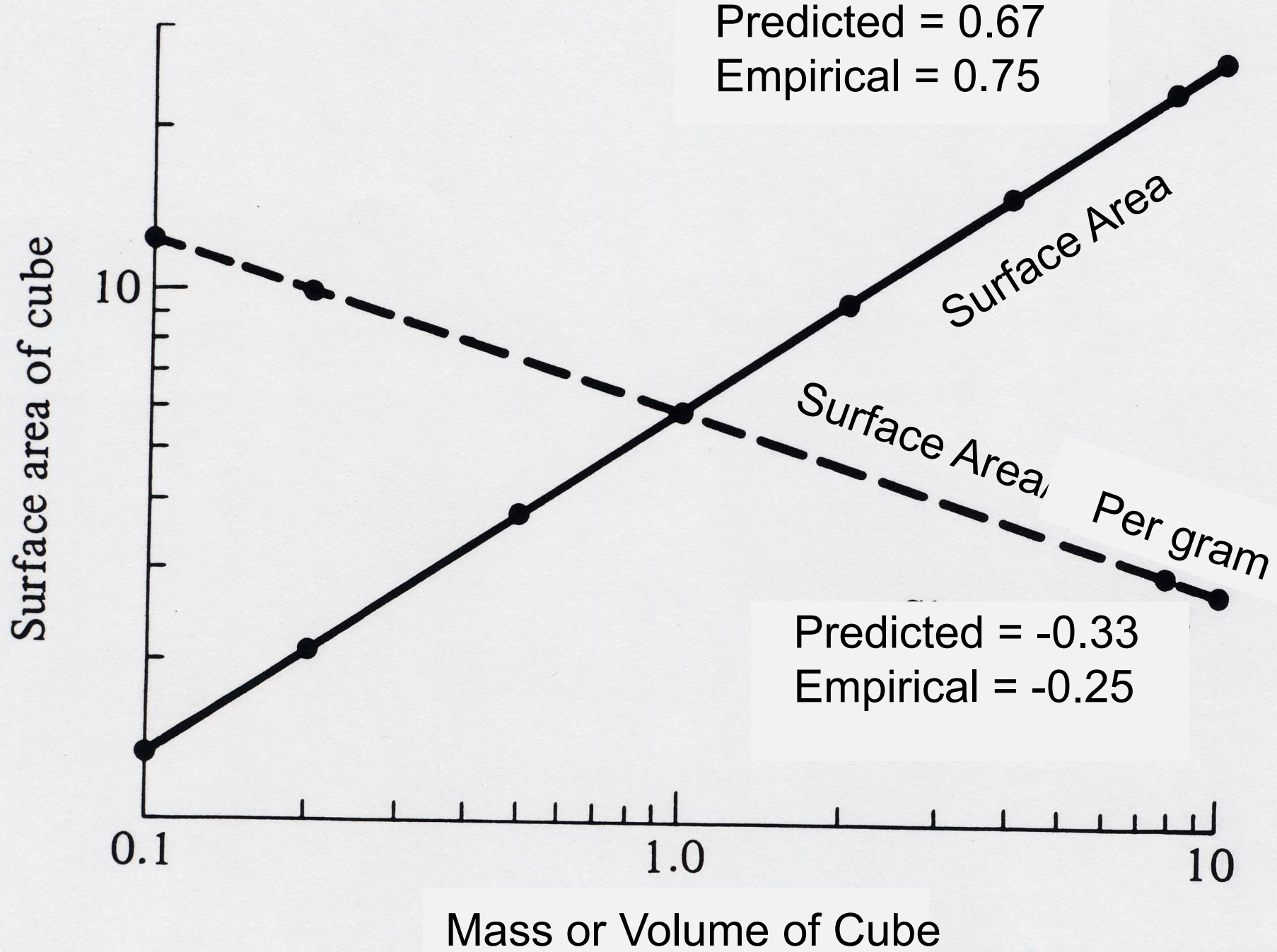
i.e. $Y = a \times X^{0.75}$ (For example, absolute respiration rate)

For specific respiration rate (i.e., oxygen consumption *per gram tissue*), the form of the equation is :

$$Y = a \times X^{-0.25}$$

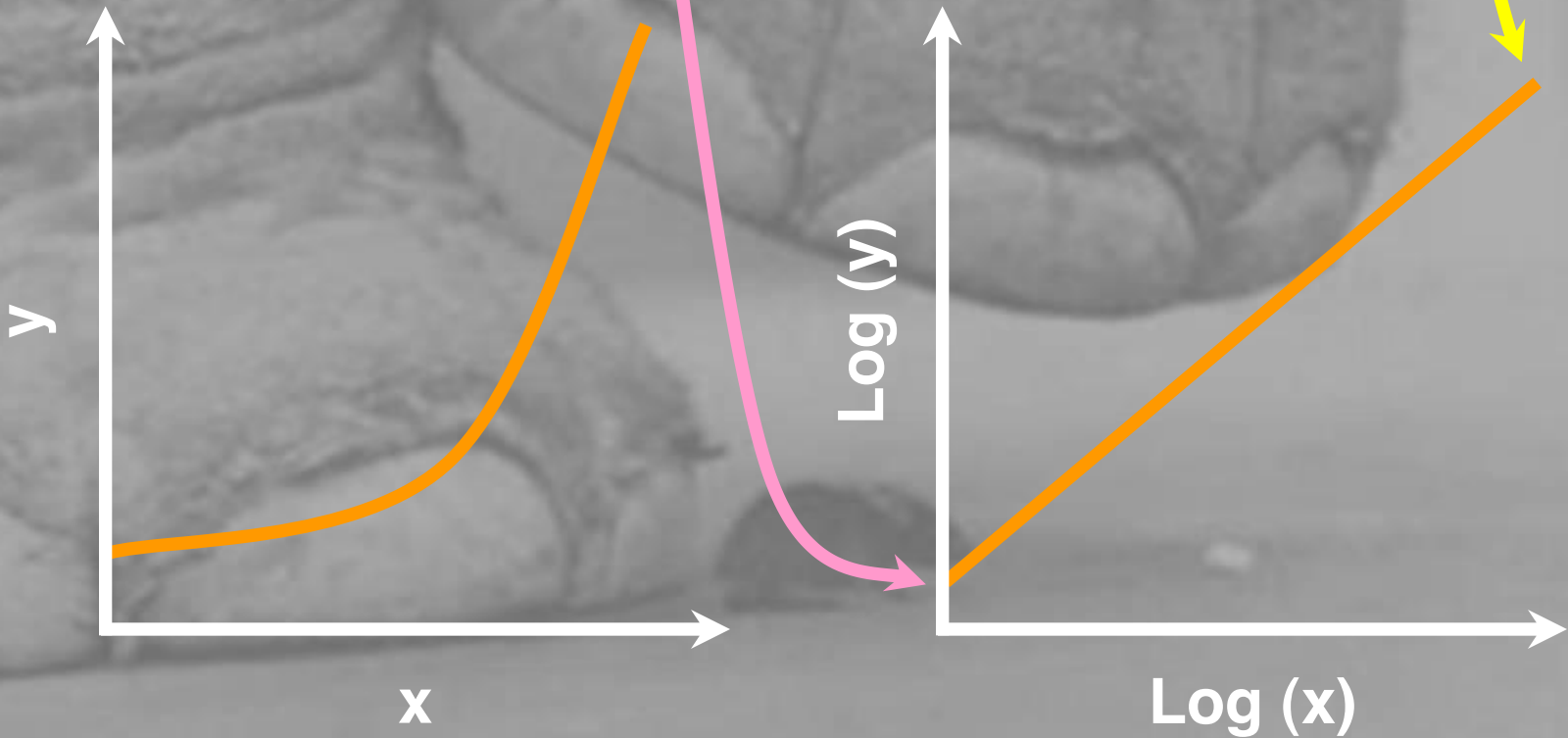
Within species (intra-specifically), the corresponding exponents (b), may have values closer to

0.66 and -0.33 respectively



Allometry INTER-SPECIFIC STUDIES OF SIZE AND LIFESPAN

$$Y \propto Y_0 \cdot x^b$$

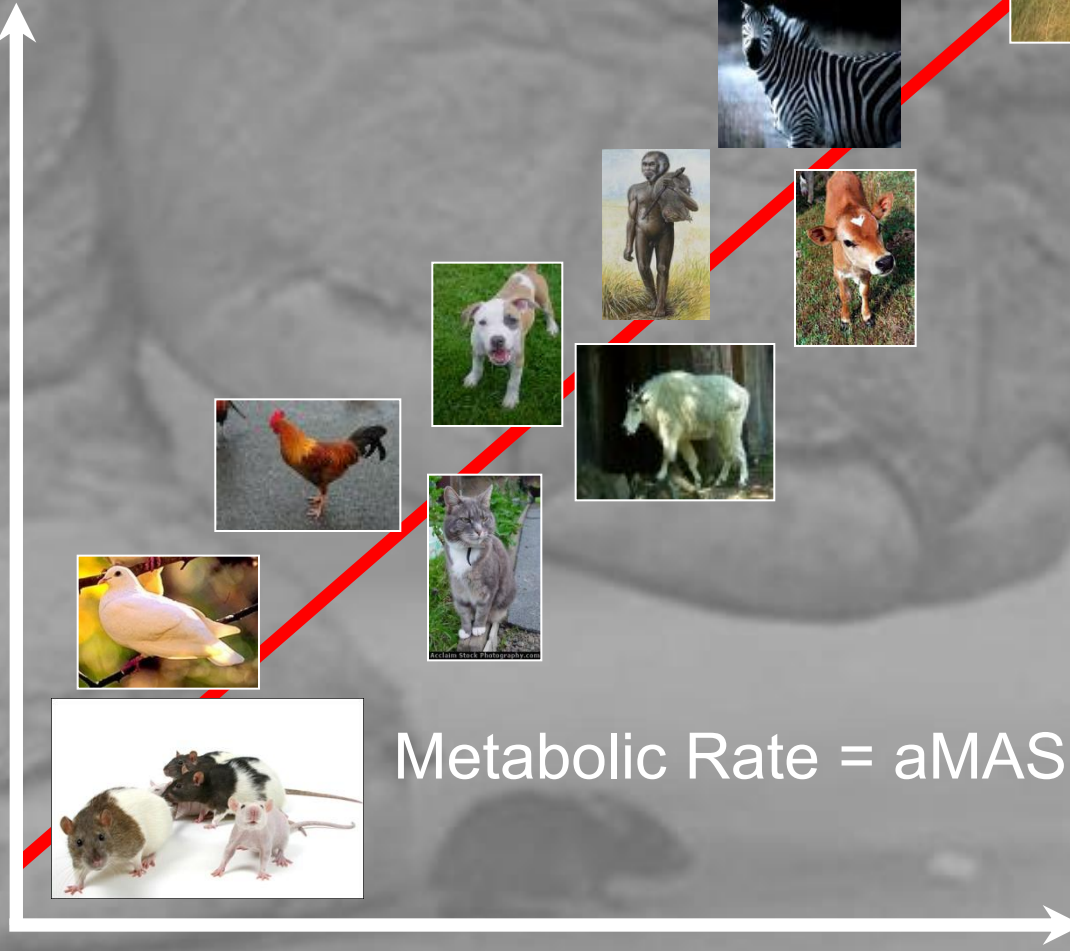


Thanks April Hayward

Kleiber, 1932

Allometry: Organism Metabolism

Log Total Metabolic Rate
($W \cdot h^{-1}$)



Log Body Mass (kg)

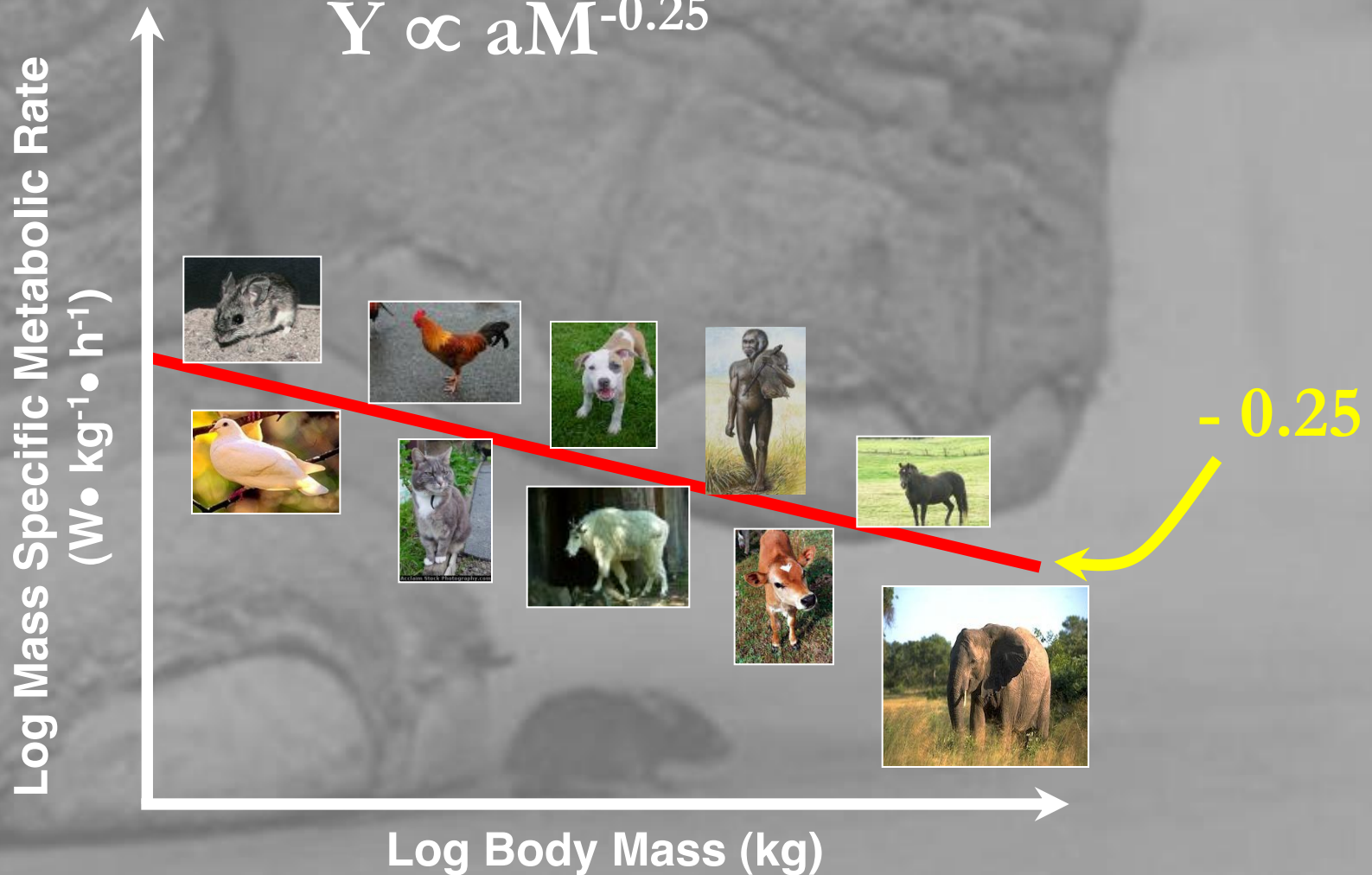
Thanks April Hayward

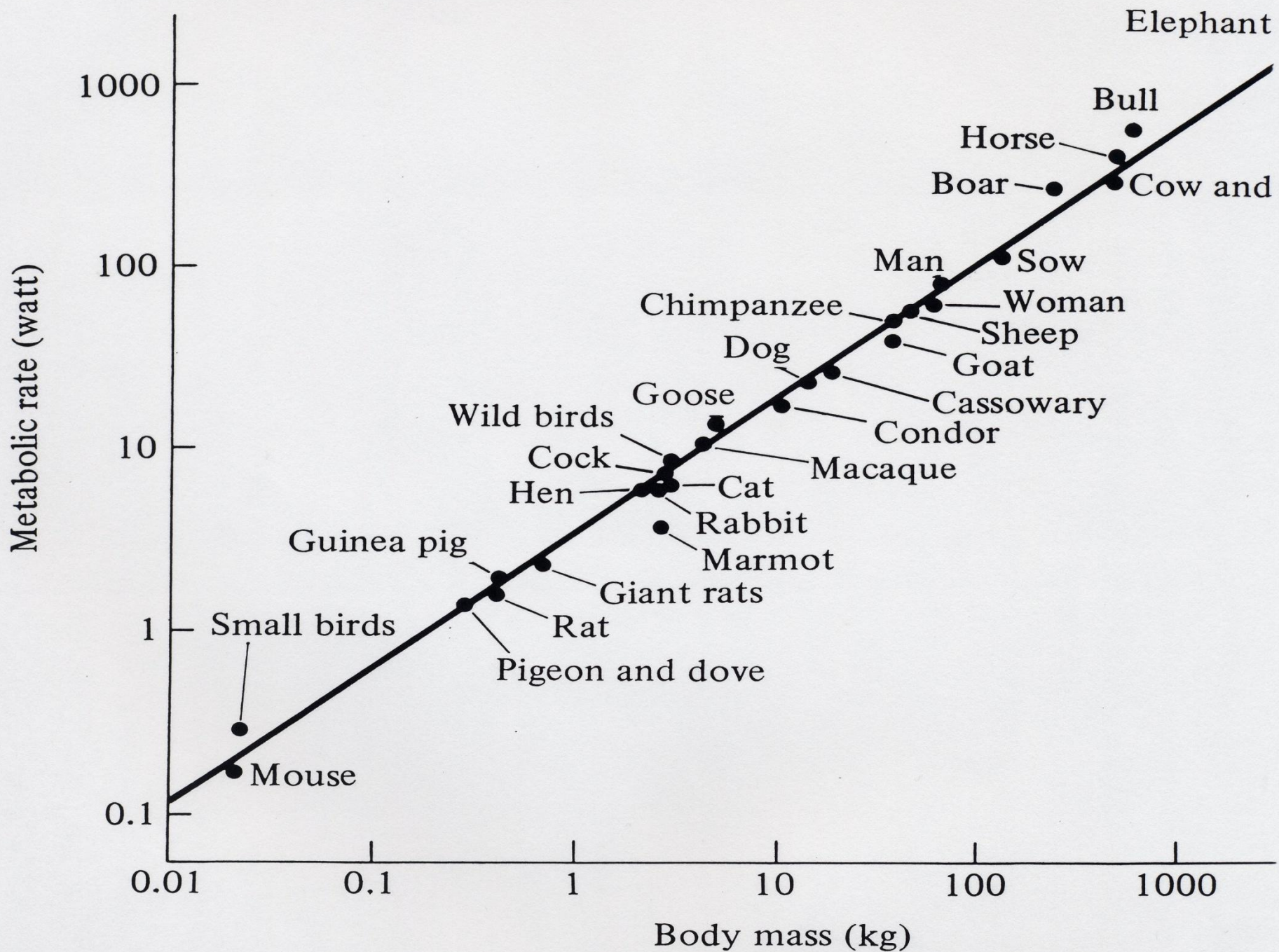
Figure adapted from Schmidt-Nielsen, 1984

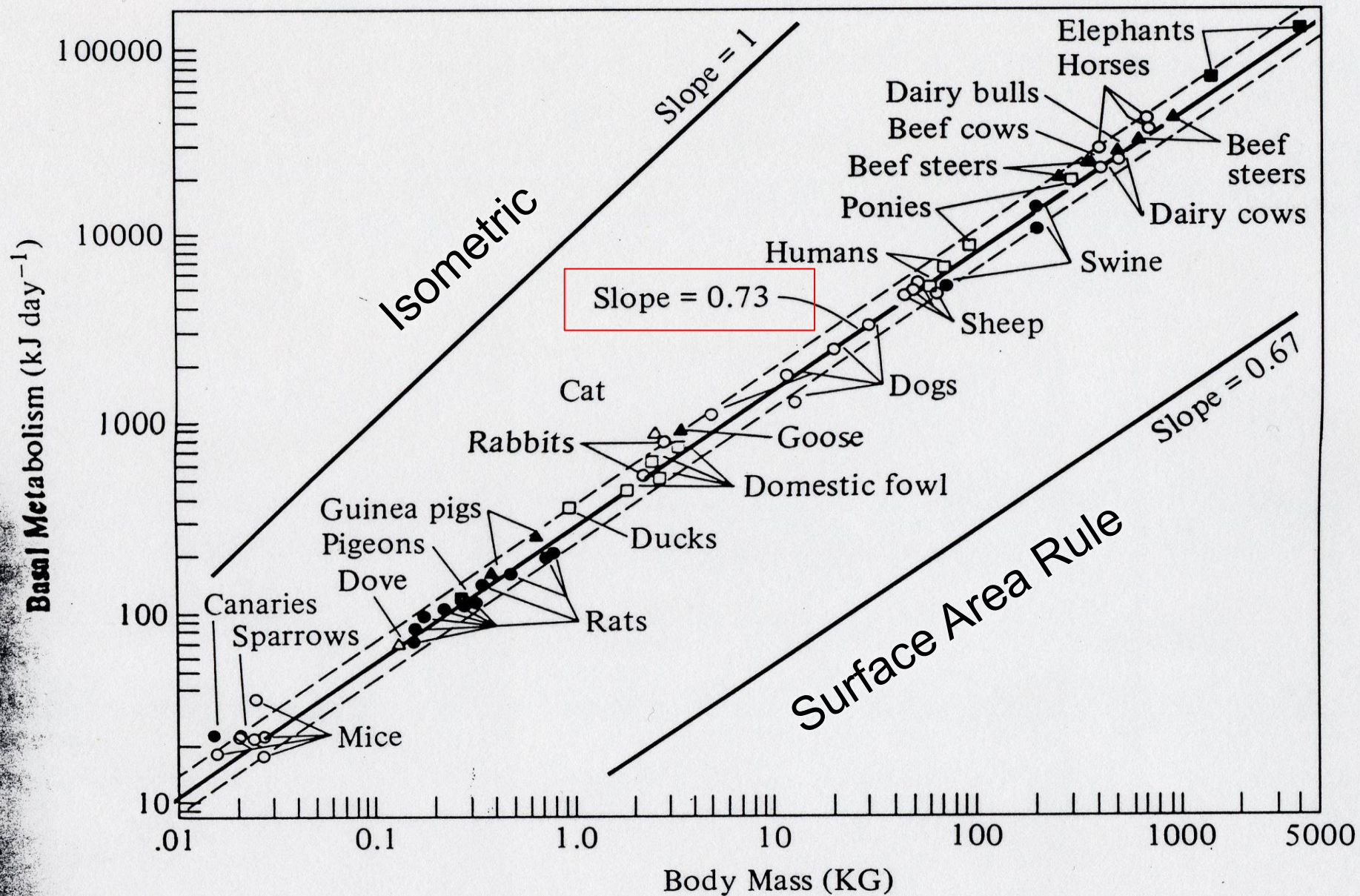
RATE OF LIVING

Organism Metabolism

$$Y \propto aM^{-0.25}$$

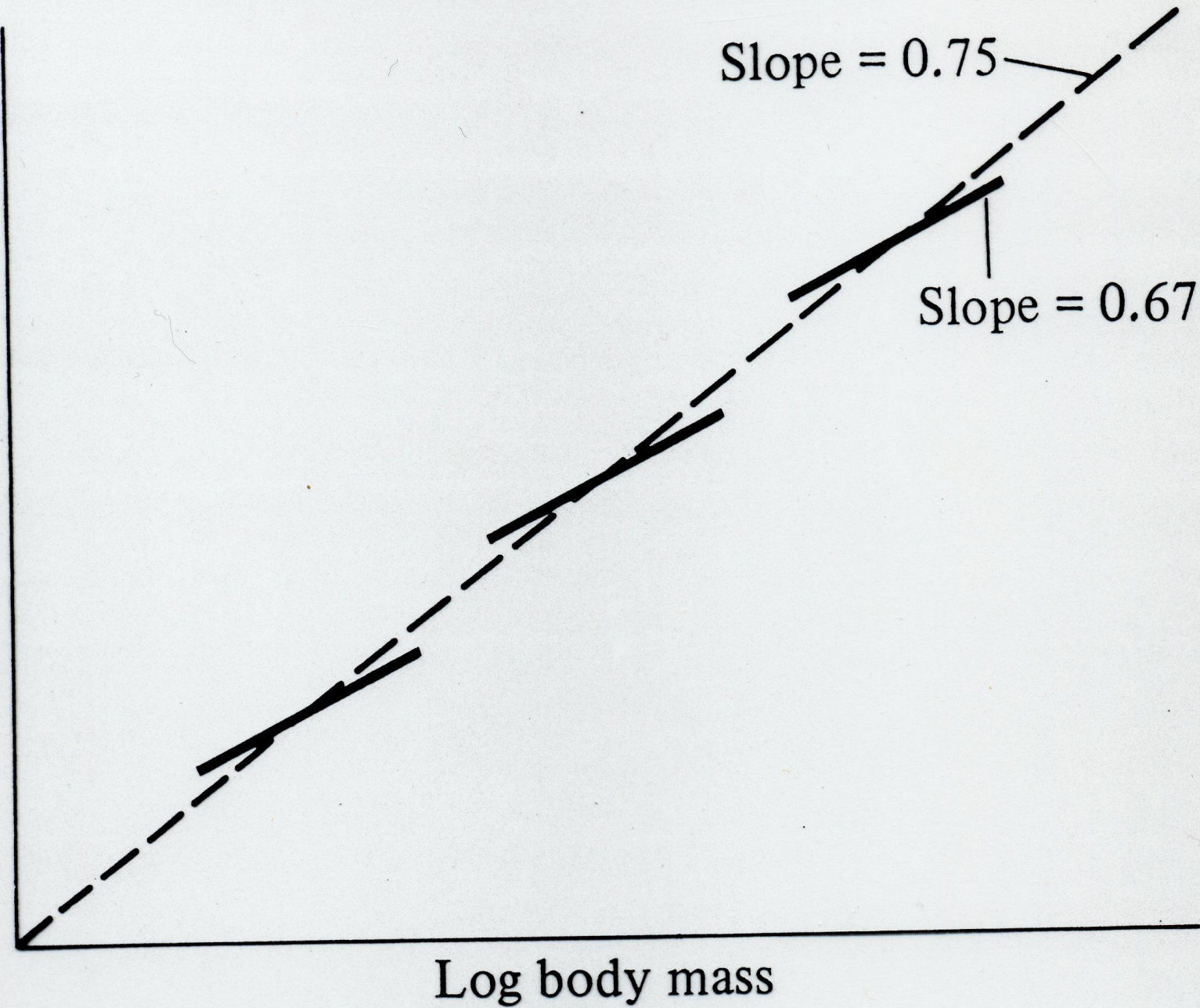






Relationship between \log_{10} metabolic rate and \log_{10} body mass for birds and mammals. (Modified from Brody 1945.)

Log metabolic rate



Review

The origin of allometric scaling laws in biology from genomes to ecosystems: towards a quantitative unifying theory of biological structure and organization

Geoffrey B. West^{1,2,*} and James H. Brown^{1,3}

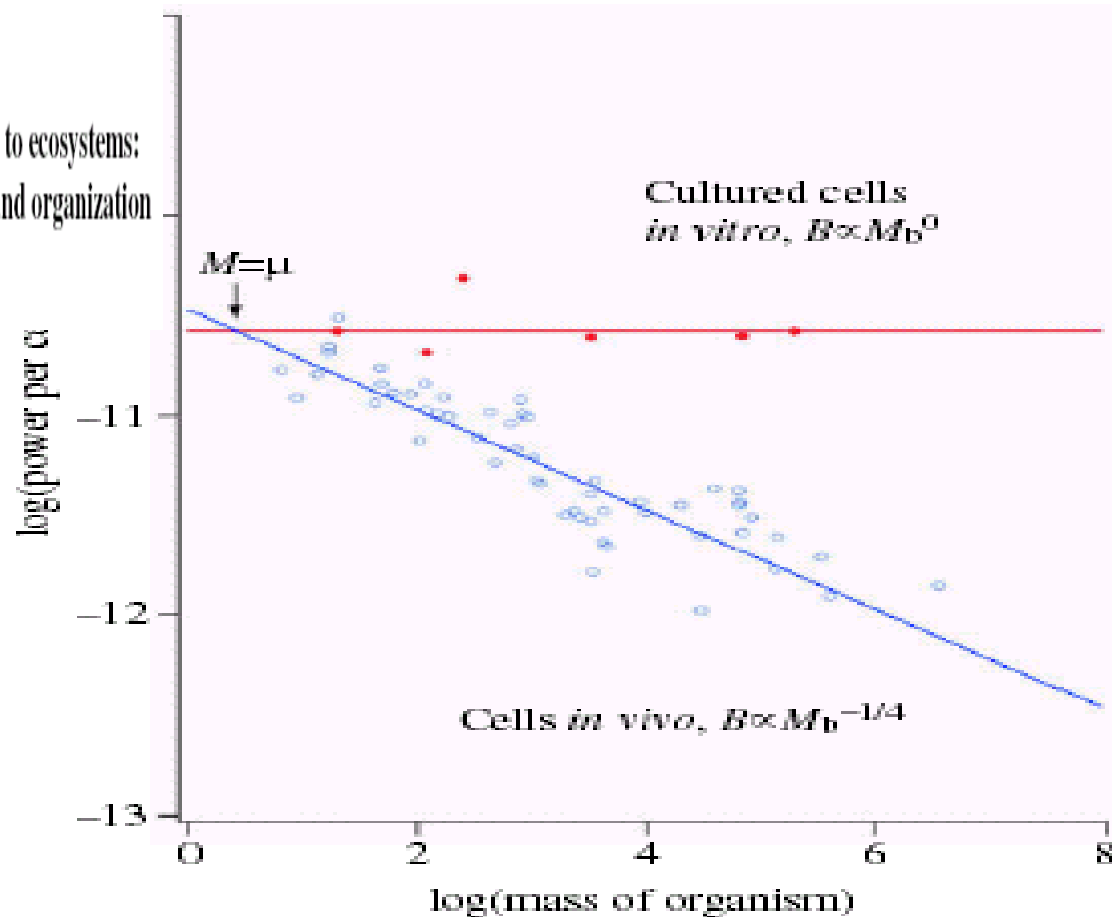
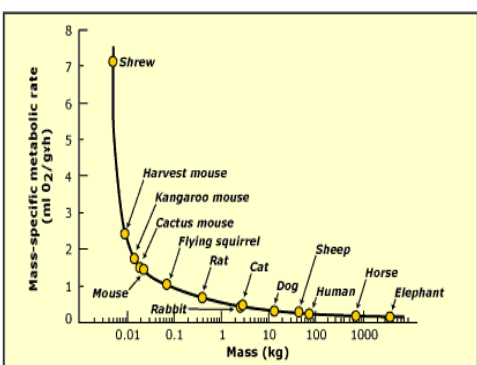
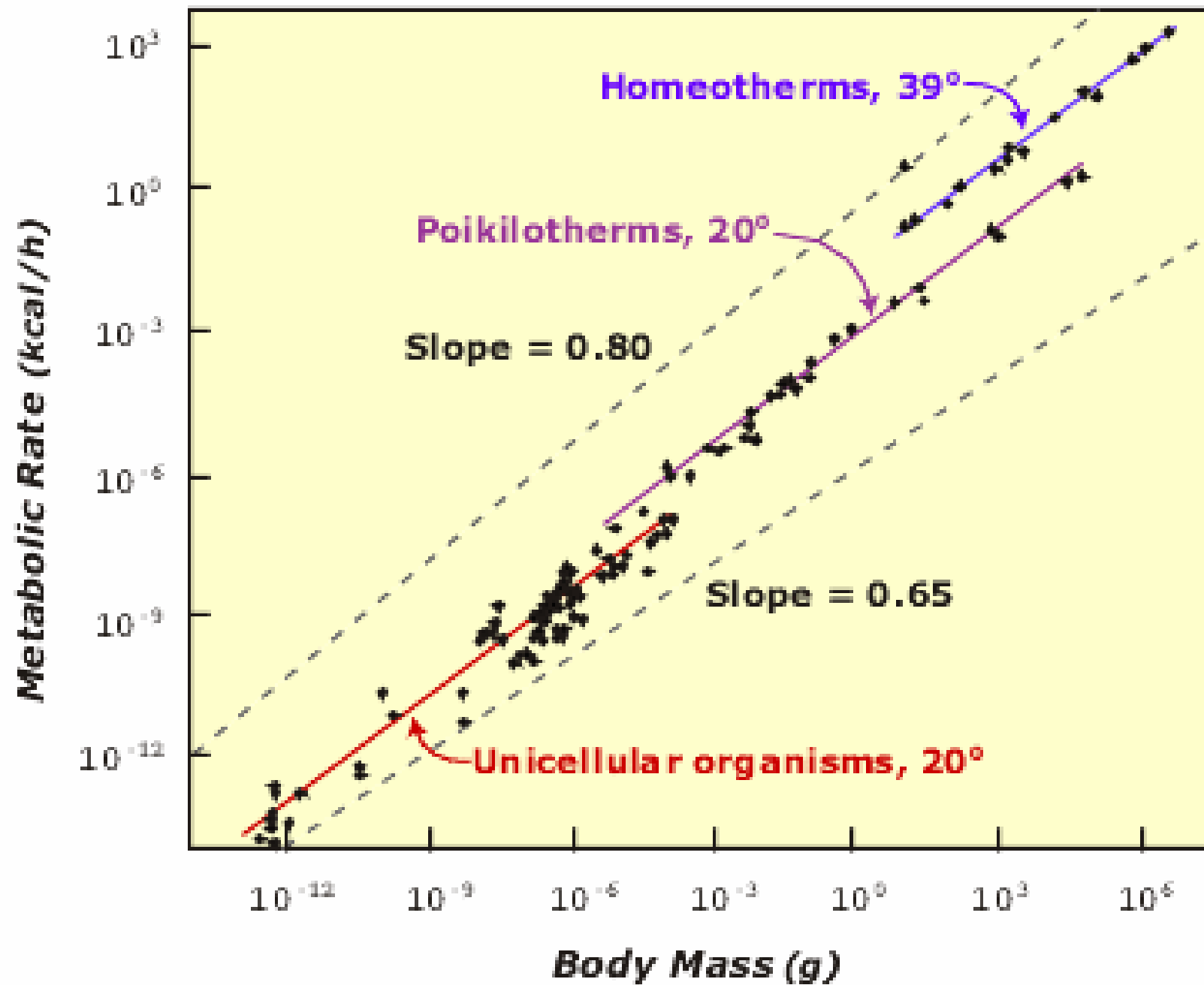


Fig. 5. Metabolic rates (in W) of mammalian cells *in vivo* (blue line) and cultured *in vitro* (red line) plotted as a function of organism mass (M_b in g) on a log–log scale. While still in the body and constrained by vascular supply networks cellular metabolic rates scale as $M_b^{-1/4}$. When cells are removed from the body and cultured *in vitro*, their metabolic rates converge to a constant value predicted by theory (West et al., 2002b). The two lines meet at the mass of the smallest mammal (the shrew with mass ~1 g, as predicted). Figure taken from West et al. (2002b) with permission.





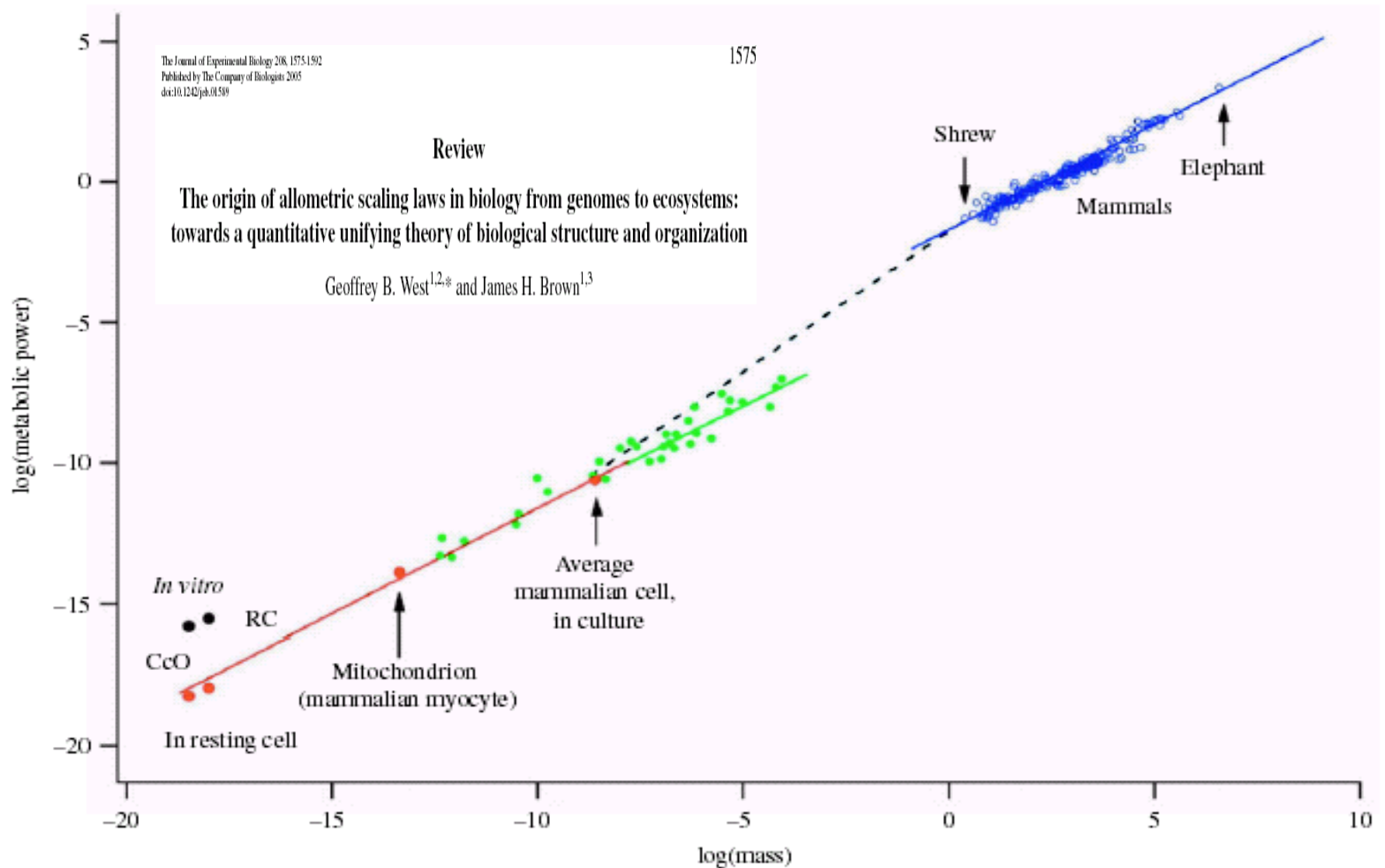
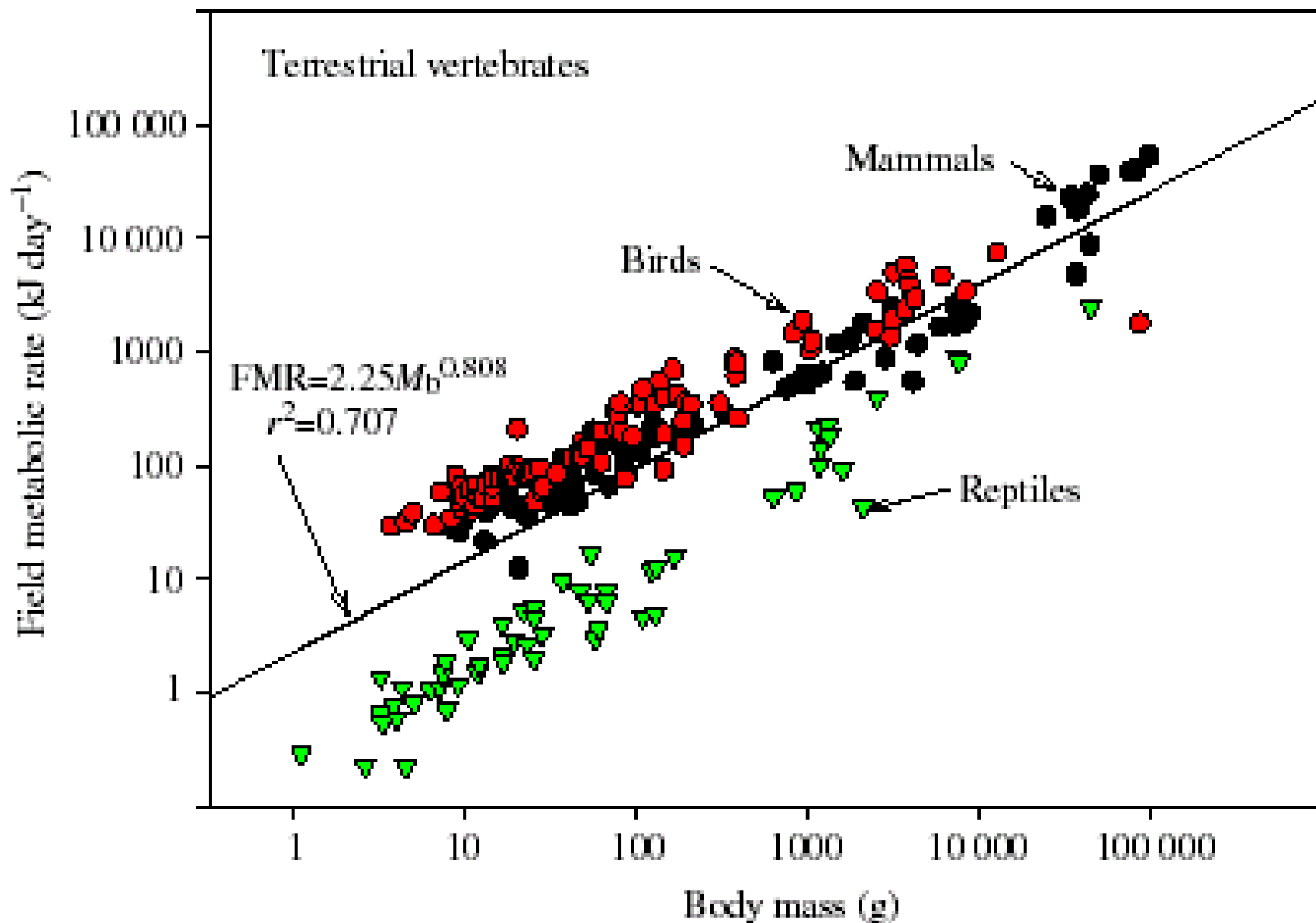
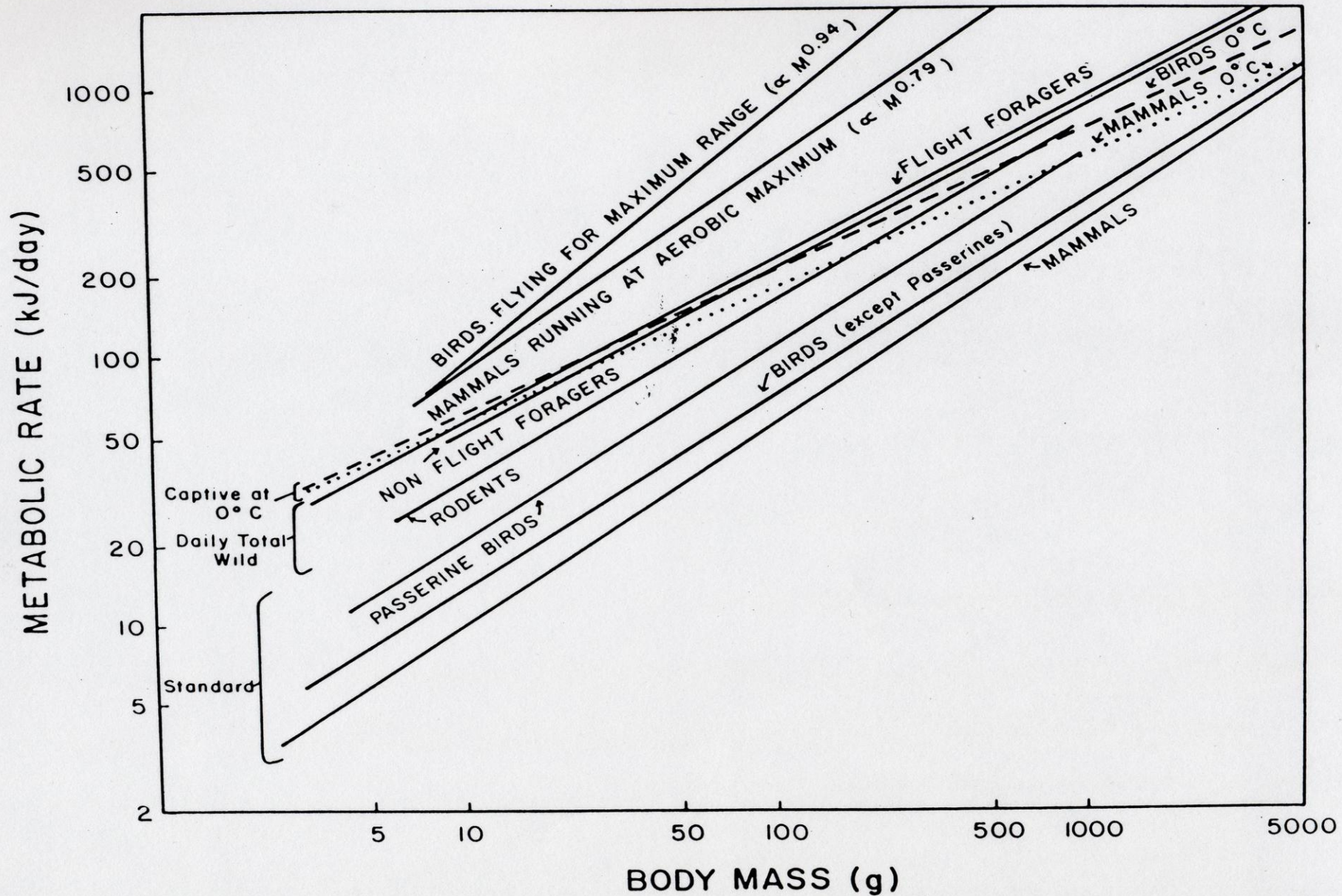
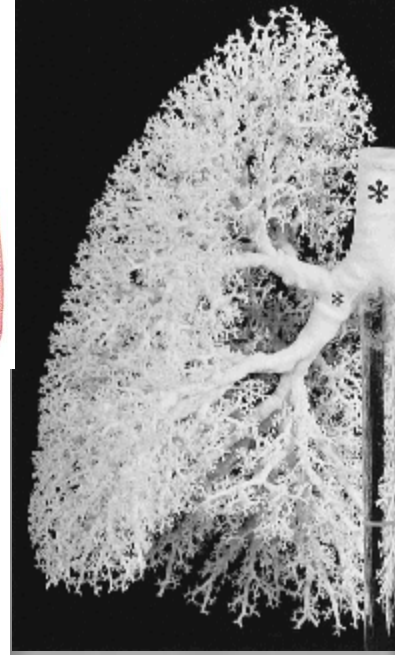
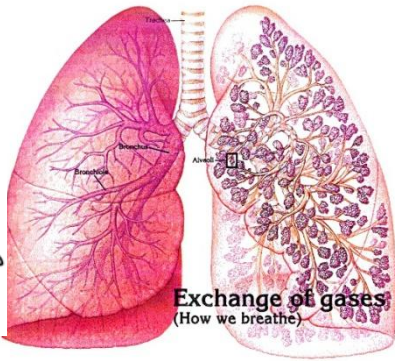
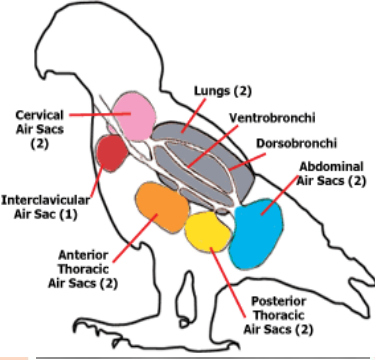
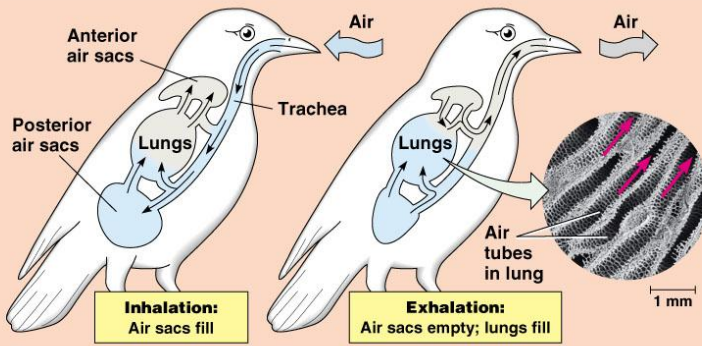


Fig. 2. Extension of Kleiber's 3/4-power law for the metabolic rate of mammals to over 27 orders of magnitude from individuals (blue circles) to uncoupled mammalian cells, mitochondria and terminal oxidase molecules, CcO of the respiratory complex, RC (red circles). Also shown are data for unicellular organisms (green circles). In the region below the smallest mammal (the shrew), scaling is predicted to extrapolate linearly to an isolated cell *in vitro*, as shown by the dotted line. The 3/4-power re-emerges at the cellular and intracellular levels. Figure taken from West et al. (2002b) with permission.

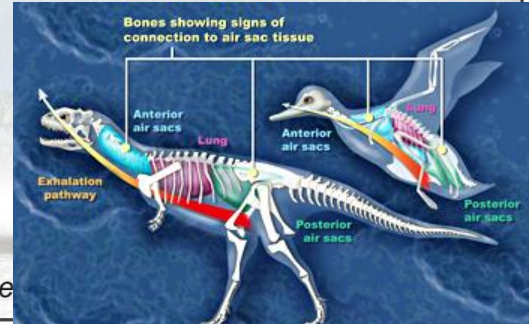
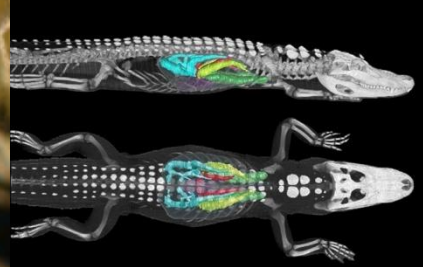
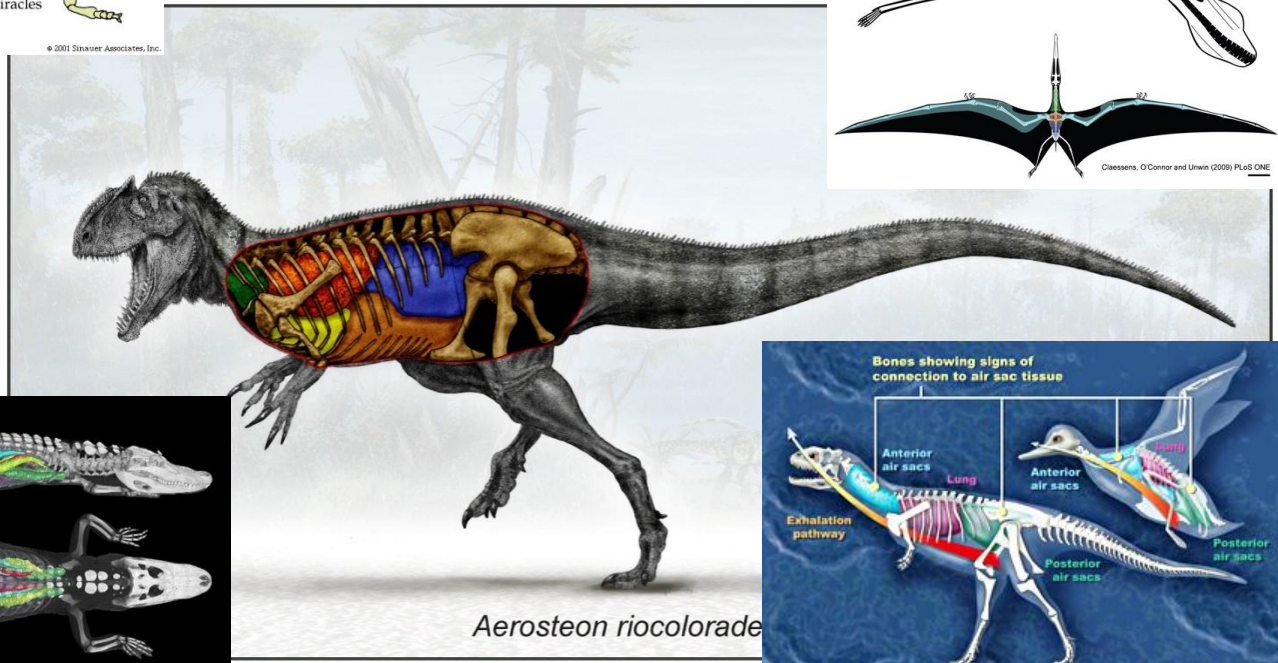
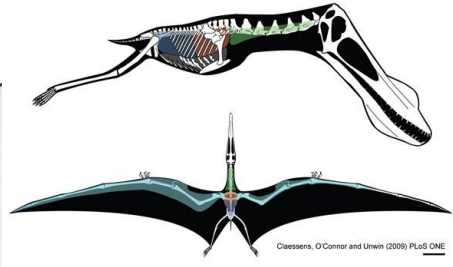
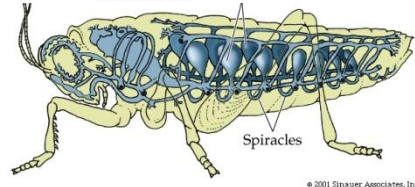
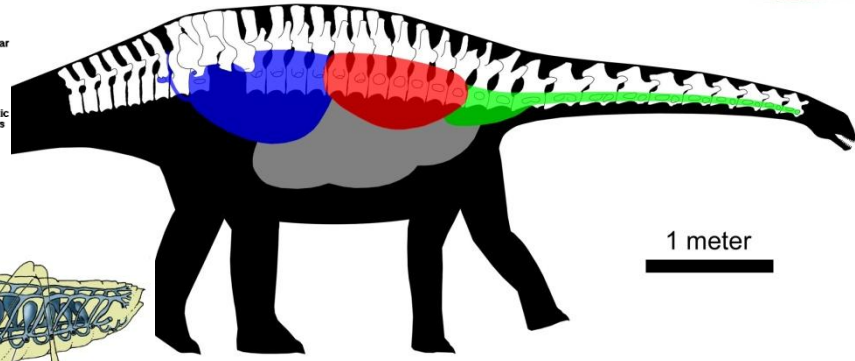
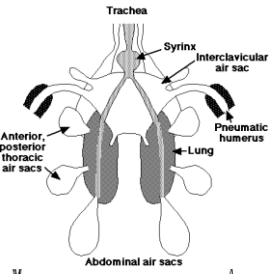
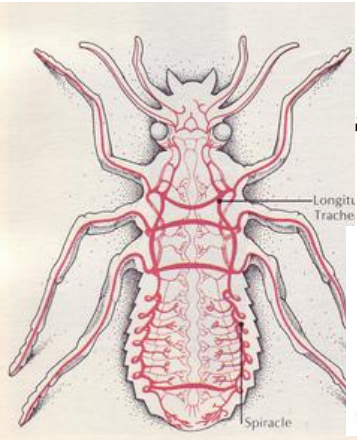


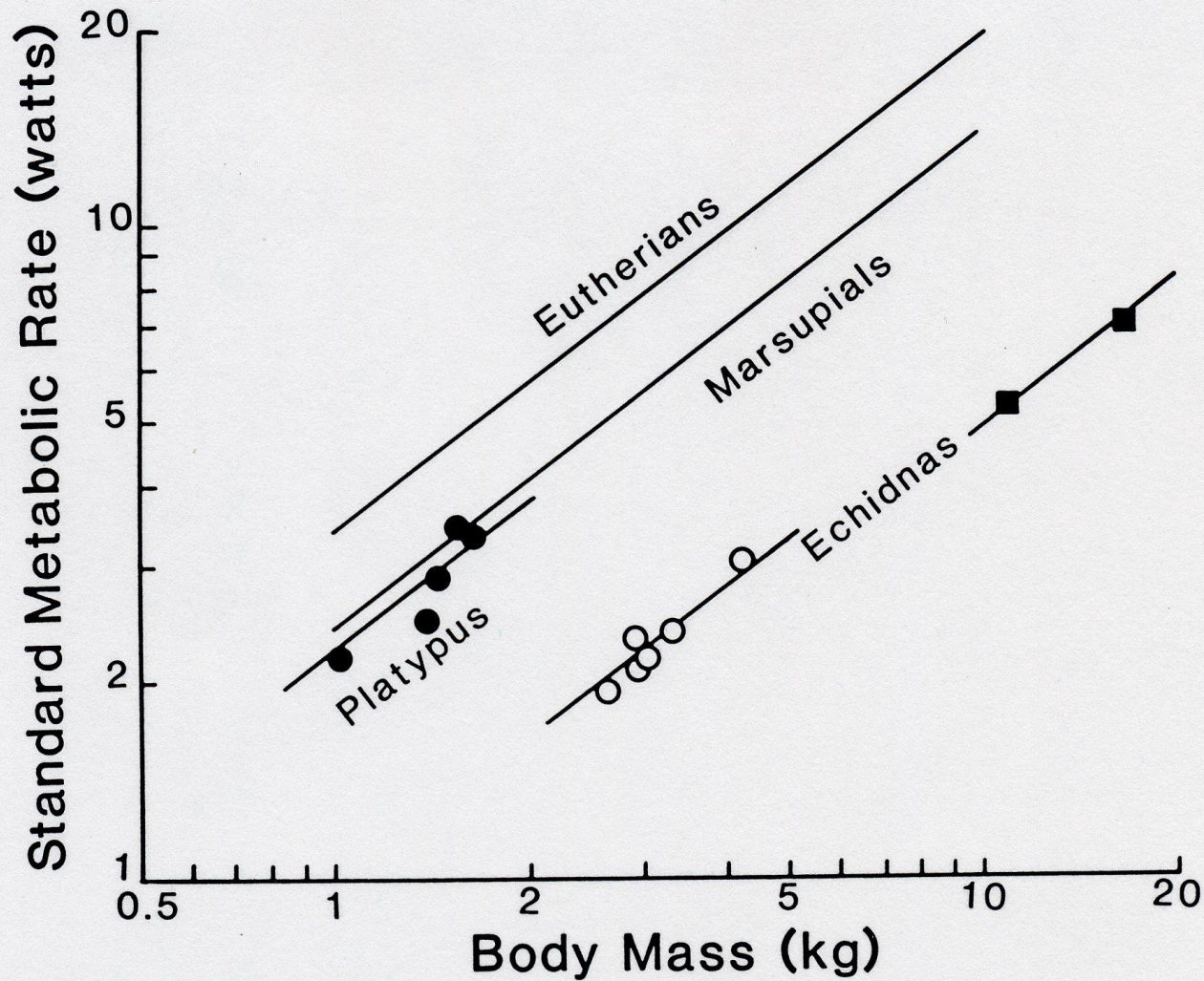


8-5 Metabolic rates as they are elevated above standard (basal) levels ($\propto M^{3/4}$), during exposure to cold ($\propto M^{0.8\text{ to }0.9}$), and during unrestrained natural activity in the field ($\propto M^{0.6\text{ to }0.7}$) are seen in this log-log plot. Equations for the lines appear in Eq. (7-45), and Tables 3-3, 7-3, 8-2, and 8-3.



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5-1 Compared on an equal-size basis, marsupials and the aquatic monotreme, the platypus (●-*Ornithorhynchus anatinus*), have standard metabolic rates only two-thirds of minimum maintenance requirements for eutherian mammals, while the echidnas (○-*Tachyglossus aculeatus* and ■-*Zaglossus bruijnii*) require energy at rates less than one-third of what the eutherians need for basal maintenance. Within these taxa, however, size has essentially the same effect, the $M^{3/4}$ scaling first described by Kleiber. (After Dawson et al., 1979; with permission.)

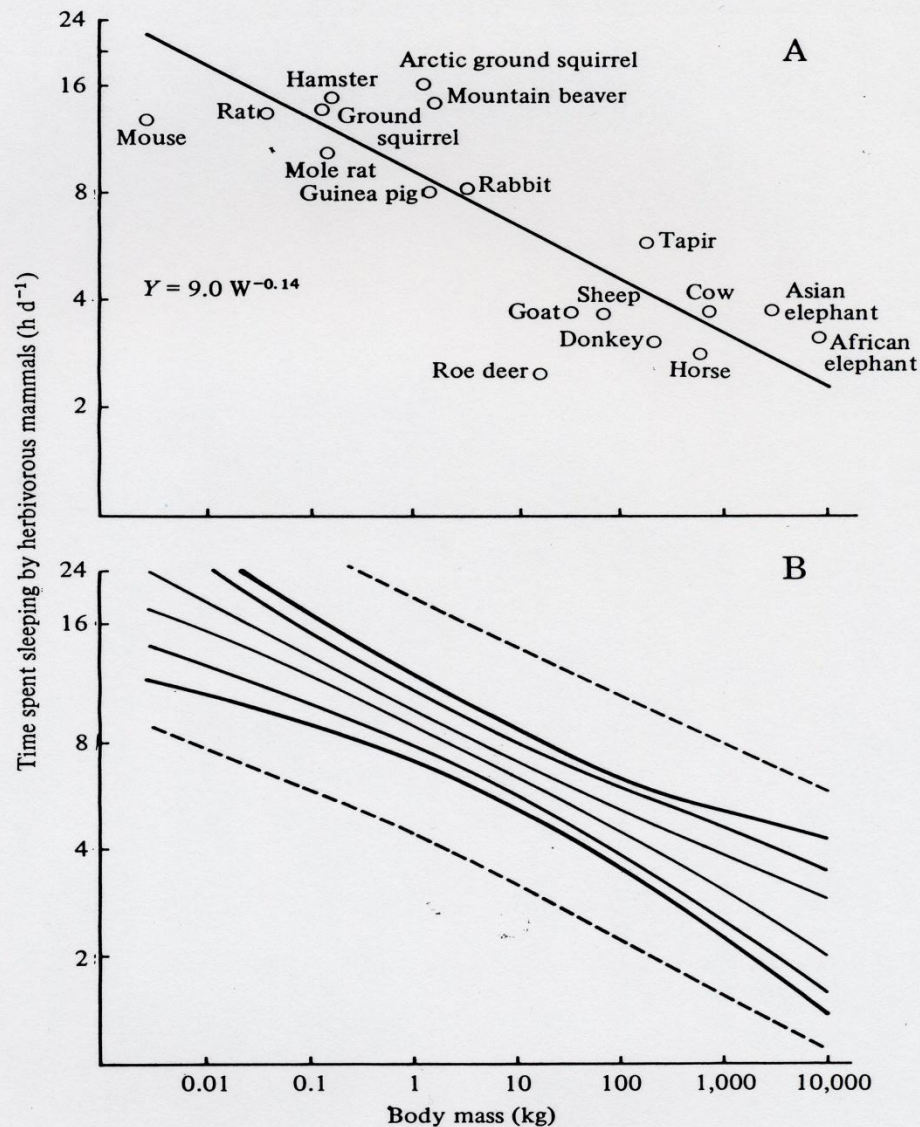


Figure 1.1. The effect of animal body size on the number of hours of daily sleep in herbivorous mammals. (A) Relevant data for 19 species reported by Zepelin and Rechtschaffen (1974). (B) A more general theory based on these data and the results of regression analysis (Appendix IIIc). The solid lines represent the 50, 95, and 99% confidence limits around the estimate of the mean. The dashed lines enclose 95% of all individual points.

The ecological implications of body size

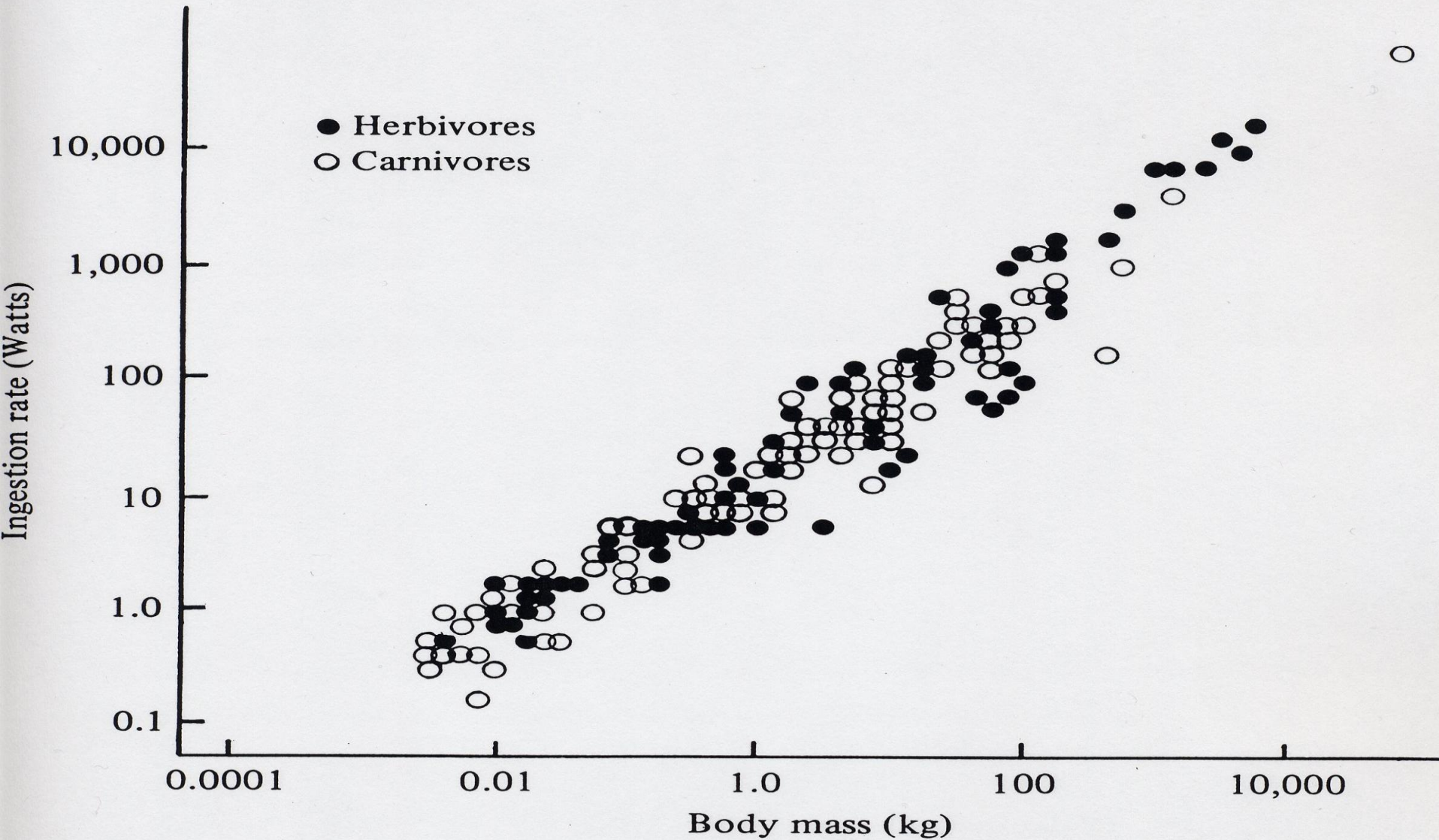
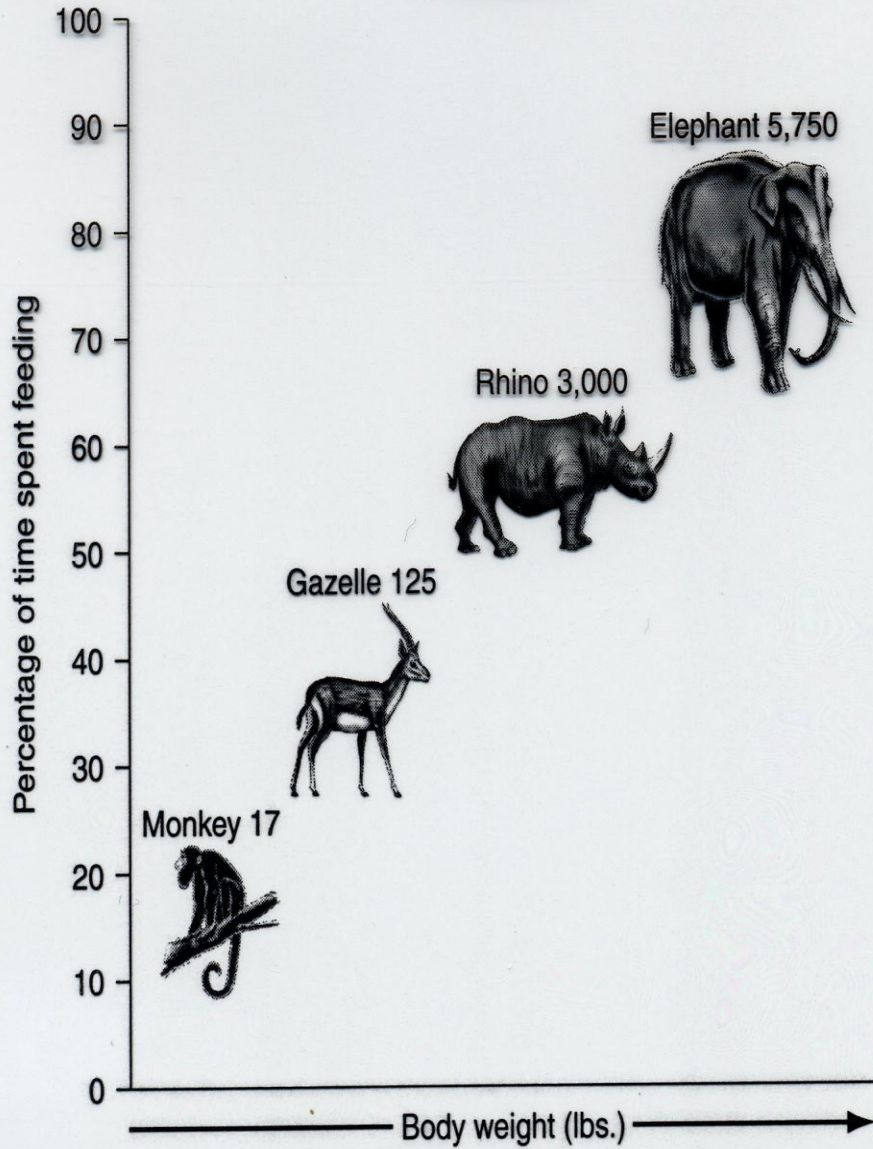


Figure 7.1. Ingestion rate of herbivorous and carnivorous endotherms as a function of animal body mass. Data from Farlow (1976). The regression line through these data is ingestion rate = $10.7W^{0.70}$.

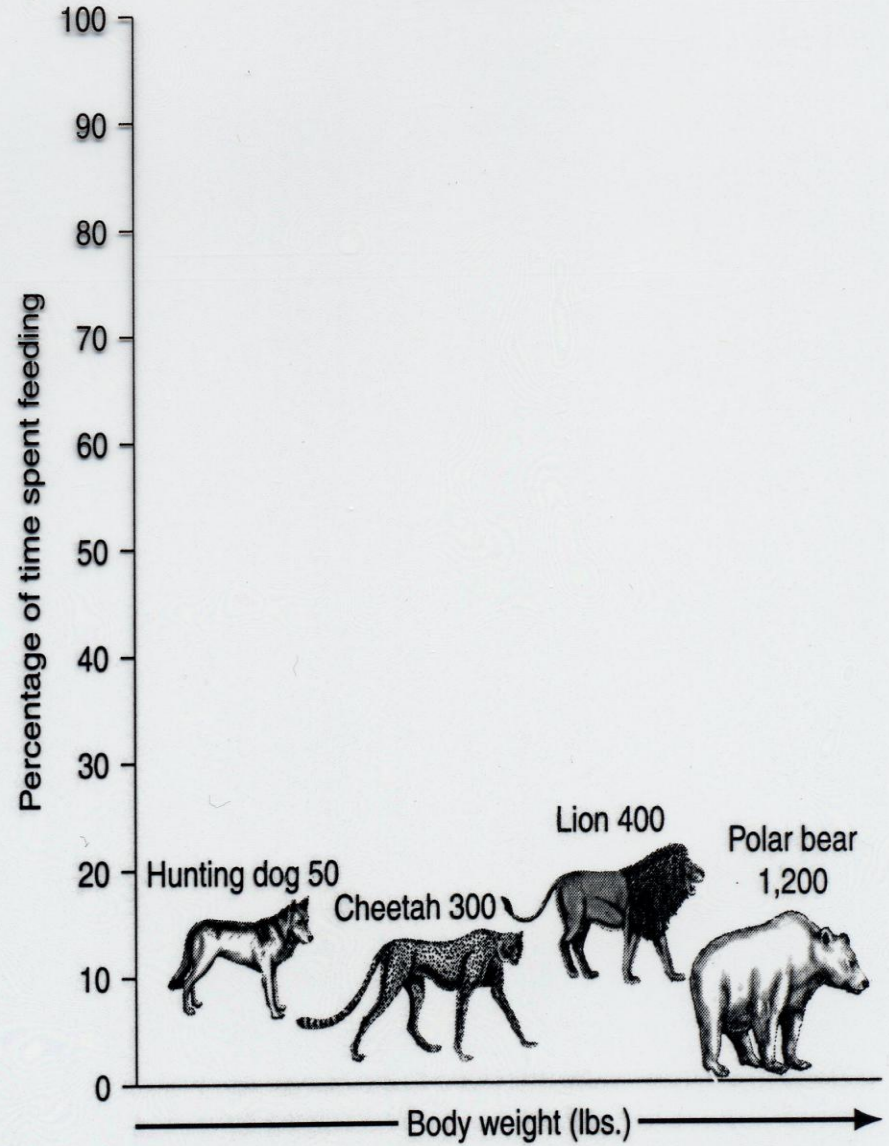
Table 5-9 Daily food intake of mammals and birds in captivity.

Group	Food intake	r^2	s_b	No. of species (n)	References
Mammals					
Vegetable and seed eating	kg food/day = $0.157 M^{0.84}$	0.971	0.046	12	Bourlière, 1969, table 9 (except bats and whales)
Animal-food eating	kg food/day = $0.234 M^{0.72}$	0.982	0.044	12	Bourlière, 1969, table 9 (except bats and whales)
Herbivores	$\text{kJ/da} = 971 M^{0.73}$	0.942	0.020	$\frac{df}{1,083}$	Farlow, 1976
Carnivores	$\text{kJ/da} = 975 M^{0.70}$	0.968	0.013	1,100	Farlow, 1976
Mean	$\propto M^{0.75}$				
Birds and mammals					
Herbivores	$\text{kJ/da} = 1,006 M^{0.72}$	0.948	0.015	1,118	Farlow, 1976
Carnivores	$\text{kJ/da} = 917 M^{0.69}$	0.958	0.012	1,148	Farlow, 1976
Mean	$\propto M^{0.71}$				
Maximum	$\text{kJ/da} = 1,713 M^{0.72}$	0.998	0.008	8 spp. mammals, 11 spp. birds	Kirkwood, 1983

Herbivores

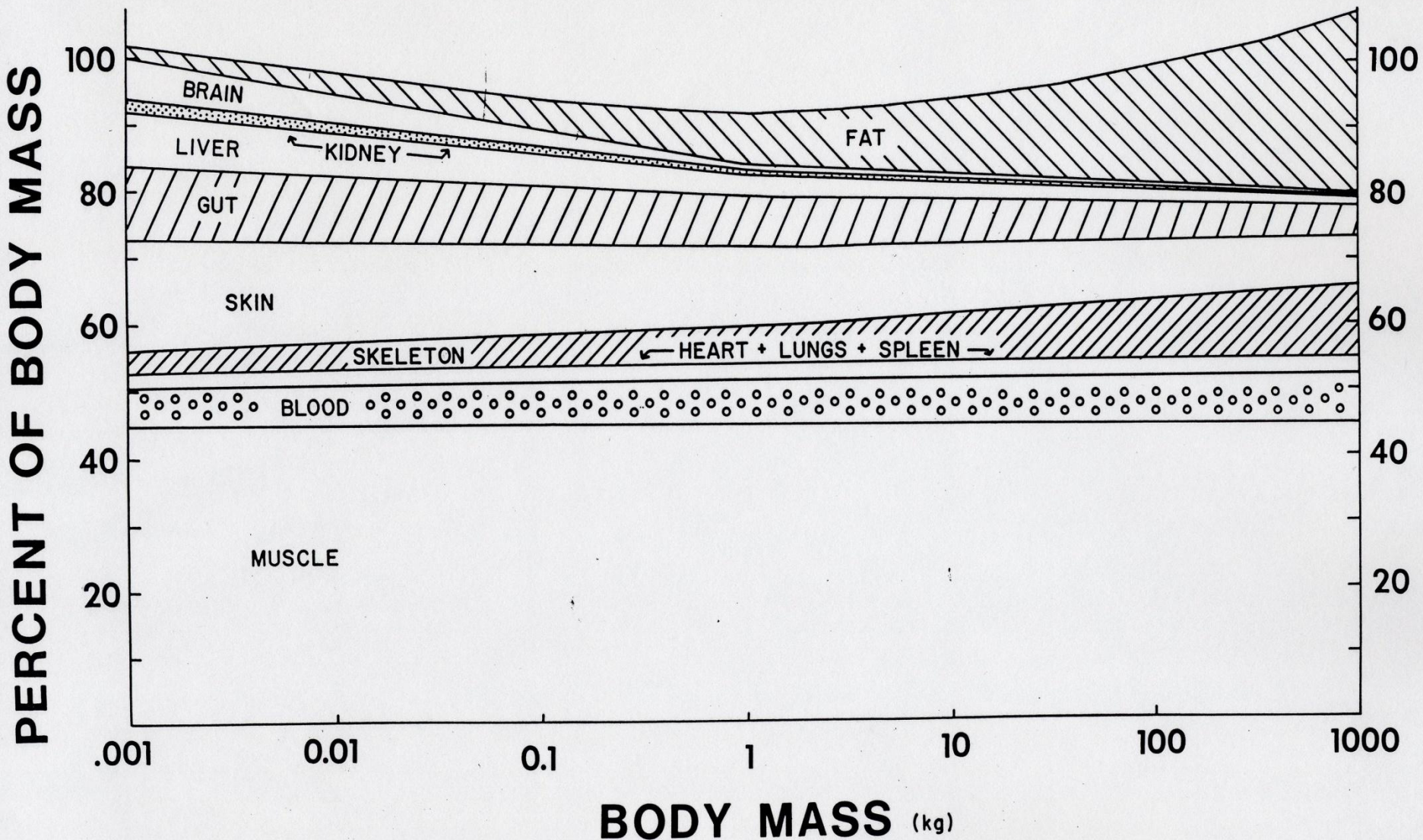


Carnivores



Although carnivores must work harder than herbivores to find a meal, a carnivore's meal is higher in nutrition than an herbivore's. Thus, carnivores spend much less of their time eating than herbivores. In addition, the larger the herbivore, the more time it needs each day simply to stay fed.

Source: Data from Shipman, "What Does it Take to be a Meateater?" in Discover Magazine, September, 1988.



2-1 The body composition of eutherian mammals, reconstituted from available allometric generalizations (equations shown in Table 3-4). Skeletal muscle, blood, heart, lungs, and spleen account for about 52% of body mass, regardless of animal size. Skeleton and fat contribute proportionately more of the total mass in larger mammals, while the skin, brain, and many organs scale up in less than linear fashion. There is considerable variation in what the graph would predict for a particular size, though there are relatively few studies that give complete analyses for one species (see Table 3-6).

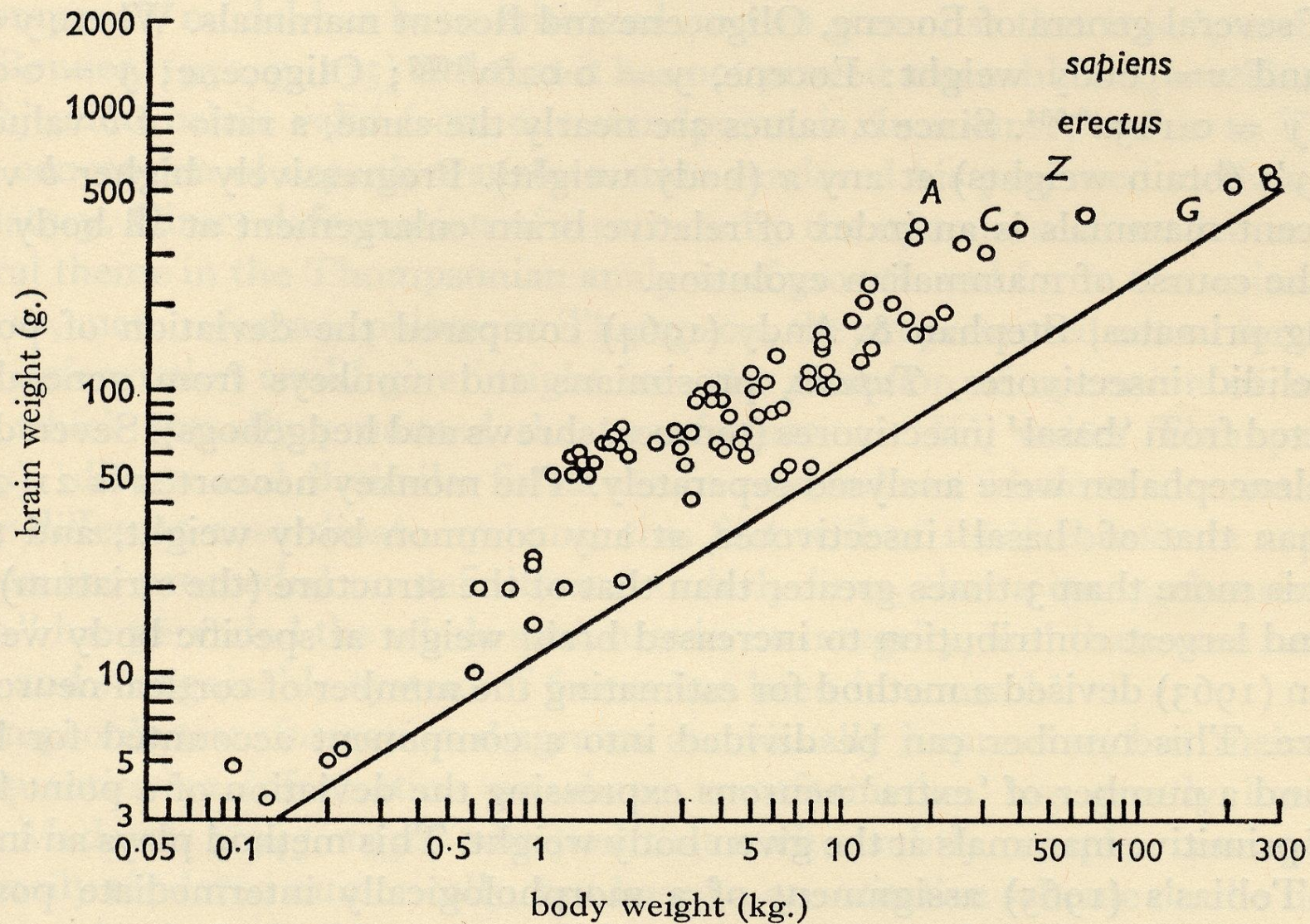
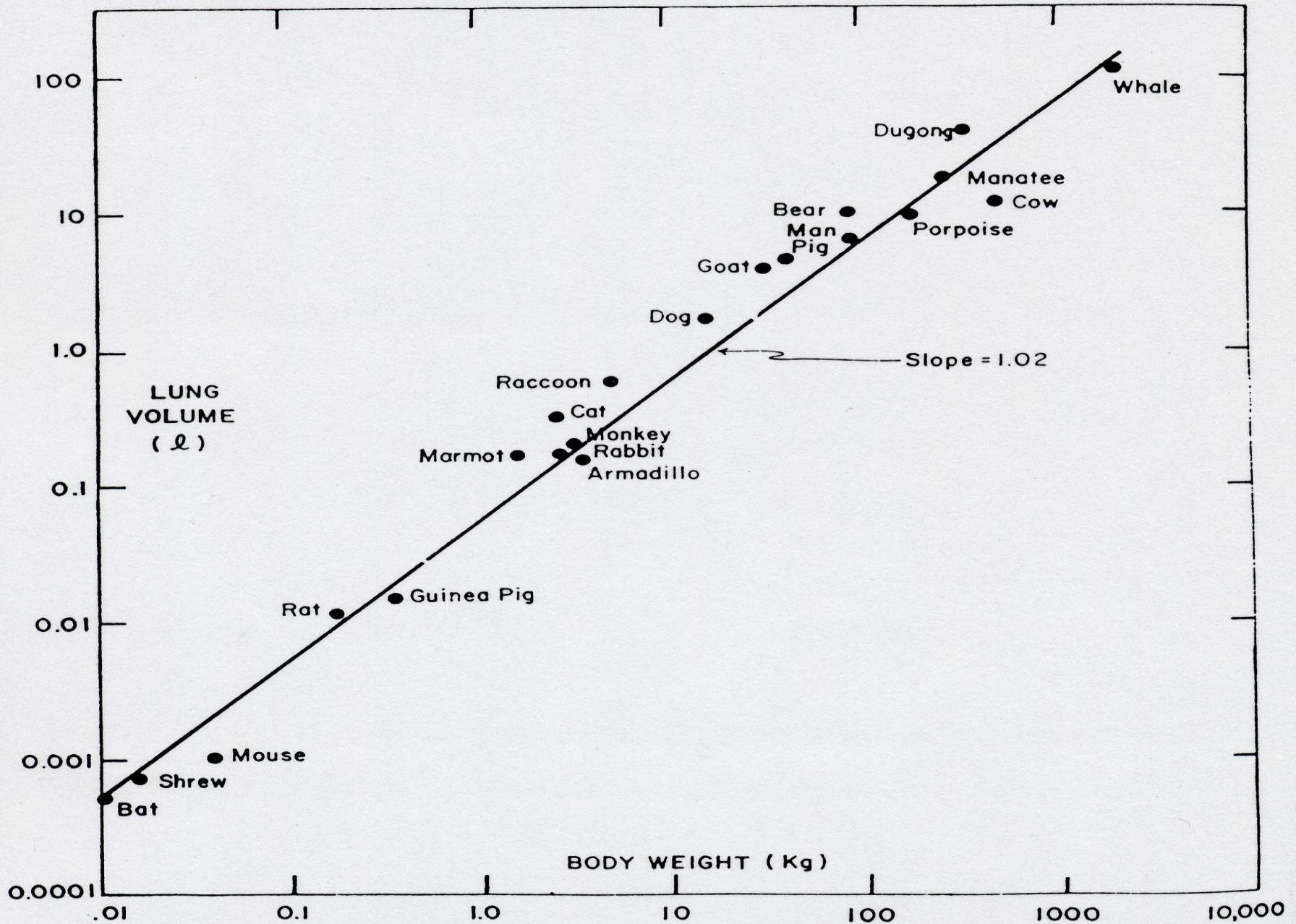


Fig. 6. Interspecific allometry of brain weight *v.* body weight in primates compared with the mean mammalian brain weight *v.* body weight regression (solid line, slope = 0.66). Primate superiority (in size) is indicated by upward deviation of points from the mean line at specific body weights. *A* = *Australopithecus africanus*; *Z* = 'Zinjanthropus'; *C* = chimpanzee, and *G* = gorilla. (From Jerison, 1963.)

Figure 9.1. Logarithmic plot of lung volume as a function of body size. From Tenney and Remmers (1963).





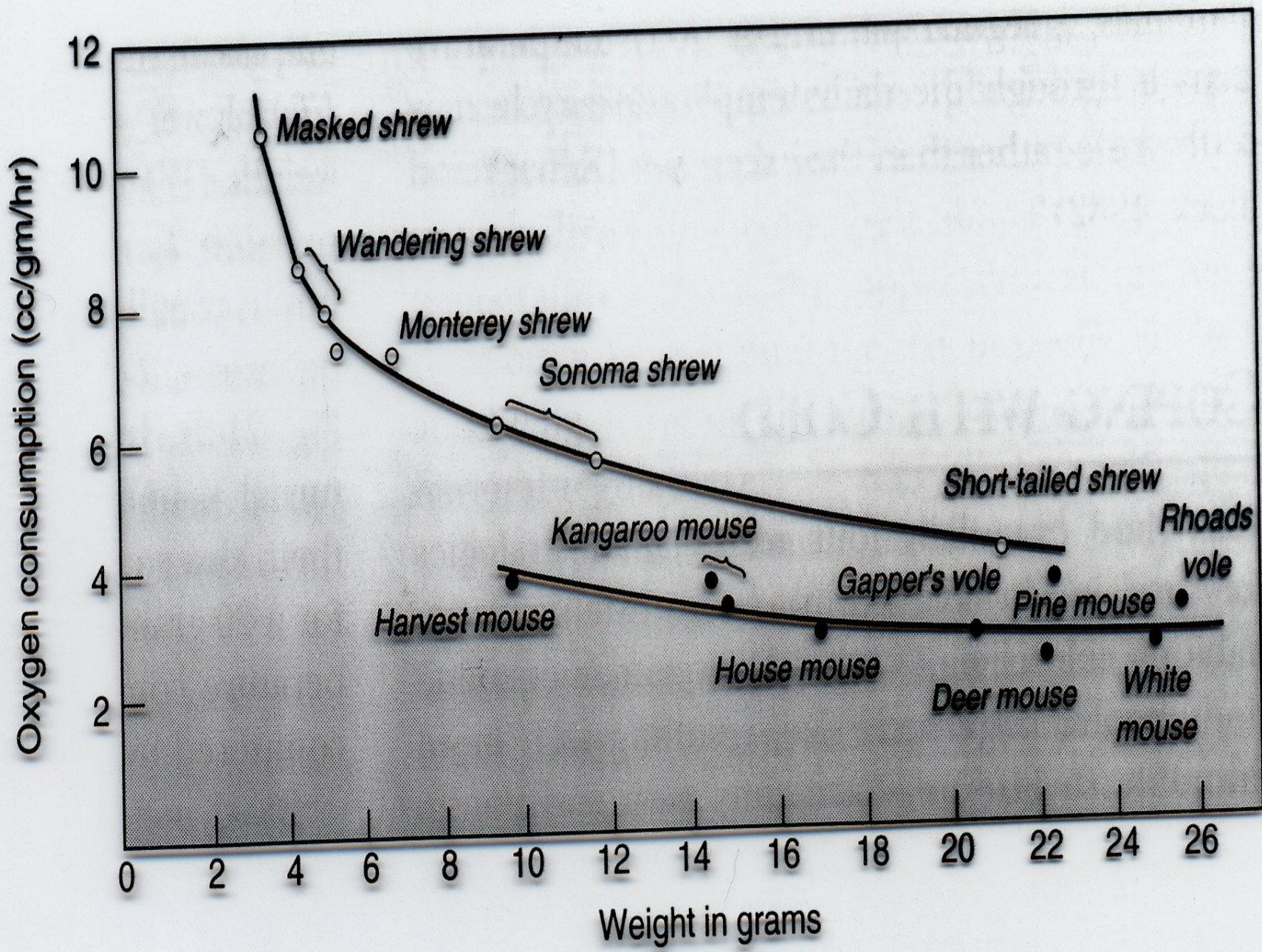
ALLOMETRY: (= different scaling)

	MEASURES BASED ON CUBES		CHANGE FROM SMALL TO LARGE	
	Small	Large	Small	Large
LENGTH (One side) $\approx L$	2	3	4	2
SURFACE AREA $\approx L^2$ (cross sectional)	4	9	16	4
$\approx L^2 \times 6$ (sides) (total for cube)	24	54	96	4
VOLUME/ MASS $\approx L^3$	8	27	64	8

Surface Area/Volume Ratio SMALL CUBE: $24/8 = 3$

LARGE CUBE: $96/64 = 1.5$

Surface area does not increase as quickly as mass or volume. Larger animals have RELATIVELY less surface area than smaller animals.



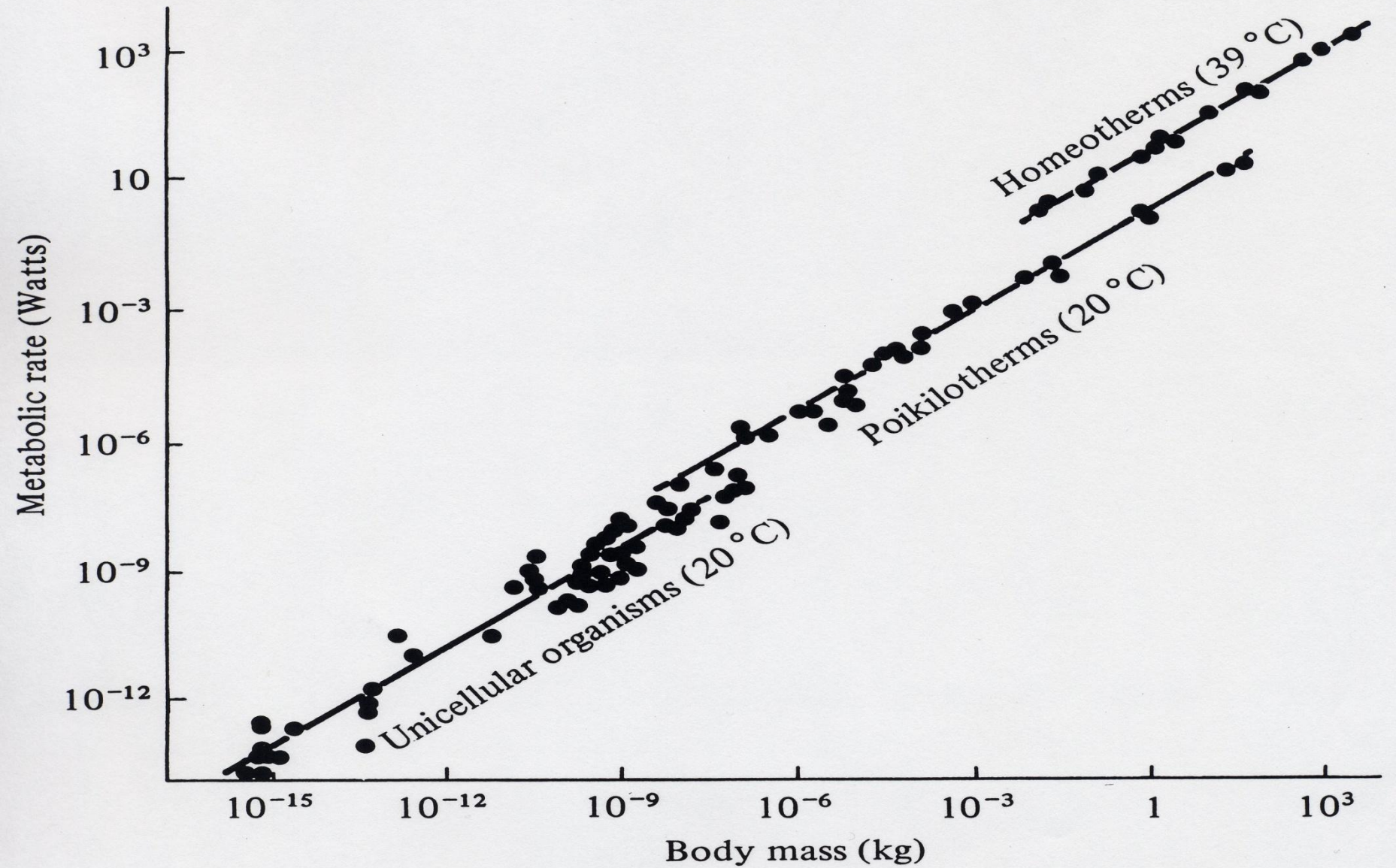


Figure 3.1. Standard metabolic rates of homeotherms, poikilotherms, and unicells presented as Equations 3.1–3.3. Modified from Hemmingsen (1960).